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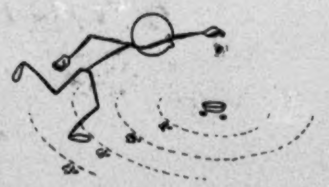
ABSTRACT

This is volume 2 of a set of mathematics materials developed for low achievers. These materials are designed to help teachers provide interesting and worthwhile learning opportunities for students in grades five through eight who have had little success in mathematics. The materials may be used in conventional classroom settings as well as in team teaching, multi-unit programs, and other organizational structures. The units are not designed to be used as a complete mathematics program for low achievers, but rather as representative segments of mathematics needed by all students. A teaching package, containing materials that are closely correlated with individual activities within each unit, is also included for this volume. Topics considered in this volume include: Tables and Change, Using Tables to Solve Problems, Ratio, Graphs, Organizing Data, Dealing with Uncertainty, and Geometry. (Author/FL)



# EXPERIENCES IN MATHEMATICAL IDEAS

VOLUME 2

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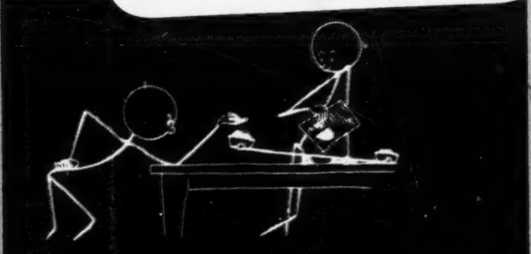

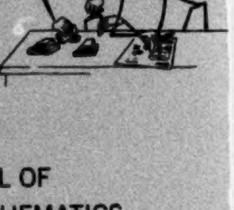

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1

# EXPERIENCES IN MATHEMATICAL IDEAS

# 2

# EXPERIENCES IN MATHEMATICAL

3

ED0 46780

Volume Two

# IDEAS

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# Preface

Curriculum projects during the late fifties and early sixties in all subject fields, but particularly in mathematics and science, produced and made available an abundance of material for average and above-average students throughout the elementary and secondary schools. Many of these curriculum projects employed the development strategy of "teaming." Teams of experts, including classroom teachers, psychologists, and representatives of the relevant subject disciplines, developed materials that were widely accepted and implemented throughout the schools of our nation. The implementation process was enhanced by private corporations and public agencies that expended millions of dollars to provide additional educational experiences for thousands of teachers. During this same period of time, however, little concern, effort, money, or talent was directed toward developing curriculum materials for slow learners and their teachers.

The challenge to develop mathematics materials for slow learners was accepted by the Board of Directors of the NCTM at the 1967 Annual Meeting when the Board approved a proposal presented by the Committee on Mathematics for the Non–College Bound. The proposal evolved into a writing project which has become known and identified as "Experiences in Mathematical Ideas" (EMI).

The successful completion of any writing project is a direct result of the dedication, talents, and enthusiasm of many individuals. The EMI project is no exception.

Members of the NCTM Committee on Mathematics for the Non–College Bound were responsible for the initial concept and beginning blueprint for the prototype units which emerged from this project—units now assembled as chapters of the present publication.

The NCTM Board of Directors, after considerable study of the original proposal, not only approved the concept but agreed to fund the project entirely from Council resources. Marguerite Brydegaard served diligently as the Board liaison representative. James Gates and the staff of the NCTM office helped immeasurably with financial matters and final production procedures. The support thus given is greatly appreciated.

Although the primary leadership and direction of the entire project was the responsibility of the EMI Executive Committee, the heart of the developmental activities was a result of the insights provided by the writing team. The EMI writing team included Charles Allen, Beth Baer, Bonnie Brooks, C. William Engel, Lowell Leake, Jr., Edith Robinson, Carolyn Smith, Donald Wiederanders, and Larry Yarck. The experience of these writers in working with teachers of slow learners and with pupils with learning problems was invaluable. Moreover, each of the writers demonstrated an unusual talent for creating new mathematical experiences. They worked cooperatively as a team throughout the entire project.

An unusual feature of the EMI project was the manner in which materials were edited. While the Executive Committee carried the overall editorial responsibility, two editors, Raphael Wagner and Kenneth Travers, offered their talents at two different stages of the project development in order that fresh points of view might effect a better product.

There are always many individuals who have been associated with a project such as EMI to whom no special recognition is given. For example, teachers in both urban and rural settings tried out preliminary drafts of the EMI materials and fed back information which was utilized in the final writing of each unit. To all these individuals a sincere "Thank you" is expressed.

Finally, special acknowledgment is given to Mary Parks and Aletha Buchert for their secretarial assistance. Without them many deadlines could not have been met.

The Executive Committee of EMI is truly indebted to and appreciative of the talents and excellent contributions of all these individuals. The ultimate reward to all who have been associated with this project will be the degree to which the basic ideas and spirit of EMI are transmitted into improved opportunities for slow learners throughout our nation.

*Executive Committee*

Arnold M. Chandler, *Director*
George Immerzeel Editor
Harold C. Trimble

# Introduction

*Experiences in Mathematical Ideas* is designed to help teachers provide interesting and worthwhile learning opportunities for those students in grades five through eight who have had little success in mathematics. It will help those teachers whose devotion to slow learners has made them sensitive to the shortcomings of present curriculum materials and methods of presentation. The EMI project materials are designed so that teachers may implement them in conventional self-contained classrooms as well as in other organizational patterns—for example, in team teaching, multi-unit programs, nongraded structures. Hence, EMI materials are dependent only on the willingness of teachers to offer pupils opportunities and freedom to learn those basic mathematical concepts that have been provided. Teachers are a crucial and central element in implementing the student-centered and activity-oriented EMI materials. They must be exceptionally kind, warm, and fair, displaying a sincere empathy for pupils with learning problems.

The EMI project, like other projects, has been carefully structured within a philosophical and psychological framework which includes certain assumptions and beliefs. The assumptions and beliefs come from what is generally known and accepted about human growth and development as it pertains to slow learners. For example, it is assumed that pupils identified as slow learners *can* learn if experiences for them grow out of their physical environment. Further, it is assumed that there are a variety of teaching strategies, not commonly known and practiced, which will aid and abet the learning process of slow learners. There is a fervent belief that objectives for pupil achievement can be stated and should serve as a focus for most mathematical learning activities. It follows naturally that certain evaluations of pupils can then be made in terms of these objectives.

The following additional assumptions of the EMI project give further insight into the orientation and purpose of the materials:

1. *Activity.* Learning is an active process. Slow learners learn mathematics by reacting to and interacting with their environment. Too often mathematical concepts have been treated initially at the symbolic level by authors in textbooks, by teachers on chalkboards, and by students in notebooks. This approach has been unsatisfactory for slow learners. The EMI materials suggest many ways in which the same mathematical concepts can be learned by these students when initiated through experiences. Only after the experiences is attention given to associating these activities with appropriate mathematical symbols.
2. *Individuality.* Learning is an individual process. Each learner must actively participate in the ongoing classroom situation. EMI materials are developed so that each pupil has an opportunity to observe, to explore relationships, to collect information, to hypothesize solutions to problems, and to test tentative hunches. Typically the classroom is busy with teacher activity. EMI materials are structured so the classroom will be dominated by pupil activity. When learning mathematics, it is not enough to sit and watch the teacher "do mathematics." Each individual must pitch in and get a piece of the action.
3. *Success.* Success breeds success. Yet too many mathematics classes offer many students little else but experiences of failure. EMI materials are designed to provide activities that will give even the slower students successful encounters with mathematics. This is done by breaking down concepts into very small components and devising simple tasks that will carefully lead the student to a grasp of the basic ideas. The teacher using EMI material will have to resist the urge to move too quickly through the experiences because they look "too easy."
4. *Meaning.* Classroom experiences must have meaning for the learner. Most students readily pick up catch phrases or rules of thumb like "Invert the divisor and multiply," but many have little understanding of what they are talking about. It is not surprising when these individuals deal with mathematics in utterly nonsensical ways, if they deal with it at all. EMI materials attempt to bring meaning to mathematics by drawing largely from situations with which the student is familiar, or in which he can readily imagine himself to be an active participant.

5. *Novelty.* A change is as good as a rest. Perhaps more than any other subject in the curriculum, mathematics poses the risk of becoming routine and dull in the classroom. The prospects are particularly bleak for the youngster who day after day faces the same kind of tasks that have repeatedly meant failure for him in the past. EMI materials attempt to approach familiar mathematics topics from a fresh point of view. At the same time, the ultimate objective is to develop a more complete understanding of the fundamental mathematical concepts included in the set of units.

The EMI project materials comprise thirteen prototype units: (1) Base and Place Value, (2) Renaming Numbers in Addition and Subtraction, (3) Physical Models for Multiplication, (4) Units of Measure, (5) Physical Models for Fractions, (6) Physical Models for Decimals, (7) Tables and Change, (8) Using Tables to Solve Problems, (9) Ratio, (10) Graphs, (11) Organizing Data, (12) Dealing with Uncertainty, and (13) Geometry. The first six units are bound in volume 1, and the remaining seven units are bound in volume 2.

The units in the EMI project do not compose a complete mathematics program for low achievers in grades five through eight, and they are not intended to be used as such. Neither are the units intended to be used solely for supplementary or enrichment purposes. Rather, the units offer a model for teaching representative segments of mathematics—segments that are usually taught because they are needed by all students. Most of the units are independent of the others and need not be taught in any specified order nor at specified grade levels within the scope of this project.

Each unit contains a sequential development of a specific mathematical topic through a series of "experiences." A brief overview of these experiences is outlined at the beginning of each unit. This overview gives the teacher a quick and understandable preview of what the entire unit is about. Generally there are five experiences in a unit. Some may take as long as three days, but many will require only one day.

The description of each experience includes (1) a statement of a pupil objective, (2) a list of materials to be used, (3) a strategy that may be employed by the teacher, (4) reproductions of student activity cards or worksheets, and (5) a suggestion about evaluation.

Another special feature of the EMI project is the "Teaching Package." A teaching package has been developed for each volume, with materials that are closely correlated with individual activities within each experience. These materials are printed on 8½-by-11-inch sheets so they can be duplicated by the teacher to make (1) overhead transparencies, (2) labels and other software needed at activity stations, and (3) copies of worksheets for students. The teaching-package materials have been punched and numbered so the teacher can keep them in order in a notebook.

The two volumes describing experiences and the associated teaching packages in the EMI project have been structured in such a way as to help many teachers provide new and different learning experiences in mathematics for students who have not achieved from teaching materials and patterns commonly employed. The Council hopes that success with some or all of the prototypes will encourage teachers to provide similar experiences throughout grades five to eight for students whose mathematical needs are not now being met.

# 7

## EXPERIENCES WITH

# Tables and Change

Seeing is believing. Seeing relations and patterns involving numbers is a giant step in problem solving. Some students can solve problems like the following without hesitation:

Give the next two numbers in the sequence 1, 3, 5, 7, __, __.
Which numbers are missing in the sequence 1, 3, 6, 10, __, 21, 28, __?
Which numbers do not belong in the sequence 1, 4, 9, 16, 21, 25, 26, 36?
John and Sam together caught 9 fish. How many did John catch?
State a rule for finding the second number in the table of pairs below:

| First Number | Second Number |
|---|---|
| 44 | 88 |
| 2 | 4 |
| 13 | 26 |
| 56 | 112 |

Many of our less able students, however, have little success with problems such as these, which require insight into numerical relations and time to test trials to see if they work. In class discussions some students quickly give answers. Curiosity about a particular problem is killed; it has been solved, and the class goes on to the next one. The slower students have another experience in failing to achieve success.

What can we do, as teachers, to break the cycle of failure? The purpose of this unit is to suggest activities we can arrange for our students which will make it possible for all of them to succeed in recognizing patterns and relations while dealing with numbers. Each student has his own set of problems, and the slower student no longer has to compete with the faster ones in a race to find answers. Building confidence is the primary outcome desired at this time.

In this unit the student handles concrete objects and makes observations. He records these observations in tables. He later studies and evaluates this recorded information. Patterns and relationships are studied by means of tables. Tables are *made, looked at* for possible patterns, and *used* to help solve problems. The tables used throughout this unit lead to graphs and formulas in the future. Tables and number pairs are but two of several devices to show how numbers are related.

In a word, the slow learner must not be relegated to the position of being a mere spectator of what takes place in the classroom. He is a *learner* and must be stimulated to become an active participant in learning activities.

We, too, learn as we watch and guide. If problems suggested in the unit do not fit the age group or the needs of your class, change the problems, add to them, and otherwise modify the suggested activities. Unless you feel comfortable with the unit and have confidence in its objectives, you cannot hope to help the learner succeed with his adventures.

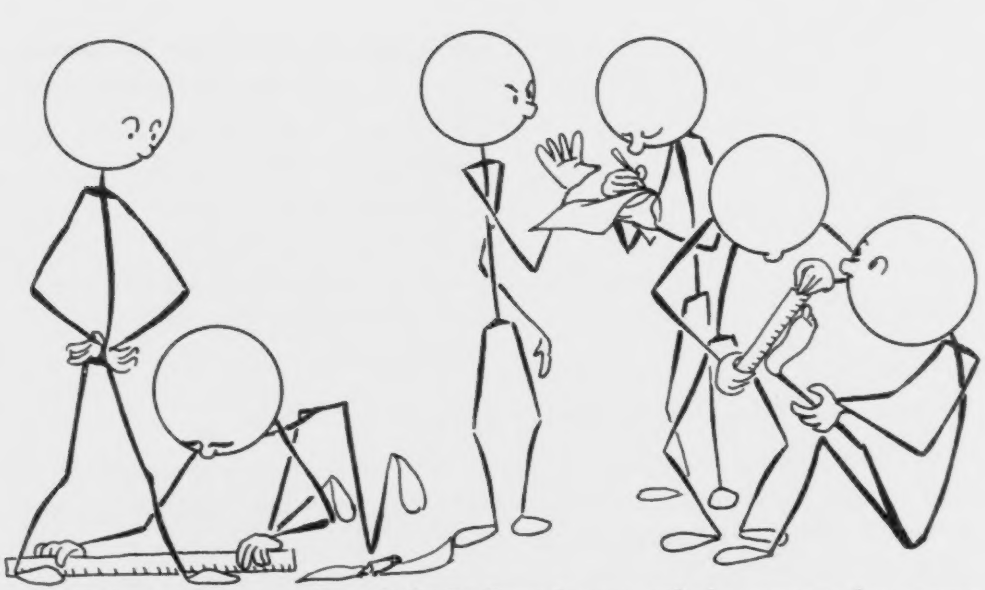
## OVERVIEW

Each of the six experiences in this unit includes a detailed section called "Teacher Strategy," which is usually one teacher's first-person account of a procedure that proved effective with his class.

The following brief summary will help you in deciding whether the experiences meet the needs of your particular class. It will also give an indication of the amount of preparation each experience involves. Sample student worksheets and other "handout" materials are provided in the Teaching Package. It is assumed that duplicating facilities are available.

### Experience 1: Making and Recording Observations

Students, in teams of two, conduct surveys of their class, their school, and their immediate surroundings. They gain experience in making simple

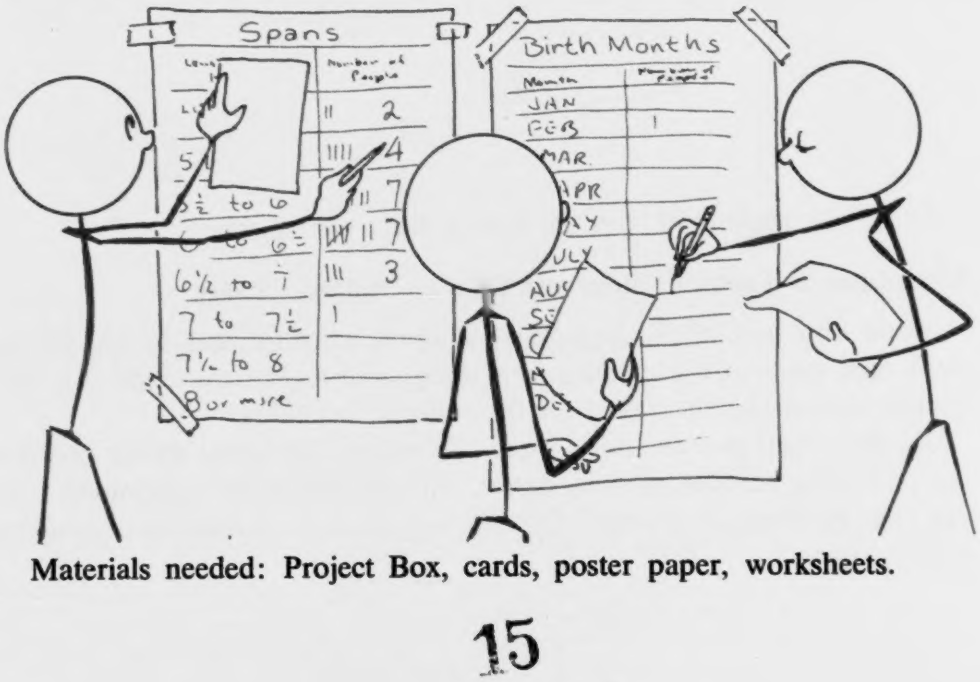


observations and recording their findings in an orderly manner by means of tables.

Materials needed: Project Box, index cards, steel tape measures or yardsticks, rulers, index cards, worksheets.

**Experience 2: Classifying Observations**

Students use the information gathered in Experience 1 to construct tally tables. They learn to organize data and answer simple questions about what they have recorded.


Spans
Number of People
2
4
5
to 6
7
6
7
6½ to 7
3
7 to 7½
7½ to 8
8 or more
Birth Months
Month
JAN
FEB
MAR
APR
AUG
Dec

Materials needed: Project Box, cards, poster paper, worksheets.

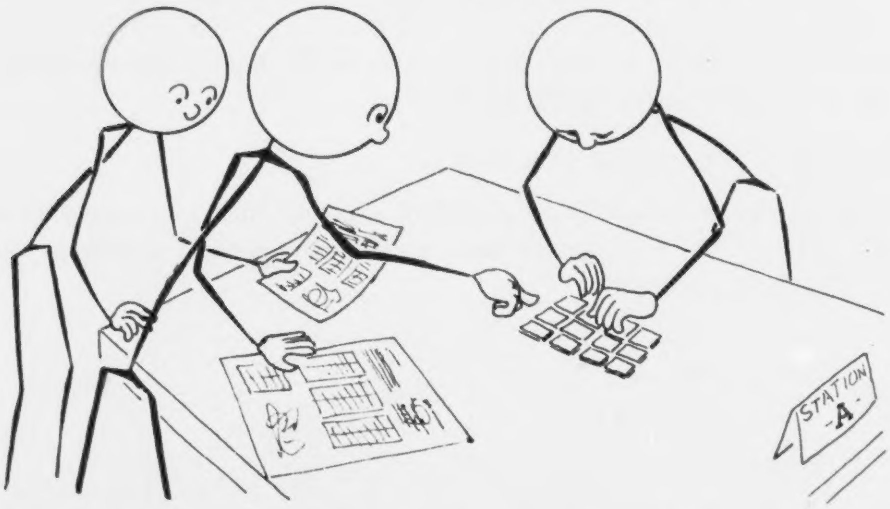
### Experience 3: Gathering, Recording, and Classifying Data

The most common length of words is guessed at. Then, in four selections of different kinds of prose, the lengths of words actually used are counted and recorded in tables. Students compare their guesses with the actual results.

Materials needed: worksheets.

### Experience 4: Handling Geometric Data

Students count the number of sides and corners of ten geometric figures and record the data in tables they make up as an introduction to the experience. Then they arrange sets of square tiles in rectangles in as many different ways as possible and record the dimensions in tables.

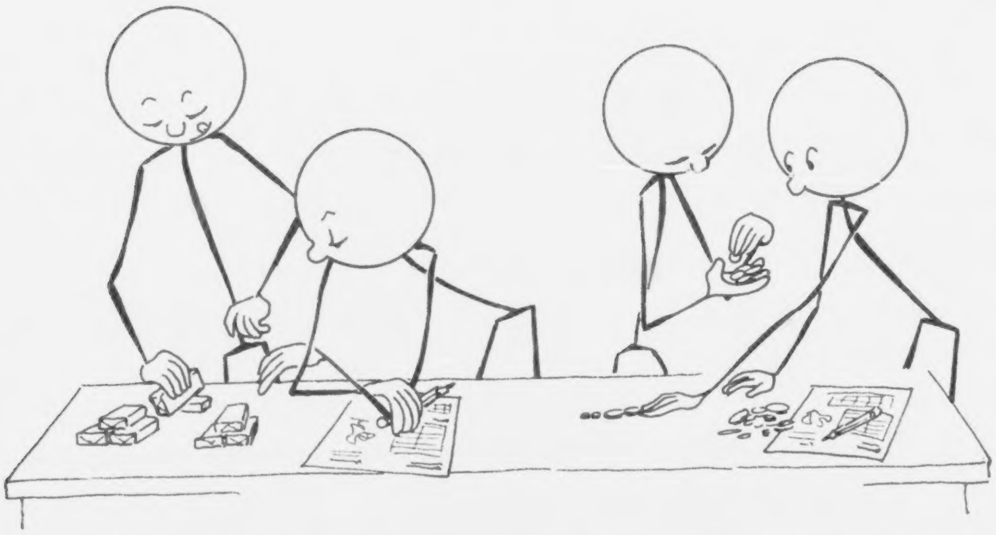


Materials needed: "Polygons" sheets, tiles, worksheets.

### Experience 5: Tables of Sums and Data on Making Change

In the first part of this experience students work in pairs to find all the ways they can share a collection of things and record the data. To give greater interest, candy is used for the collection of things.

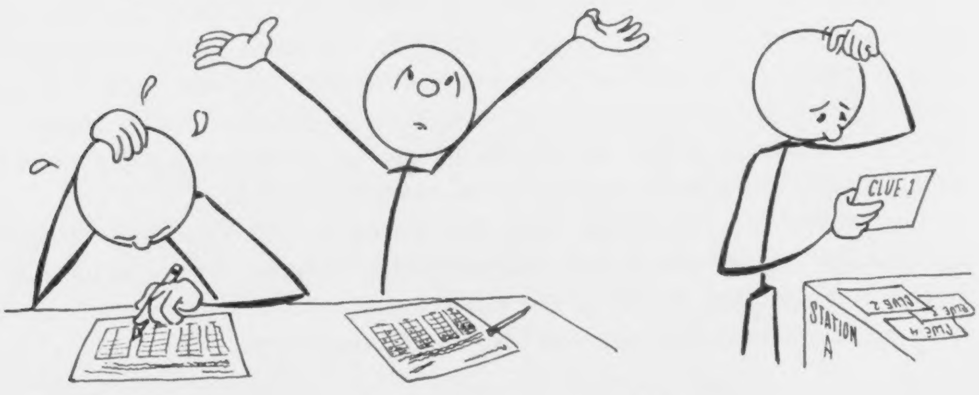
In the second part the same pairs of students determine all the possible ways of using pennies, nickels, dimes, and quarters to form amounts from one cent to twenty-five cents. The students record the number of possible combinations.



Materials needed: numbered pieces of paper, candy, real or fabricated coins, worksheets.

**Experience 6: Related Number Pairs**

Students examine incomplete tables and complete them by guessing and testing or by using clues.


CLUE 1
STATION A
CLUE 2
CLUE 3
CLUE 4

Materials needed: envelopes, clue slips, worksheets.

# EXPERIENCE 1

## Making and Recording Observations

### OBJECTIVE

The student should be able to gather information and record it in the form of a table.

### MATERIALS

Box labeled Project Box
Steel tape measures or yardsticks
12-in. rulers
Index cards
1 worksheet for each 2 students

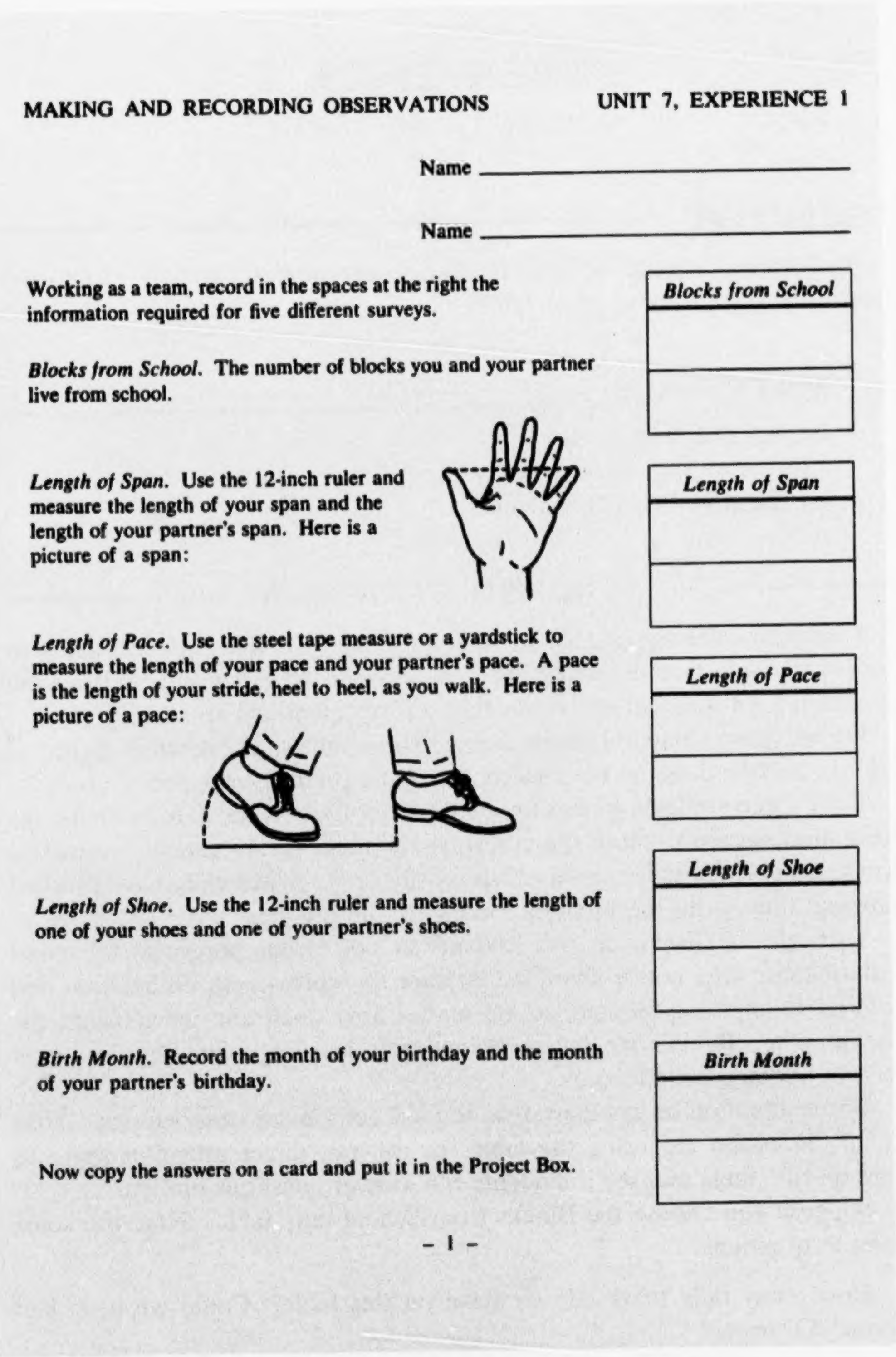
### TEACHER STRATEGY

I like to introduce this experience by telling the students that they are going to gather some data about themselves and learn how to record the data in tables. I ask them to select partners so that they can work in pairs; then I give each pair a copy of the worksheet, calling it a "survey sheet." I tell the students to follow the directions on the sheet, asking questions if necessary but working by themselves as much as possible.

The worksheet, duplicated from the master in the Teaching Package, asks for information about five different things. The results are to be transferred to a card and placed in the Project Box. (The worksheet is shown here without overprinted answers because the data vary.)

### EVALUATION

With very little help from you, the students should be able to follow the directions in making and recording the observations. You need not be concerned about the accuracy of the measurements.



**MAKING AND RECORDING OBSERVATIONS** **UNIT 7, EXPERIENCE 1**

Name ______________________________

Name ______________________________

Working as a team, record in the spaces at the right the information required for five different surveys.

*Blocks from School.* The number of blocks you and your partner live from school.

| *Blocks from School* |
|---|
|  |
|  |

*Length of Span.* Use the 12-inch ruler and measure the length of your span and the length of your partner's span. Here is a picture of a span:

| *Length of Span* |
|---|
|  |
|  |

*Length of Pace.* Use the steel tape measure or a yardstick to measure the length of your pace and your partner's pace. A pace is the length of your stride, heel to heel, as you walk. Here is a picture of a pace:

| *Length of Pace* |
|---|
|  |
|  |

*Length of Shoe.* Use the 12-inch ruler and measure the length of one of your shoes and one of your partner's shoes.

| *Length of Shoe* |
|---|
|  |
|  |

*Birth Month.* Record the month of your birthday and the month of your partner's birthday.

| *Birth Month* |
|---|
|  |
|  |

Now copy the answers on a card and put it in the Project Box.

# EXPERIENCE 2
## Classifying Data

### OBJECTIVE

The student should be able to record information on tally tables and answer questions about these tables.

### MATERIALS

Project Box and cards
3 pieces of light-colored poster paper
1 worksheet for each student

### TEACHER STRATEGY

I introduce this experience by telling the students that they are going to record the data that the whole class gathered in Experience 1 and put the data in a form that will enable them to answer questions about it.

On the poster paper I begin making three tables, as shown in figure 1. (This could be done on the chalkboard, but I prefer poster paper.)

I ask three students to use the cards from Experience 1 to tally the information needed to fill in the tables, giving them grease pencils or crayons to use so that the tally marks can be easily seen. When they have finished tallying, they write the numeral indicating the number.

To begin the discussion, ask students to look at one particular table and tell anything they notice about it. Be sure to express your enthusiasm and interest in anything they notice, no matter how small and insignificant the fact may be. Remember that participation in learning adventures is a vital step in building confidence.

Focus attention on another table and ask for similar observations. After some discussion regarding the tables in general, direct attention again to one specific table and see if students can answer questions orally.

Suppose you choose the Blocks from School tally table. Here are some sample questions:

How many tally marks do we have on this table? Could we have had more? Or fewer?

SPANS

| Length in Inches | Number of People |
|---|---|
| Less than 5 | |
| 5 up to 5½ | |
| 5½ up to 6 | |
| 6 up to 6½ | |
| 6½ up to 7 | |
| 7 up to 7½ | |
| 7½ up to 8 | |
| 8 or more | |

BIRTH MONTHS

| Month | Number of People |
|---|---|
| Jan. | |
| Feb. | |
| Mar. | |
| Apr. | |
| May | |
| June | |
| July | |
| Aug. | |
| Sept. | |
| Oct. | |
| Nov. | |
| Dec. | |

BLOCKS FROM SCHOOL

| Number of Blocks | Number of People |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| More than 7 | |

Fig. 1

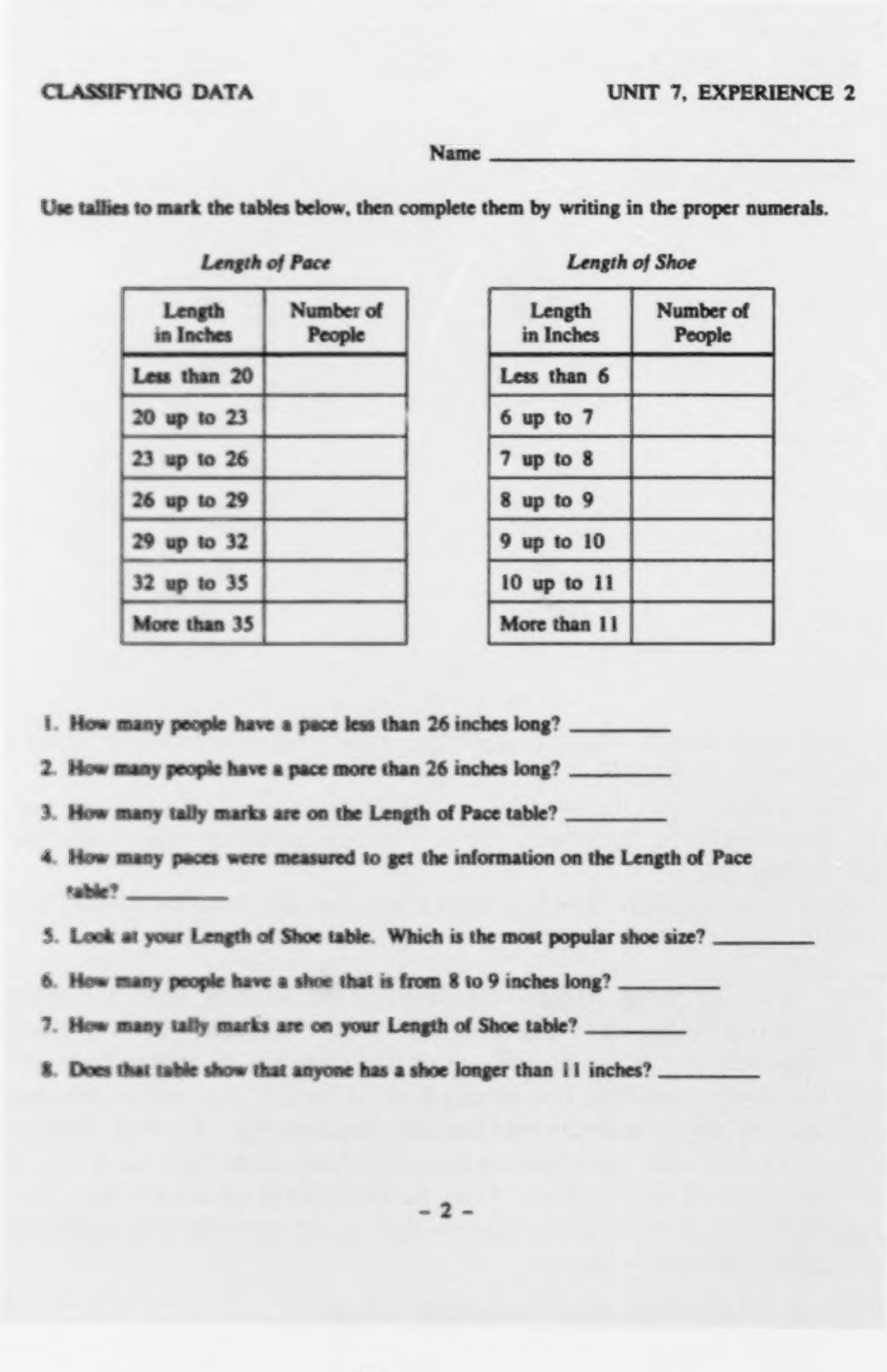
How many people have we surveyed? Could we have surveyed (questioned) more people? Fewer?

How many people live two blocks or less from school? How do we find this from looking at the table? (Have someone go to the table and show how to find this.)

How many live more than two blocks from school? How do we find this from the table?

> *Warning.* Some student may notice he can tell how many live more than two blocks from school without using the table. (Total number minus number living two blocks or less from school equals the number living more than two blocks from school.) This is fine thinking and should be commended, but not emphatically. Nothing shatters confidence more than hearing that a problem could have been solved by a method you don't see. Here the emphasis is on using tables. The slow learner can learn to rely on the use of tables as a tool if he is given a chance to do so.

What is the farthest anyone lives from school?



**CLASSIFYING DATA** **UNIT 7, EXPERIENCE 2**

Name ______________________________

Use tallies to mark the tables below, then complete them by writing in the proper numerals.

*Length of Pace*

| Length in Inches | Number of People |
|---|---|
| Less than 20 | |
| 20 up to 23 | |
| 23 up to 26 | |
| 26 up to 29 | |
| 29 up to 32 | |
| 32 up to 35 | |
| More than 35 | |

*Length of Shoe*

| Length in Inches | Number of People |
|---|---|
| Less than 6 | |
| 6 up to 7 | |
| 7 up to 8 | |
| 8 up to 9 | |
| 9 up to 10 | |
| 10 up to 11 | |
| More than 11 | |

1. How many people have a pace less than 26 inches long? ________
2. How many people have a pace more than 26 inches long? ________
3. How many tally marks are on the Length of Pace table? ________
4. How many paces were measured to get the information on the Length of Pace table? ________
5. Look at your Length of Shoe table. Which is the most popular shoe size? ________
6. How many people have a shoe that is from 8 to 9 inches long? ________
7. How many tally marks are on your Length of Shoe table? ________
8. Does that table show that anyone has a shoe longer than 11 inches? ________

- 2 -

How many live more than five blocks from school? Less than five blocks? Exactly five blocks?

After this discussion I pass out copies of the worksheet and have each student complete the tally tables and answer the questions.

## EVALUATION

Students should be able to answer the questions suggested for group discussion and to complete the worksheet.

# EXPERIENCE 3

## Gathering, Recording, and Classifying Data

### OBJECTIVE

The student should be able to use tally tables to record the number of words of different lengths from four different samples of selected prose.

### MATERIALS

2-page worksheet for each student

### TEACHER STRATEGY

Before handing out the worksheets you might arouse the students' interest by a discussion like the following:

"Suppose we think for a few minutes about the words we use when we speak, write, or read. How many letters do these words usually have?" Give time for thinking, and accept answers from all volunteers.

"Which do you think are the most popular—two-letter words? three-letter words? four-letter words? six? I'd like to have each of you write down a guess on the outside of the activity sheets I am about to hand out. Then we can see what is inside."

[*Continued on page 14*]

## GATHERING, RECORDING, AND CLASSIFYING DATA

**UNIT 7, EXPERIENCE 3**

**Tally Tables**

Name ______________________________

Tally the length of the words in the writing samples below. Do not write out any answers to the questions at the end of each sample, but think about the questions so that you will be ready for class discussion.

### SAMPLE FROM A GEOGRAPHY BOOK

Let us now take the train for Paris. What funny trains we do find in Europe. They are not at all like ours. Their carriages, as the cars are called, are of three kinds, first, second, and third class, and each car is divided into little rooms which hold six, eight or ten persons. A door and two small windows are in each end of a compartment. The first and second classes have cushioned seats, but there are only wooden benches in the third.

*Do you notice anything about the table at the right? What?*

| Number of Letters | Number of Words |
|---|---|
| 1 | 2 |
| 2 | 12 |
| 3 | 24 |
| 4 | 14 |
| 5 | 17 |
| 6 | 7 |
| 7 | 5 |
| More | 3 |

### SAMPLE FROM A NEWSPAPER EDITORIAL

Of all the reforms most desired in an essentially agricultural nation, such as South Vietnam, the most valuable symbolically and actually is land reform. Give the farmer land and he is on the road to independence and democracy. The cry for the redistribution of large land holdings is as old as the revolt led by Spartacus against Rome and as new as any reform in Latin America.

*What do you notice about the table you have just made?*

| Number of Letters | Number of Words |
|---|---|
| 1 | 0 |
| 2 | 17 |
| 3 | 17 |
| 4 | 8 |
| 5 | 3 |
| 6 | 5 |
| 7 | 5 |
| More | 10 |

[*Continued*]

## GATHERING, RECORDING, AND CLASSIFYING DATA

UNIT 7, EXPERIENCE 3

Tally Tables–*Continued*

Name ______________________________

Tally the length of the words in the writing samples below. Do not write out any answers to the questions at the end of each sample, but think about the questions so that you will be ready for class discussion.

### SAMPLE FROM A SPORTS STORY

The Colt touchdowns came hard. The Rams either scored easily or they did not move the ball at all. As the third quarter opened, Gabriel hoisted another long, towering pass to Jack Snow. Well covered by Boyd, Snow tipped the ball into the air, juggled it momentarily, then secured it and outran everybody for an 80-yard touchdown.

*What do you notice about the length of words in sports writing?*

| Number of Letters | Number of Words |
|---|---|
| 1 | 0 |
| 2 | 8 |
| 3 | 12 |
| 4 | 18 |
| 5 | 1 |
| 6 | 6 |
| 7 | 7 |
| More | 5 |

### SAMPLE FROM A NOVEL

The sun was up so high when I waked that I judged it was after eight o'clock. I laid there in the grass and the cool shade thinking about things, and feeling rested and ruther comfortable and satisfied. I could see the sun out at one or two holes, but mostly it was big trees all about, and gloomy in there amongst them. There was freckled places on the ground where the light sifted down through the leaves, and the freckled places swapped about a little, showing there was a little breeze up there. A couple of squirrels set on a limb and jabbered at me very friendly.

*Look at your table. Do you notice anything? What?*

| Number of Letters | Number of Words |
|---|---|
| 1 | 8 |
| 2 | 14 |
| 3 | 30 |
| 4 | 9 |
| 5 | 18 |
| 6 | 16 |
| 7 | 5 |
| More | 8 |

In talking about the directions on the worksheets, which contain four samples of prose writing,[1] the following agreements will have to be made: (1) Any word containing a hyphen or an apostrophe will count as a single word. (2) Numerals will not be counted. Emphasize that the questions underneath the tables are to be answered in class discussion.

After the tally tables have been completed, ask, "Did anyone's results agree with his original guess as to the most popular word length?" Then lead a discussion based on the questions underneath the tables.

### EVALUATION

The student's individual work on his tally tables should be an indication of his ability to gather and record the data with reasonable accuracy, within two or three words in each count.

## EXPERIENCE 4

## Handling Geometric Data

### OBJECTIVE

The student should be able to count and record the number of sides and corners for each of ten plane figures. He should also be able to rearrange sets of squares into all possible rectangular patterns and record the dimensions of each rectangle.

### MATERIALS

1 "Polygons" sheet from the Teaching Package for each student
1 worksheet and 64 square tiles for each team of 2 students

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[1] The geography book quoted is *True Stories about Children of All Ages,* by Lindley Smyth (Philadelphia: John C. Winston Co., 1920). The editorial quotation is from "Senators, Remember Machiavelli," by Frank Getlein, *Washington Star,* 8 November 1967. The sample of sports writing is from "A Pair Fit to Be Tied," by Tex Maule, *Sports Illustrated Magazine,* 23 October 1967. The novel quoted is *The Adventures of Huckleberry Finn,* by Mark Twain, printed in many editions.

**Stations.** At each station place enough sets of tiles for each team of students—at Station A, 12 tiles in a set; Station B, 24; Station C, 15; Station D, 13. If regular tiles are not available, you can substitute squares cut from cardboard or other material.

## TEACHER STRATEGY

Give each student a copy of "Polygons for Use in Making a Table" (seen in fig. 2) and have him make tables on a separate sheet. Ask students to decide on the headings, which would be like the following:

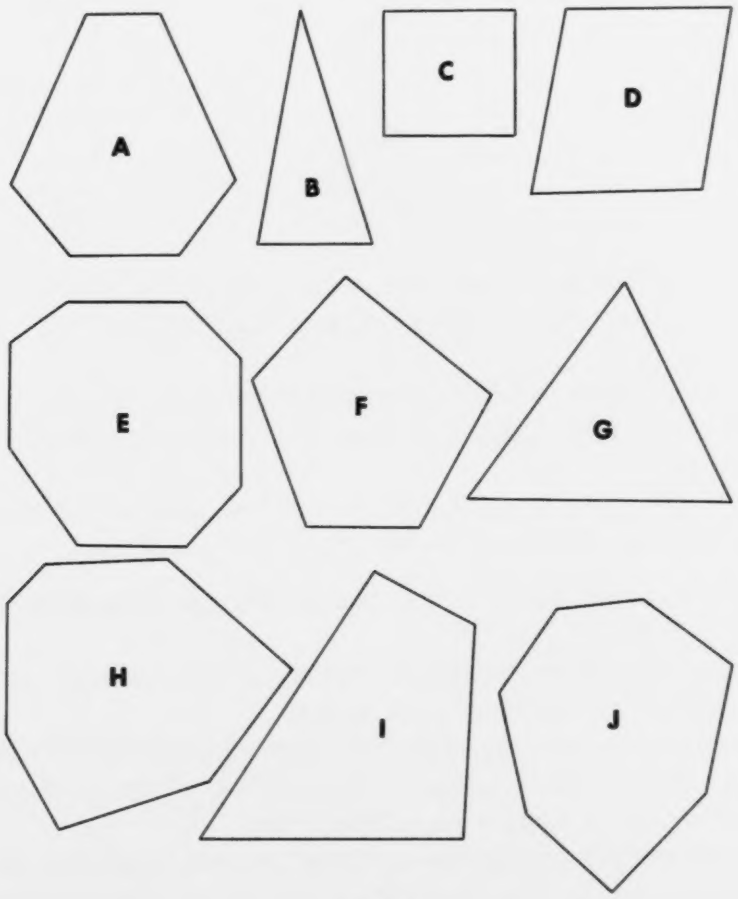


Fig. 2

Letter of Polygon
Number of Sides
Number of Corners

The students, working individually, should be able to complete this activity in a very short time. (See table 1.)

TABLE 1

| Letter of Polygon | Number of Sides | Number of Corners |
|---|---|---|
| A | 6 | 6 |
| B | 3 | 3 |
| C | 4 | 4 |
| D | 4 | 4 |
| E | 8 | 8 |
| F | 5 | 5 |
| G | 3 | 3 |
| H | 7 | 7 |
| I | 4 | 4 |
| J | 7 | 7 |

Discuss the students' results with them and generalize the findings by asking: "How many corners do you think a 9-gon would have? A 15-gon? A 20-gon?"

For the next activity have the students select partners so that they can work in teams of two. Each team is given a worksheet and told to follow the directions.

As the teams complete their visits to the four stations, have the results transferred to the chalkboard, to be helpful in a discussion:

Did all teams find the same number of different arrangements at each station?

If not, can we find the maximum number of rectangles at each station? Who has the highest results for each station?

Does anyone see anything about the pairs of numbers we have in any one of the tables? Hint: multiply. The product of the numbers in each pair is the number of squares you started with.

Look at all the numbers in one table and compare them with the number of squares you were working with. What can we see about these numbers? Hint: divide. They are all divisors of the original number.

## EVALUATION

The students should be able to discover and record the dimensions of all the rectangles possible with each set of tiles.

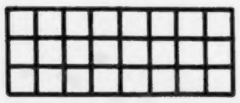
## HANDLING GEOMETRIC DATA

**UNIT 7, EXPERIENCE 4**

Name ______________________________

Name ______________________________

Here is a picture of a rectangle that is 8 blocks long and 3 blocks high.



Go to each of the four stations in any order.

At each station use *all* the tiles in front of you to make as many *different* rectangles as you can. Fill in the table for that station, showing how the rectangles are formed.

*Station A—12 Tiles*

| Number in Length | Number in Height |
|---|---|
| 1 | 12 |
| 2 | 6 |
| 3 | 4 |
| 4 | 3 |
| 6 | 2 |
| 12 | 1 |
| | |

*Station B—24 Tiles*

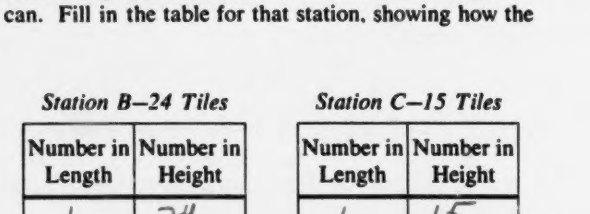
| Number in Length | Number in Height |
|---|---|
| 1 | 24 |
| 2 | 12 |
| 3 | 8 |
| 4 | 6 |
| 6 | 4 |
| 8 | 3 |
| 12 | 2 |
| 24 | 1 |

*Station C—15 Tiles*

| Number in Length | Number in Height |
|---|---|
| 1 | 15 |
| 3 | 5 |
| 5 | 3 |
| 15 | 1 |
| | |
| | |
| | |

*Station D—13 Tiles*

| Number in Length | Number in Height |
|---|---|
| 1 | 13 |
| 13 | 1 |
| | |
| | |
| | |



# EXPERIENCE 5

## Tables of Sums and Data on Making Change

### OBJECTIVE

The student should be able to record all possible combinations of pennies, nickels, dimes, and quarters that make up the amounts from one cent to twenty-five cents.

### MATERIALS

1 piece of paper for each team of 2 students, with 6, 7, 8, 9, or 10 written on it

Wrapped pieces of candy, enough to equal the total of the numerals on the pieces of paper

25 pennies, 5 nickels, 2 dimes, and 1 quarter, in actual coins or in substitute form, for each team of 2

1 worksheet for each team of 2

### TEACHER STRATEGY

Have the class divide into pairs of students to work as teams in this experience. Place the slips of paper in a box. Have each team draw one and then count out as many pieces of candy as the paper indicates. Warn them not to eat any of it until the experience is completed!

I like to introduce this experience by asking, "How many different ways could you and your partner share this candy so that each of you would get at least one piece?" After receiving a few answers I distribute a copy of the worksheet to each pair of students and tell them to complete the table for Activity 1.

As each team finishes its table, I have the students record their results on the chalkboard. When all teams have recorded their results, I lead the class along the following lines:

I examine the tables, with the students helping, to see if they are really complete. We work at this until they are.

I focus attention on any one table and ask, "How do the shares in any one combination compare with the number of pieces of candy the team has?" (The sum of the shares is the number of pieces of candy.)

"How many combinations are there for each separate table?" I record the number on the board beneath each table. "Can we see any pattern here?

"How many combinations do you suppose we would have if two people shared fourteen pieces? Sixteen pieces? Twenty-six pieces? Thirty pieces?" Sometimes I have the students construct, on the board, the table for fourteen pieces.

"How many combinations if we had seven pieces?" (Six.) "Nine pieces?" (Eight.) "Twenty-one pieces?" (Twenty.) This is a good time to construct on the board, for all to see, the table for seven.

"How many combinations would we get if we had four pieces and each person was *not* guaranteed a piece?" (Five.) Then, to one of the teams that had six pieces, I ask, "How about six?" I continue with eight pieces, ten pieces, and so on.

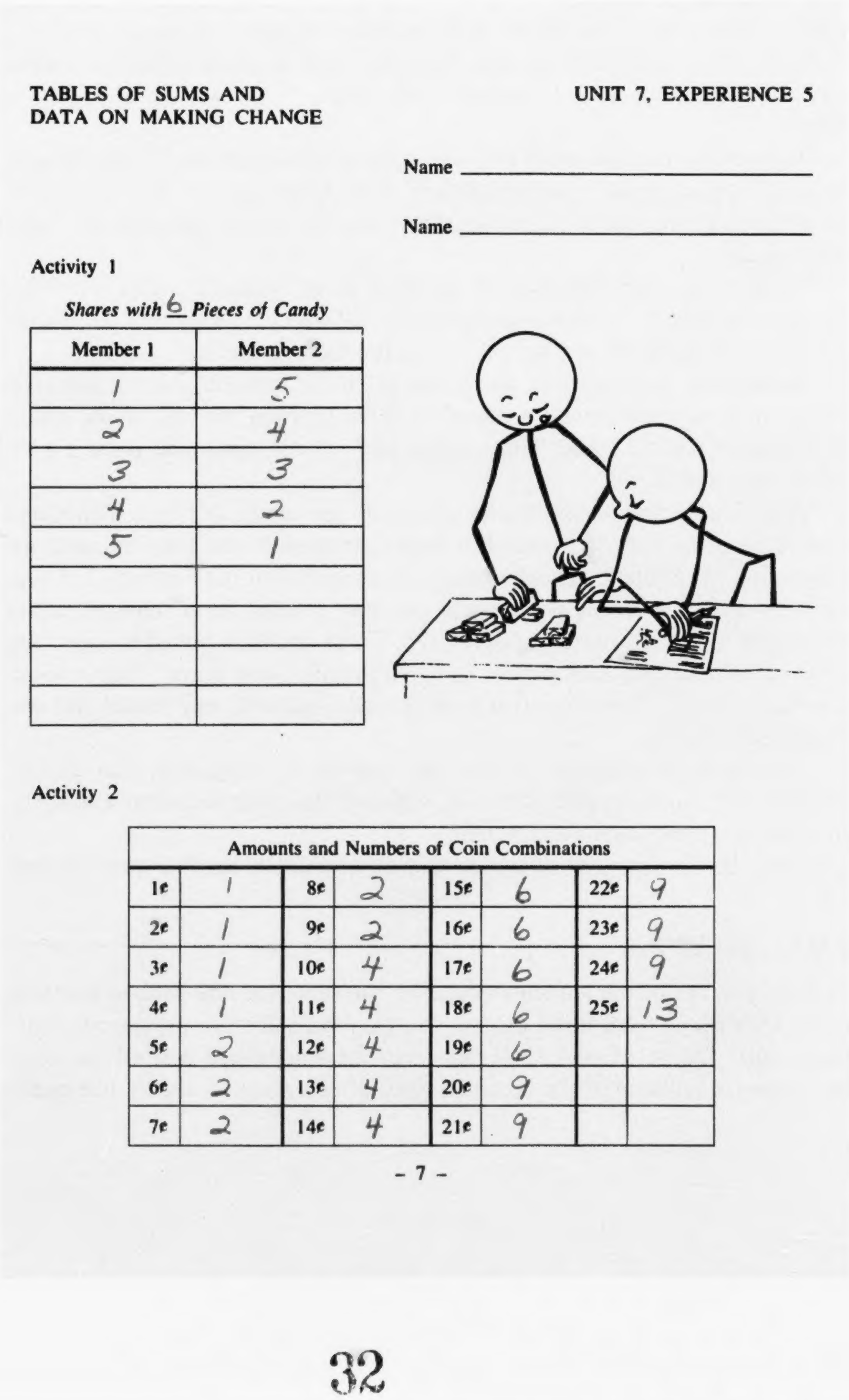
After this discussion the teams return all the candy (to be redistributed and eaten when the experience has been completed) and take an envelope containing the "coins" already listed. As preparation for Activity 2, I ask, "In how many different ways could you pay a clerk for a ten-cent candy bar if you gave him the exact change?" There are four possible ways: ten pennies; two nickels; one nickel and five pennies; one dime. "How about a six-cent item?" There are two possible combinations: one nickel and one penny; six pennies.

I now ask the students to start on Activity 2, suggesting that as one member of each team puts the coins together the other member record the number of arrangements in the table.

It may be necessary to complete this experience in another class period.

## EVALUATION

If 60 percent of the students complete the table for one cent to fourteen cents with fewer than three errors, you may consider the experience quite successful. Do not expect more than half the students to find all the combinations for any *one* of the amounts from fifteen cents to twenty-five cents.



**TABLES OF SUMS AND
DATA ON MAKING CHANGE**

**UNIT 7, EXPERIENCE 5**

**Name** ______________________________

**Name** ______________________________

**Activity 1**

***Shares with 6 Pieces of Candy***

| Member 1 | Member 2 |
|---|---|
| 1 | 5 |
| 2 | 4 |
| 3 | 3 |
| 4 | 2 |
| 5 | 1 |
| | |
| | |
| | |
| | |
| | |

**Activity 2**

| Amounts and Numbers of Coin Combinations | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1¢ | 1 | 8¢ | 2 | 15¢ | 6 | 22¢ | 9 |
| 2¢ | 1 | 9¢ | 2 | 16¢ | 6 | 23¢ | 9 |
| 3¢ | 1 | 10¢ | 4 | 17¢ | 6 | 24¢ | 9 |
| 4¢ | 1 | 11¢ | 4 | 18¢ | 6 | 25¢ | 13 |
| 5¢ | 2 | 12¢ | 4 | 19¢ | 6 | | |
| 6¢ | 2 | 13¢ | 4 | 20¢ | 9 | | |
| 7¢ | 2 | 14¢ | 4 | 21¢ | 9 | | |

# EXPERIENCE 6

## Related Number Pairs, with Clues

### OBJECTIVE

The student should be able to examine four incomplete tables and complete them, using clues if necessary.

### MATERIALS

4 envelopes at each of 4 stations
4 sets of 4 clue slips (and duplicates)
1 worksheet for each student

Envelopes should be labeled with the name of the station and the number of the clue the envelope contains. To avoid confusion, have several slips in each envelope. The clues, to be reproduced from two pages in the Teaching Package, are listed below.

**Station A.** (1) Can you multiply? (2) Add this pair to your table: 10, 20. (3) Add these pairs to your table: 15, 30; 20, 40; 25, 50. (4) Multiply by 2.

**Station B.** (1) Can you add? (2) Add this pair to your table: 10, 1. (3) Add these pairs to your table: 20, 2; 21, 3; 22, 4. (4) Add digits.

**Station C.** (1) Can you multiply? (2) Add this pair to your table: 11, 1. (3) Add these pairs to your table: 21, 2; 22, 4; 23, 6. (4) Multiply all the digits.

**Station D.** (1) Can you multiply? (2) Add this pair to your table: 5, 20. (3) Add these pairs to your table: 3, 12; 30, 120; 35, 140. (4) Multiply by 4.

### TEACHER STRATEGY

I have found that a good way to introduce this experience is to put on the chalkboard the incomplete table shown in figure 3 on the next page and ask the students to guess the missing entries.

There are many correct starts. One student may notice that the sum of

| | |
|---|---|
| 1 | 6 |
| 2 | 3 |
| | 2 |
| 6 | |

Fig. 3

the first pair of numbers is seven and of the next pair is five and guess that the sum of the third pair should be three. This would lead him to use a one for the missing entry, to get 1 + 2 = 3. He wouldn't know what to do in the next step, however, so this scheme has to be abandoned and something else tried.

Soon some student will observe that the product of the pairs of numbers is six. Testing this scheme shows how the table should be completed.

I put the three tables shown in figure 4 on the chalkboard and ask the students to copy and complete them. The missing entries, inserted downward in each table, turn out to be 18, 9, 3; 2, 14; 1, 2, 5.

| | |
|---|---|
| 3 | 6 |
| 1 | |
| | 2 |
| 6 | |
| 18 | 1 |

| | |
|---|---|
| 2 | 7 |
| 7 | |
| 14 | 1 |
| 1 | |

| | |
|---|---|
| 4 | 2 |
| 3 | 3 |
| 5 | |
| | 4 |
| | 1 |

Fig. 4

When most of the students have completed the tables or have gone as far as they can without help, I have a discussion that brings out the rules—for the third table, that of adding the pairs of numbers to keep the sum of six. Some students may suggest adding the pairs 6, 0 and 0, 6 to complete the table.

I now distribute the worksheets and ask one-fourth of the students to begin with the Station A table, one-fourth with the Station B table, and so on. This is to minimize confusion at the stations for the students who need the help of clues. I stress the importance of not using clues unless they are needed and point out that students who do need clues are to check, at the bottom of the table, the clues used.

## EVALUATION

Most students should be able to complete correctly three of the four tables without using the fourth clue.

**RELATED NUMBER PAIRS, WITH CLUES** **UNIT 7, EXPERIENCE 6**

Name ____________________

Try to finish each of the tables on this sheet.

If you need some help, go to the station named at the top of the table and use the clues you will find there. Use the clues in order, with Clue 1 first, and don't use any more than you need to.

Check, at the bottom of the table, the numbers of the clues you have used.

| Station A | |
|---|---|
| 22 | 44 |
| 8 | 16 |
| 9 | 18 |
| 16 | 32 |
| 31 | 62 |
| 91 | 182 |
| | |
| | |
| | |
| | |
| | |
| | |

Clues Used:

1_ 2_ 3_ 4_

| Station B | |
|---|---|
| 265 | 13 |
| 14 | 5 |
| 82 | 10 |
| 146 | 11 |
| 75 | 12 |
| 89 | 17 |
| 888 | 24 |
| | |
| | |
| | |
| | |
| | |

Clues Used:

1_ 2_ 3_ 4_

| Station C | |
|---|---|
| 13 | 3 |
| 214 | 8 |
| 223 | 12 |
| 75 | 35 |
| 64 | 24 |
| 127 | 14 |
| 84 | 32 |
| Impossible | 11 |
| | |
| | |
| | |
| | |

Clues Used:

1_ 2_ 3_ 4_

| Station D | |
|---|---|
| 2 | 8 |
| 50 | 200 |
| 9 | 36 |
| 7 | 28 |
| 40 | 160 |
| 24 | 96 |
| 120 | 480 |
| | |
| | |
| | |
| | |
| | |

Clues Used:

1_ 2_ 3_ 4_

# 3

### EXPERIENCES WITH

## Using Tables to Solve Problems

The activities described in this unit are designed to give students the opportunity to use tables to solve problems. Problem solving is a major portion of mathematics, and organization of data in the form of tables is one of many tools we can use to remove much of the mystery of problem solving.

No attempt is made to exhaust any one type of problem situation. A variety of experiences are presented in laboratory settings with data collection, organization, and examination a major part of the task. We recognize that table-making leads to graphs, equations, and formulas, but we use tables here primarily as a means of problem solving. Some of the problem situations presented may, at first glance, appear difficult. With the table as his tool, the student will find his way through the problem to a successful solution. For example, this table might be started:

| Number of Quarts of Milk | Total Cost |
|---|---|
| 1 | 32¢ |
| 2 | 64¢ |
| 3 | 96¢ |
| 4 | |

The student is asked to examine the table and fill in the blank spaces. The following sample problems might then be presented:

If you have $1.50, how many quarts of milk can you buy?

If four quarts of liquid are the same as one gallon of liquid, what is the cost of a gallon of milk?

Which costs more, six quarts of milk or two gallons of milk?

You have 80 cents. Draw a picture that shows the number of quarts of milk you can buy.

If a herd of twenty dairy cattle produces on the average three gallons of milk per day per cow, what is the value at the store of the milk produced by the herd in one day?

Initial experiences familiarize the student with reading tables and making them. After these experiences he is encouraged to use tables of his own design to help him solve problems.

Your role is like that of a supporting actor in a play. The students are the starring participants. They examine and handle objects and collect, record, and analyze data. Your task is to introduce the experiences, provide the physical props, then remain in the background while the students carry out the experience. Each experience is concluded by your leading a class discussion of what took place. These discussions serve to clarify ideas. They also serve as a means for evaluating the experience.

Fear is a major obstacle for students to overcome in problem solving. The activities of this unit give each student the opportunity to tackle problems and see them through to a successful conclusion. Success breeds success. If we can provide a few successful ventures in handling mathematical problems, we may open doors hitherto closed in the minds of these students.

## OVERVIEW

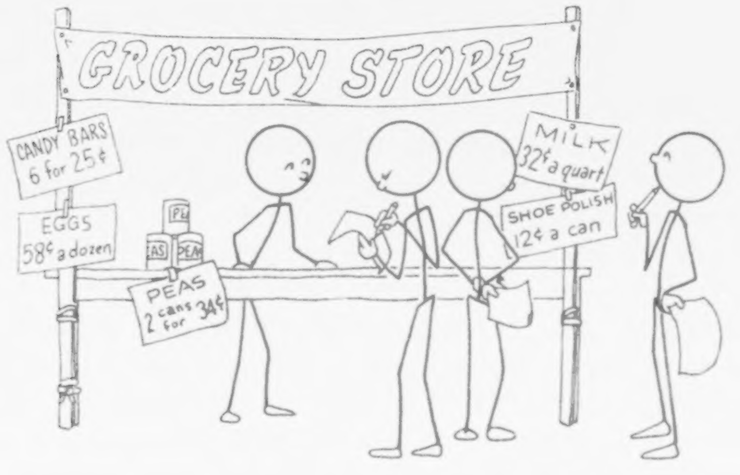
Each of the five experiences in this unit includes a detailed section called "Teacher Strategy" which is usually one teacher's first-person account of a procedure that proved effective with his class.

The following brief summary will help you decide whether these experiences meet the needs of your own class and will also give you some indication of the amount of preparation involved.

Student worksheets and other "handout" materials are provided in the Teaching Package. It is assumed that duplicating facilities are available to you.

**Experience 1: Matching Tables and Everyday Situations**

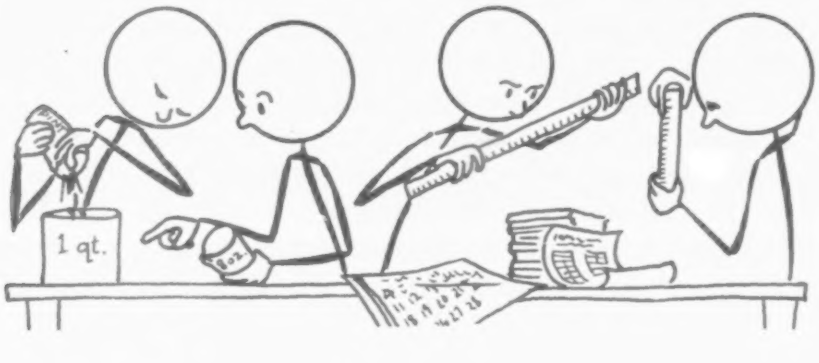
At each of four stations students select from five signs four that match tables of numbers, and they make up appropriate headings for these tables. In the discussion that follows the unused sign at each station is identified and listed on the chalkboard. The students then match these signs to tables that appear on a worksheet. On four other worksheets students solve problems using appropriate tables from the four stations.


GROCERY STORE
CANDY BARS 6 for 25¢
EGGS 58¢ a dozen
PEAS 2 cans for 34¢
MILK 32¢ a quart
SHOE POLISH 12¢ a can

Materials needed: station labels and signs, worksheets.

**Experience 2: Finding Patterns in Tables and Using Them to Solve Problems**

Students find the patterns in eight partial tables and complete them. Next they use four of the tables to solve problems involving such measurements as cups, pints, quarts, ounces, inches, feet, months, years, days, and weeks. Two stations are set up in this experience.

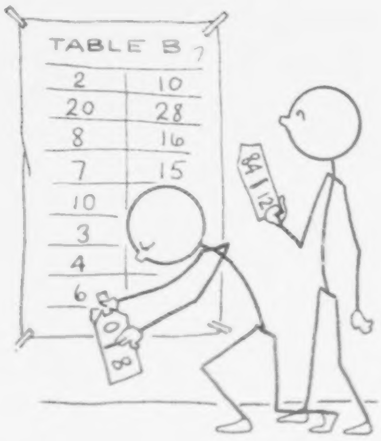


Materials needed: measuring cups, pint and quart containers, rulers, yardsticks, calendars, water, worksheets.

**Experience 3: Matching Number Pairs with Tables**

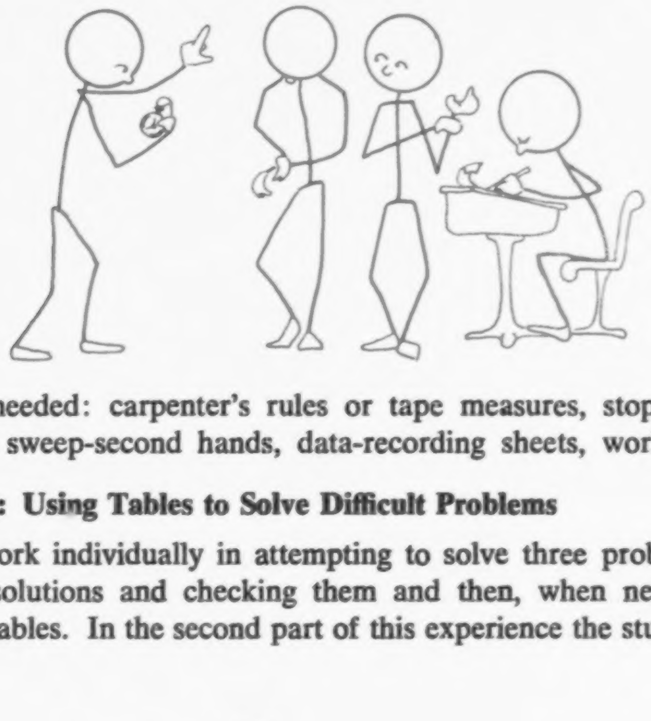
Students get further experience in finding tables to which additional numbers belong. They divide into two teams and have some competition in a game. They may also help in the preparation of the posters and tags used in the game.

Materials needed: magic markers, poster paper, paper tags, large envelope, pins.



**Experience 4: Collecting and Organizing Data in Tables**

Students work in four teams to collect such data as the number of their heartbeats in a certain time, lengths of city blocks, time to walk a set fraction of a mile, and lengths of standard-sized and small cars. They organize their data in tables and apply the data in solving problems on a worksheet.



Materials needed: carpenter's rules or tape measures, stopwatches or watches with sweep-second hands, data-recording sheets, worksheets.

**Experience 5: Using Tables to Solve Difficult Problems**

Students work individually in attempting to solve three problems—first by guessing solutions and checking them and then, when necessary, by constructing tables. In the second part of this experience the students work in four teams, each with a difficult puzzle problem ("stumper"). They have materials with which they can construct physical models of the problems. Tables showing their results at various steps of the problems are helpful.

Materials needed: pint jar, lima beans, sheet of newspaper, scissors, calendar sheet, checkerboard, squares of paper, copies of stumpers, worksheets.

# EXPERIENCE 1
## Matching Tables with Everyday Situations

### OBJECTIVE

The student should be able to match tables with the situation each table describes. In addition to being able to identify column headings on twenty tables, he should be able to use these tables to help answer questions.

## MATERIALS

4 station labels
20 signs for stations (from Teaching Package)
12 worksheets for each student

**Stations.** Labels for the stations—Grocery Store, Bakery, Post Office, and Hardware Store—may be reproduced from the Teaching Package. Also in the Teaching Package are 5 signs for each station and 4 worksheets for the students. The worksheets contain 4 sets of tables.

One of the signs for the grocery store is for peas, "2 cans for 34¢." Several cans of peas placed under that sign would lend a realistic touch to the station. Use a magic marker to write "2/34¢" on the top of each can.

## TEACHER STRATEGY

I like to introduce the experience by saying: "Let's see if you can be mind readers today. Look at the table I am starting here on the board and see if you can guess what numbers I'm thinking of."

| | |
|---|---|
| 1 | 2 |
| 4 | |
| | |

Let your students try to "read your mind." You might be thinking that the missing number is 8 (multiplying the 4 by 2) or that the missing number is 5 (adding 1 to the 4). One class decided to use 8.

| | |
|---|---|
| 1 | 2 |
| 4 | 8 |
| 7 | |
| | |

I usually make further entries in either column until the pattern is clear to all students. My second table started like this:

| | |
|---|---|
| 5 | 6 |
| 2 | |
| | |

The missing entry might be a 3 (adding 1 to the first-column entry each time), or it might be 15 (first-column entry times second-column entry yields 30). If my students don't guess 15, I write in that entry and continue like this:

| | |
|---|---|
| 5 | 6 |
| 2 | 15 |
| 3 | |
| | |

I like to start another table like the one shown below, where the missing number we want is a 5 (the sum of the first and second entries is 7).

| | |
|---|---|
| 3 | 4 |
| 2 | |
| | |

After five or ten minutes of warm-up, looking at several types of tables and giving just about all students a chance to guess, I proceed as follows:

"Around the room you see four stations: Grocery Store, Bakery, Post Office, and Hardware Store. For each station you will be given four tables with no headings for their columns."

I give the students sets of the four worksheets for Activity 1, each worksheet having a station identification at the top and a series of four tables with the column headings blank.

"Look at the signs at each station and try to match the tables with the signs. Then write in headings for the columns on each of your tables."

To provide an example, I might write the following table on the board and say: "Suppose you see a sign reading 'Pencils—5¢ Each' and one of your tables looks like this. Then the headings for the columns of your table might be 'Number of Pencils' and 'Cost of Pencils.' "

| | |
|---|---|
| 1 | 5 |
| 2 | 10 |
| 3 | 15 |
| 4 | 20 |

Now I divide the class into four groups and send each group to one of the stations. I circulate among the students at the different stations, making sure everyone gets a correct start. Remember—nothing succeeds like *success!*

This examination of the signs and tables usually takes most of the remainder of one period.

When the students have finished the task of labeling columns (or at the beginning of the next class period), I bring them back together as a group and examine their results orally. The following summations of discussion show the responses I try to get from the students.

Two of the tables are different from all the others. Which are they? They are the candy-bar table for the Grocery Store and the rope table for the Hardware Store. Why are they different? They describe the common situation of allowing the customer a reduced price if he buys in large quantities—six candy bars, forty-eight feet of rope, and so on.

At each station there is one sign for which there is no table. Which are they? [I list them on the board as students suggest them.]

**Shoe Polish—12¢ a can**
**Special Delivery—30¢**
**24 Cookies—1 tray**
**Sandpaper—6¢ a sheet**

## EVALUATION

For the evaluation of Activity 1, discussed under Teacher Strategy, I pass to each student a copy of the first worksheet for Activity 2. This has four tables with blank headings, and the students are asked to write in the appropriate headings from the four signs listed above, which are now written on the board. I usually find that 90 percent or more of the students can do this without error. For further evaluation and reinforcement I have the students complete as many of the next four worksheets as time permits.

**MATCHING TABLES AND EVERYDAY SITUATIONS UNIT 8, EXPERIENCE 1**

Activity 1, *GROCERY STORE* Name ______________________________

For each table, find the sign that matches and write headings in the spaces over the columns.

**TABLE 1**

Milk – 32¢ a quart

| Quantity | Cost |
|---|---|
| 1 pt. | 16 |
| 2 pt. or 1 qt. | 32 |
| 2 qt. | 64 |
| 3 qt. | 96 |
| 4 qt. or 1 gal. | 128 |

**TABLE 2**

Eggs – 58¢ per dozen

| Dozens | Cost |
|---|---|
| 1 | 58 |
| 2 | 116 |
| 3 | 174 |
| 4 | 232 |
| 5 | 290 |

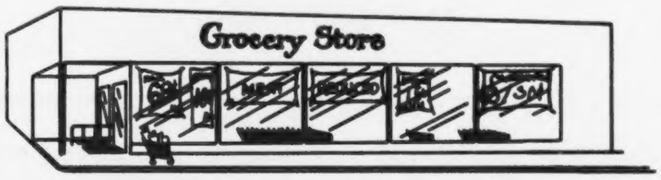


**TABLE 3**

Candy Bars – 6 for 25¢

| Number | Cost |
|---|---|
| 3 | 15 |
| 6 | 25 |
| 9 | 40 |
| 12 | 50 |
| 15 | 65 |
| 16 | 70 |

**TABLE 4**

Peas – 2 cans for 34¢

| Number | Cost |
|---|---|
| 2 | 34 |
| 4 | 68 |
| 6 | 102 |
| 8 | 136 |
| 10 | 170 |
| 1 case | 408 |

MATCHING TABLES AND EVERYDAY SITUATIONS UNIT 8, EXPERIENCE 1

Activity 1, *BAKERY* Name ______________________

For each table, find the sign that matches and write headings in the spaces over the columns.

TABLE 1

3 Teaspoons 1 Tablespoon

| Teaspoons | Tablespoons |
|---|---|
| 3 | 1 |
| 6 | 2 |
| 9 | 3 |
| 12 | 4 |
| 15 | 5 |

TABLE 2

2 Tablespoons 1 Ounce

| Tablespoons | Ounces |
|---|---|
| 2 | 1 |
| 4 | 2 |
| 6 | 3 |
| 8 | 4 |
| 10 | 5 |

TABLE 3

16 Tablespoons 1 Cup

| Tablespoons | Cups |
|---|---|
| 16 | 1 |
| 32 | 2 |
| 48 | 3 |
| 64 | 4 |
| 80 | 5 |

TABLE 4

6 Cupcakes per Pan

| Cupcakes | Pans |
|---|---|
| 6 | 1 |
| 12 | 2 |
| 24 | 4 |
| 36 | 6 |
| 48 | 8 |
| 60 | 10 |
| 144 | 24 |

MATCHING TABLES AND EVERYDAY SITUATIONS UNIT 8, EXPERIENCE 1

*Activity 1, POST OFFICE* Name ______________________

For each table, find the sign that matches and write headings in the spaces over the columns.

TABLE 1

Stamps - Regular Mail 6¢

| Number | Cost |
|---|---|
| 1 | 6 |
| 10 | 60 |
| 20 | 120 |
| 100–1 sheet | 600 |
| 500–1 roll | 3,000 |

TABLE 2

Postcards – 5¢ each

| Number | Cost |
|---|---|
| 5 | 25 |
| 10 | 50 |
| 25 | 125 |
| 100 | 500 |

TABLE 3

Stamps - Air mail 10¢

| Number | Cost |
|---|---|
| 1 | 10 |
| 10 | 100 |
| 100–1 sheet | 1,000 |
| 500–1 roll | 5,000 |

TABLE 4

Parcel Post

| Weight | Cost |
|---|---|
| 2 lb. or less | 40 |
| 3-6 lb. | 45 |
| 7-8 lb. | 50 |
| 9-11 lb. | 55 |
| 12-15 lb. | 60 |

– 17 –

**MATCHING TABLES AND EVERYDAY SITUATIONS UNIT 8, EXPERIENCE 1**

*Activity 1, HARDWARE STORE* Name ______________________

For each table, find the sign that matches and write headings in the spaces over the columns.

TABLE 1

White Paint - 1 qt. $2.60

| Amount | Cost |
|---|---|
| 1 pt. | 130 |
| 2 pt. or 1 qt. | 260 |
| 3 pt. | 390 |
| 4 pt. or 2 qt. | 520 |
| 1 gal. | 1,040 |

TABLE 4

Nails - 1 lb. 20¢

| Pounds | Cost |
|---|---|
| 1 | 20 |
| 2 | 40 |
| 5 | 100 |
| 10 | 200 |
| 20 | 400 |

TABLE 3

Topsoil - 50 lb. $1.50

| Pounds | Cost |
|---|---|
| 50 | 150 |
| 100 | 300 |
| 200 | 600 |
| 300 | 900 |

TABLE 6

Rope - 48 ft. 69¢

| Feet | Cost |
|---|---|
| 12 | 18 |
| 24 | 35 |
| 48 | 69 |
| 96 | 138 |
| 144 | 207 |

MATCHING TABLES AND EVERYDAY SITUATIONS UNIT 8, EXPERIENCE 1

Activity 2 Name ____________________

Look at the four signs we have just written on the board. Match the tables on this sheet with the signs. In each table, write headings in the spaces over the columns.

TABLE 1

Special Delivery 30¢

| Number | Cost |
|---|---|
| 1 | 30 |
| 2 | 60 |
| 3 | 90 |
| 4 | 120 |
| 5 | 150 |

TABLE 2

Sandpaper 6¢/sheet

| Cost | Number |
|---|---|
| 6 | 1 |
| 12 | 2 |
| 18 | 3 |
| 24 | 4 |
| 30 | 5 |

TABLE 3

Shoe Polish – 12¢/can

| Number | Cost |
|---|---|
| 1 | 12 |
| 2 | 24 |
| 3 | 36 |
| 4 | 48 |
| 5 | 60 |

TABLE 4

24 Cookies 1 Tray

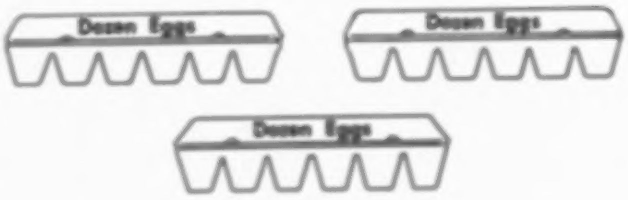
| Cookies | Trays |
|---|---|
| 24 | 1 |
| 36 | 1½ |
| 48 | 2 |
| 60 | 2½ |
| 72 | 3 |

MATCHING TABLES AND EVERYDAY SITUATIONS UNIT 8, EXPERIENCE 1

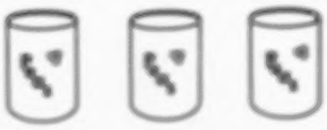
Activity 2. *GROCERY STORE* Name ______________________

Use your tables about the Grocery Store to help you solve these problems.

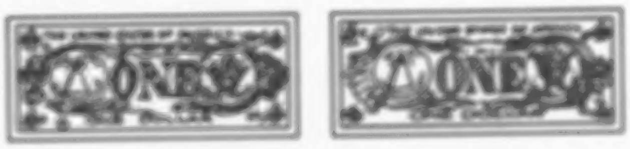
1. How much do these eggs cost? $1.74

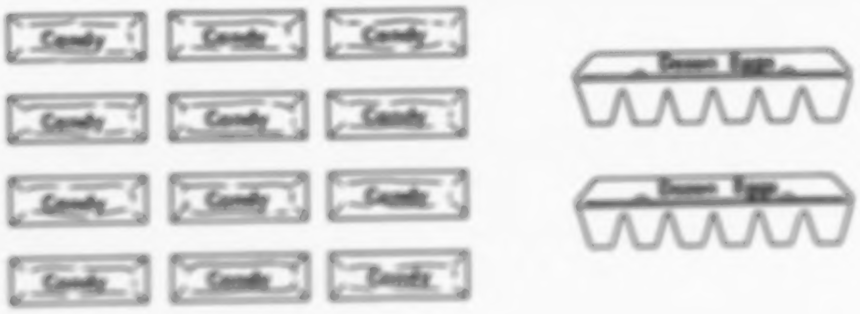

Dozen Eggs
Dozen Eggs
Dozen Eggs

2. Find the total cost of these peas. 51¢



3. You have the money shown in the picture. You buy candy and eggs:




Candy
Candy
Candy
Candy
Candy
Candy
Candy
Candy
Candy
Candy
Candy
Candy
Dozen Eggs
Dozen Eggs

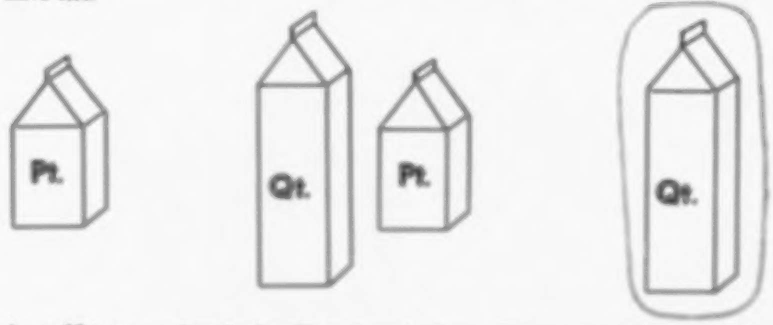
How much money do you have left? 24¢

(Continued)

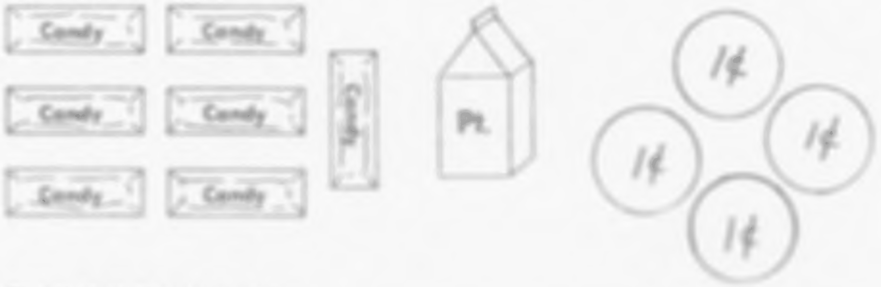
MATCHING TABLES AND EVERYDAY SITUATIONS UNIT 8. EXPERIENCE 1

Activity 2. *GROCERY STORE–Continued* Name ______________________________

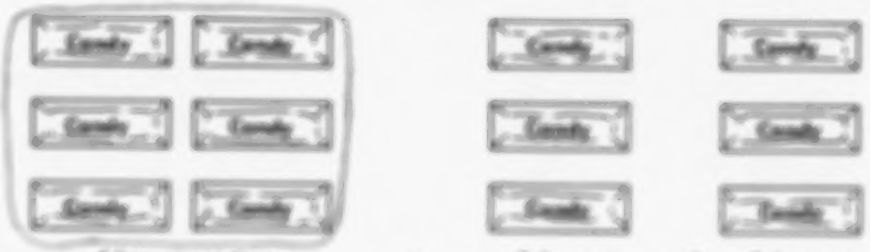
Continuing with the same problem, circle how much milk you can buy with the money you have left.



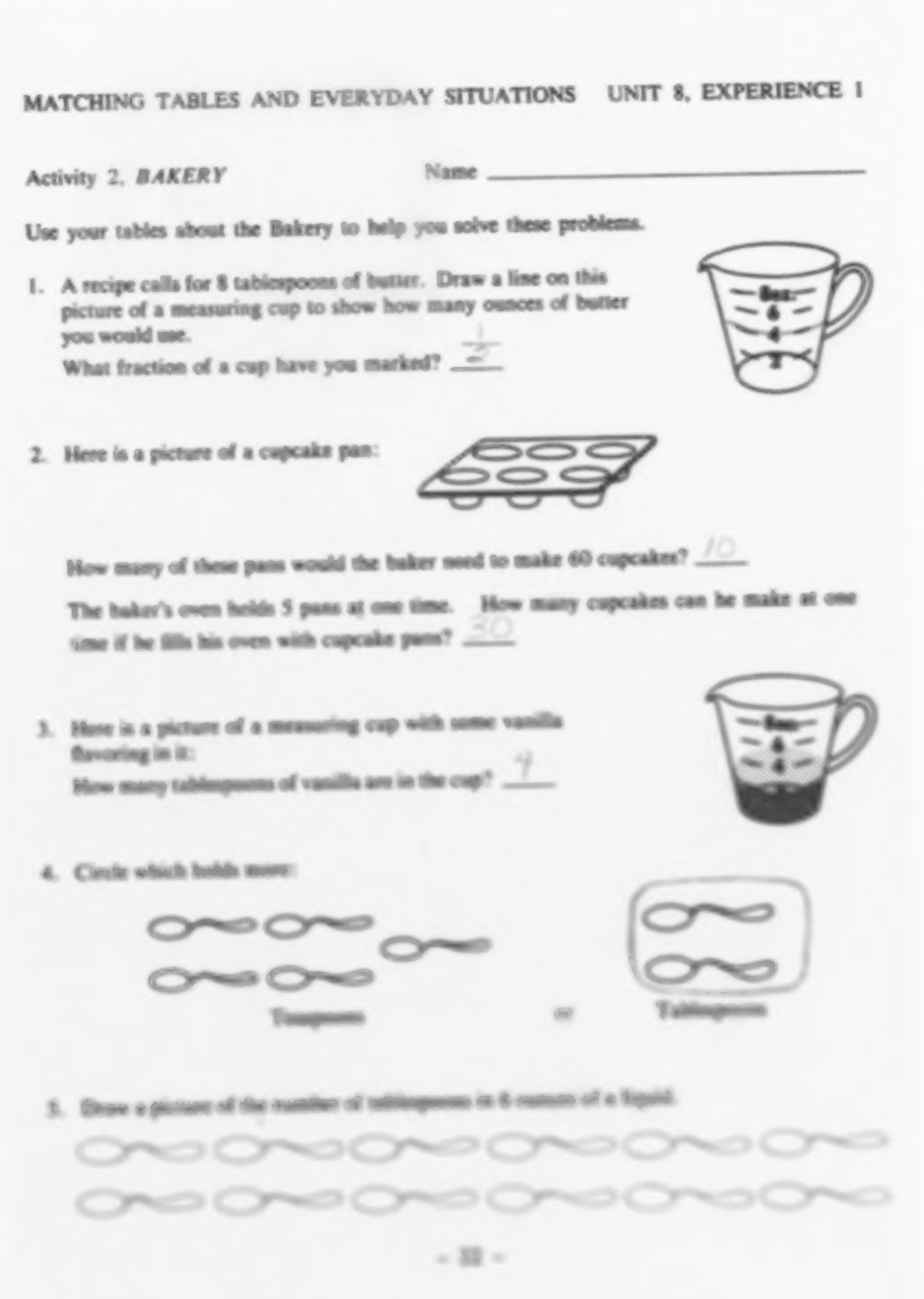
4. You have 50¢ to spend on both milk and candy bars. Draw some pictures to show one way you could buy some milk and some candy bars with your 50¢.


Candy
Candy
Candy
Candy
Candy
Candy
Candy
Pt.
1¢
1¢
1¢
1¢

5. Circle which is the better buy:


Candy
Candy
Candy
Candy
Candy
Candy
Candy
Candy
Candy
Candy
Candy
Candy

6 bars at one time or 3 bars at one time and 3 bars at another time



MATCHING TABLES AND EVERYDAY SITUATIONS UNIT 8, EXPERIENCE 1

Activity 2. *BAKERY* Name ______________________

Use your tables about the Bakery to help you solve these problems.

1. A recipe calls for 8 tablespoons of butter. Draw a line on this picture of a measuring cup to show how many ounces of butter you would use.
   What fraction of a cup have you marked? $\frac{1}{2}$

2. Here is a picture of a cupcake pan:
   How many of these pans would the baker need to make 60 cupcakes? 10
   The baker's oven holds 5 pans at one time. How many cupcakes can he make at one time if he fills his oven with cupcake pans? 30

3. Here is a picture of a measuring cup with some vanilla flavoring in it:
   How many tablespoons of vanilla are in the cup? 4

4. Circle which holds more:
   Teaspoons or [✓ Tablespoons]

5. Draw a picture of the number of tablespoons in 6 ounces of a liquid.

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MATCHING TABLES AND EVERYDAY SITUATIONS UNIT 8, EXPERIENCE 1

Activity 2, *POST OFFICE*

Name ______________________

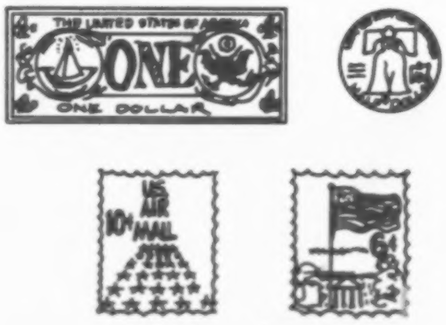
Use your tables about the Post Office to help you solve these problems.

1. You have the money in the picture.

   You want to buy the stamps shown.

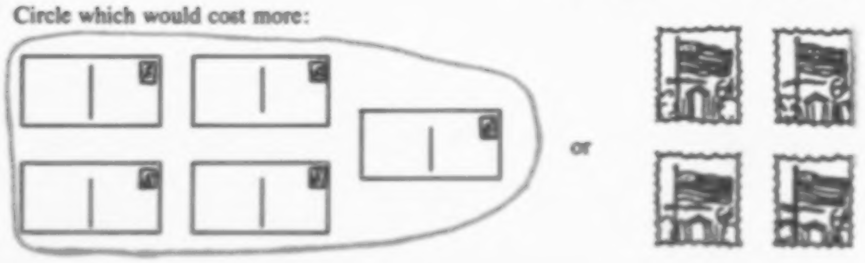
   Do you have enough money?

   No


THE UNITED STATES OF AMERICA
ONE
ONE DOLLAR
US AIR MAIL
10¢

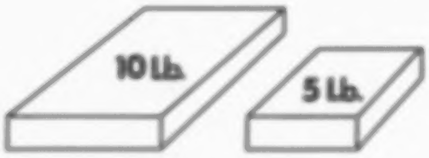
10 Airmail and 10 Regular

2. Circle which would cost more:



3. How much would it cost to mail these two packages at the same time?

   $1.00



4. Circle the total cost of 2 rolls of regular stamps and 1 sheet of airmail stamps. There are 500 stamps in a roll and 100 stamps in a sheet.

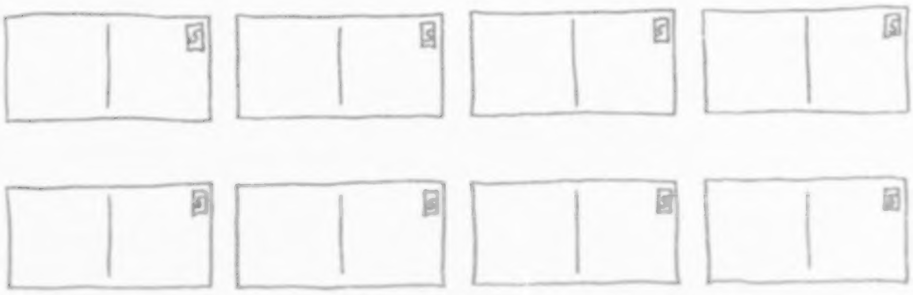
   $16 [✓ $70] $46 $10

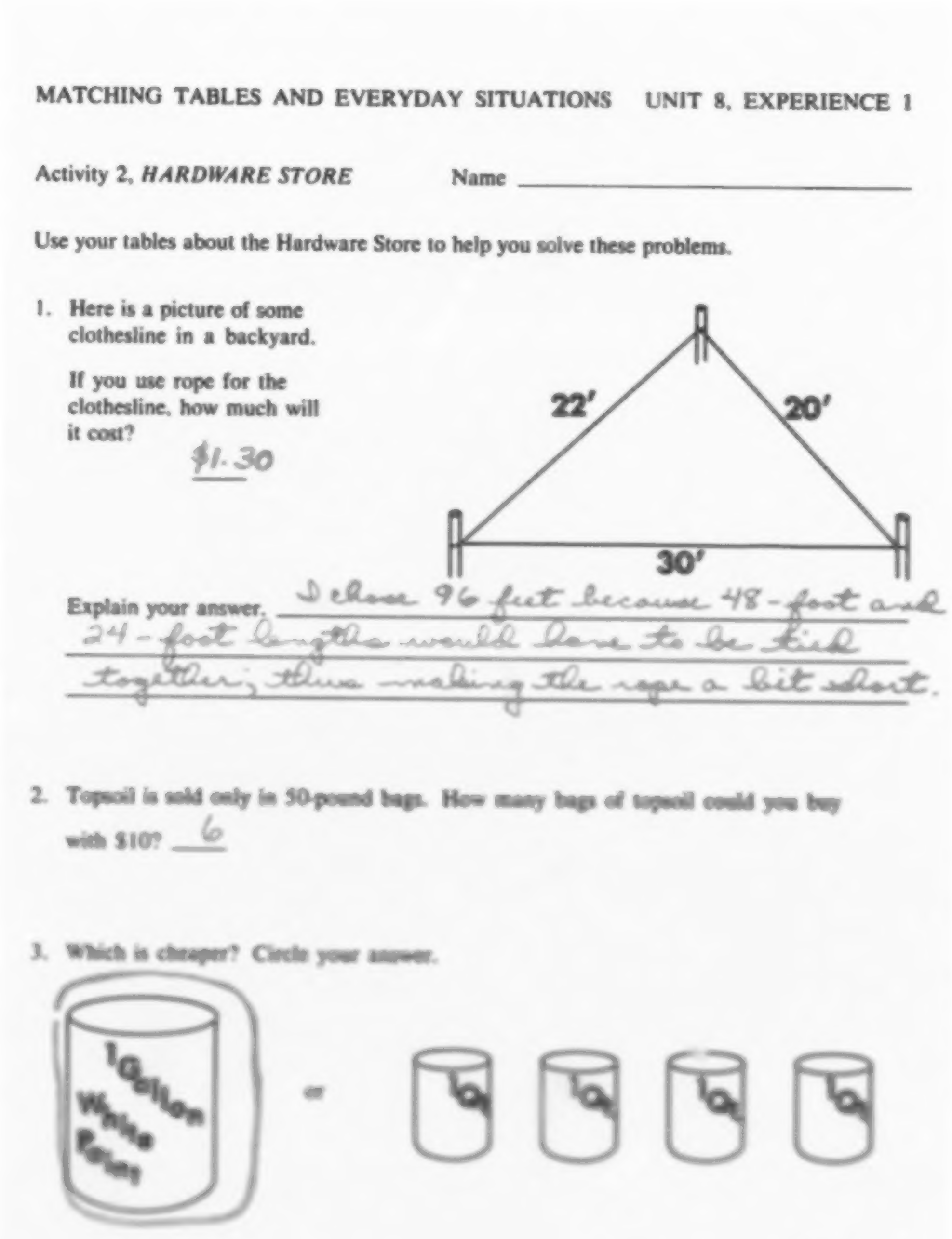
*[Continued]*

## MATCHING TABLES AND EVERYDAY SITUATIONS UNIT 8, EXPERIENCE 1

Activity 2. *POST OFFICE–Continued* Name ______________________________

5. You went to the post office with $1.00. You bought 6 airmail stamps, and the clerk gave you your change in postcards. Draw a picture of the correct number of postcards the clerk gave you.





MATCHING TABLES AND EVERYDAY SITUATIONS UNIT 8, EXPERIENCE 1

Activity 2, *HARDWARE STORE* Name ____________________________

Use your tables about the Hardware Store to help you solve these problems.

1. Here is a picture of some clothesline in a backyard.

   If you use rope for the clothesline, how much will it cost? $1.30

   Explain your answer. I chose 96 feet because 48-foot and 24-foot lengths would have to be tied together, thus making the rope a bit short.

2. Topsoil is sold only in 50-pound bags. How many bags of topsoil could you buy with $10? 6

3. Which is cheaper? Circle your answer.

(Continued)

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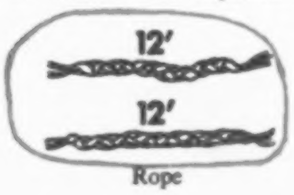
MATCHING TABLES AND EVERYDAY SITUATIONS UNIT 8, EXPERIENCE 1

Name ________________________________

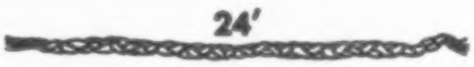
Activity 2, *HARDWARE STORE–Continued*

4. Which is more expensive? Circle your answer.



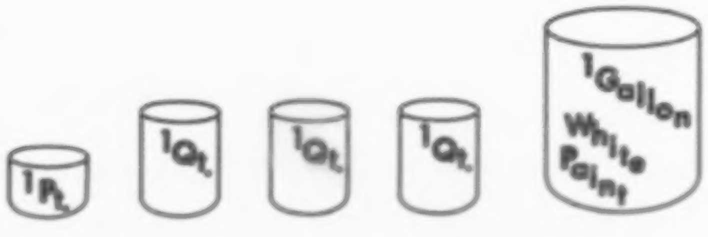
Rope

or



Rope

5. Find the total cost of the paint shown here. $19.10


1 Pt.
1 Qt.
1 Qt.
1 Qt.
1 Gallon White Paint

## EXPERIENCE 2

## Finding Patterns in Tables and Using Them to Solve Problems

### OBJECTIVE

The student should be able to demonstrate, by finishing incomplete tables, that he has discovered the key patterns.

### MATERIALS

2 measuring cups marked in ounces
2 pint containers, labeled Pint
2 quart containers, labeled Quart
2 12-in. rulers
2 yardsticks
2 calendars (any year)
Water
7 worksheets for each student

**Stations.** At each of two locations in the room set up a table with the measuring equipment, calendars, and water.

### TEACHER STRATEGY

My introduction to this experience is similar to the one for Experience 1. I put the tables shown in figure 1 on the board, one at a time, and in each case ask students to supply the missing entry. If necessary, I extend a table until someone discovers the pattern. Occasionally my students see an entirely different pattern than the one I intended; in that case, we may complete the table in more than one way.

After a warm-up of from five to ten minutes with the tables on the board, I give each student a set of worksheets for Activity 1. These contain eight tables, identified by A through H, to be completed.

When some students have completed all the tables, I have them copy their tables on the chalkboard for others in the group to examine. When all students seem finished, I ask those who have put their work on the board to tell the rest of the class how they discovered the missing entries.

| | |
|---|---|
| 1 | 2 |
| 2 | 4 |
| 3 | |
| | |

| | |
|---|---|
| 8 | 1 |
| 16 | 2 |
| | 3 |
| | |

| | |
|---|---|
| 5 | 1 |
| 6 | 2 |
| | 3 |
| | |

| | |
|---|---|
| 3 | 4 |
| 5 | 2 |
| 6 | |
| | |

| | |
|---|---|
| 6 | 4 |
| 8 | 3 |
| 2 | |
| | |

Fig. 1

The students should now change any wrong entries in their tables. At this stage I do not expect or stress accuracy in the original entries; but the tables are used in Activity 2, and for that purpose they *must* be correct.

Before we begin Activity 2, there is further discussion. I have students arrange their worksheets so that they can look at tables A, D, F, and H, and continue somewhat as follows:

"Look at this table." I put on the board the table shown in figure 2.

| | |
|---|---|
| 1 | 12 |
| 2 | 24 |
| 3 | 36 |
| 4 | 48 |

Fig. 2

"This table is not only a table of numbers; it is also a table about measurement." I hold up a yardstick. "Can anyone tell me what this yardstick has to do with my table?"

If I get no response, I may ask, "Who knows how many inches there are in one foot? In two feet? In three feet?" This questioning should result in the suggestion that we label the columns of the table on the board to show that they relate feet and inches.

"The tables you are studying now—tables A, D, F, and H—involve measurement. Go to one of the stations where you see the measuring

[Continued on page [illegible]]

## FINDING PATTERNS IN TABLES AND USING THEM TO SOLVE PROBLEMS

UNIT 8, EXPERIENCE 2

Activity 1

Name ______________________________

Use the objects at the stations to help you decide what the headings for each table should be, then write them in. Complete the tables.

TABLE A

| Inches | Feet* |
|---|---|
| 12 | 1 |
| 24 | 2 |
| 36 | 3 |
| 48 | 4 |
| 60 | 5 |
| 72 | 6 |
| 84 | 7 |
| 96 | 8 |
| 108 | 9 |
| 120 | 10 |

* or Months Years

TABLE B

| a Number | 8 More |
|---|---|
| 2 | 10 |
| 20 | 28 |
| 8 | 16 |
| 7 | 15 |
| 10 | 18 |
| 3 | 11 |
| 4 | 12 |
| 6 | 14 |
| 1 | 9 |
| 15 | 23 |

*[Continued]*

**FINDING PATTERNS IN TABLES**
**AND USING THEM TO SOLVE PROBLEMS**

**UNIT 8, EXPERIENCE 2**

**Activity 1—*Continued***

**Name** ______________________________

**Use the objects at the stations to help you decide what the headings for each table should be, then write them in. Complete the tables.**

**TABLE C**

| Factors | of 36 |
|---|---|
| 2 | 18 |
| 12 | 3 |
| 6 | 6 |
| 4 | 9 |
| 9 | 4 |
| 1 | 36 |
| 3 | 12 |
| 18 | 2 |
| 36 | 1 |

**TABLE D**

| Days | Weeks |
|---|---|
| 7 | 1 |
| 14 | 2 |
| 21 | 3 |
| 28 | 4 |
| 35 | 5 |
| 42 | 6 |
| 49 | 7 |
| 56 | 8 |
| 63 | 9 |
| 70 | 10 |

***[Continued]***

## FINDING PATTERNS IN TABLES AND USING THEM TO SOLVE PROBLEMS

UNIT 8, EXPERIENCE 2

Activity 1—*Continued*

Name ______________________________

**Use the objects at the stations to help you decide what the headings for each table should be, then write them in. Complete the tables.**

**TABLE E**

| Sum Is 18 | |
|---|---|
| 1 | 17 |
| 2 | 16 |
| 3 | 15 |
| 4 | 14 |
| 5 | 13 |
| 6 | 12 |
| 7 | 11 |
| 8 | 10 |
| 9 | 9 |
| 10 | 8 |

**TABLE F**

| Cups | Pints* |
|---|---|
| 2 | 1 |
| 4 | 2 |
| 6 | 3 |
| 8 | 4 |
| 10 | 5 |
| 12 | 6 |
| 14 | 7 |
| 16 | 8 |
| 18 | 9 |
| 20 | 10 |

* or
Twice the Number / A Number

*[Continued]*

## FINDING PATTERNS IN TABLES AND USING THEM TO SOLVE PROBLEMS

UNIT 8, EXPERIENCE 2

Activity 1—*Continued*

Name ______________________________

Use the objects at the stations to help you decide what the headings for each table should be, then write them in. Complete the tables.

TABLE G

| Sum | Is 25 |
|---|---|
| 20 | 5 |
| 23 | 2 |
| 18 | 7 |
| 22 | 3 |
| 17 | 8 |
| 16 | 9 |
| 21 | 4 |
| 24 | 1 |
| 19 | 6 |
| 15 | 10 |

TABLE H

| Ounces | Cups |
|---|---|
| 8 | 1 |
| 16 | 2 |
| 24 | 3 |
| 32 | 4 |
| 40 | 5 |
| 48 | 6 |
| 56 | 7 |
| 64 | 8 |
| 72 | 9 |
| 80 | 10 |

FINDING PATTERNS IN TABLES
AND USING THEM TO SOLVE PROBLEMS

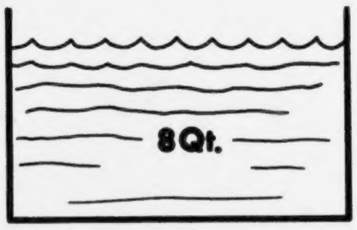
UNIT 8, EXPERIENCE 2

Activity 2

Name ____________________

Use tables A, D, F, and H to help you solve these problems.

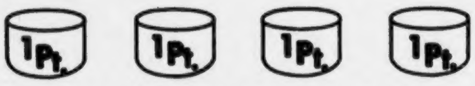
1. Joe's fish tank holds 8 quarts of water.
   Joe takes out half the water.
   How many quarts of water are now left in Joe's tank? 4
   How many *pints* of water are now left in Joe's tank? 8



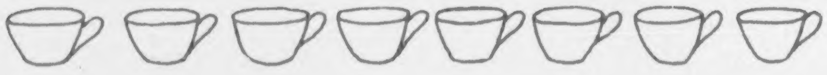
2. Sara has a 12-ounce bottle of coke. Circle how many cups of coke she has in her bottle.

   1 cup
   [✓ Between 1 and 2 cups]
   2 cups
   More than 2 cups

3. John has strawberries as pictured below.



Draw cups to show the number of cups of strawberries John has.



4. Which contains less milk, 11 cups or 5 pints? 5 pints
   11 cups or 3 quarts? 11 cups

[*Continued*]

FINDING PATTERNS IN TABLES
AND USING THEM TO SOLVE PROBLEMS

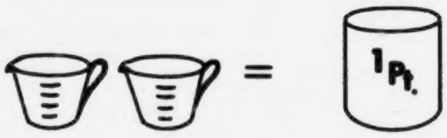
UNIT 8, EXPERIENCE 2

Activity 2–*Continued*

Name ______________________________

5. Following the example at the top, connect the equations below to make true statements.

2 cups = 2 pints / [✓ 1 pint] / 2 quarts / 1 quart



4 quarts = [✓ 8 pints] / 4 pints / 6 cups / 10 cups

24 ounces = 2 cups / 4 cups / [✓ 3 pints] / 3 cups

10 pints = 2 quarts / [✓ 20 cups] / 4 quarts / 40 cups

6 cups = 1 quart / 1 pint / [✓ 3 pints] / 24 ounces

[*Continued*]

FINDING PATTERNS IN TABLES
AND USING THEM TO SOLVE PROBLEMS

UNIT 8, EXPERIENCE 2

Activity 2—*Continued*

Name ______________________________

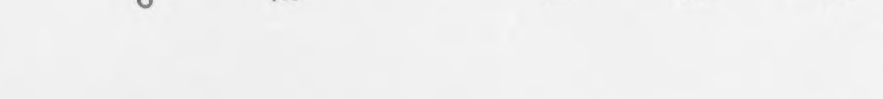
Use tables A, D, F, and H to help you solve these problems:

6. A customer on your paper route will be gone on his vacation 6 weeks and 3 days.
   How many days will he be gone? 45

7. Becky's dad wants to put screen around their back porch. He measured and found that the porch is 40 feet around. If the store sells screen by the running inch, how many inches of screen should Becky's dad buy? 480

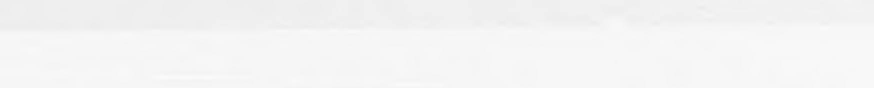
8. John is 8 years and 4 months old; Mike is 7 years and 18 months old.
   Who is the older of the two? Mike

9. Here is a diagram of a yardstick:



Draw a line on it to show a measure of 2 feet, 3 inches.

10. Draw another diagram of a yardstick and show a measure of $1\frac{1}{2}$ feet on it.



equipment and see if you can discover names for the columns of your tables. Write the names in where the space is left for them."

I circulate between the two stations to see that the students understand how to use water to measure the number of pints in a quart, and so on. This part of the experience may have to be completed on another day.

As students finish identifying column names they return to their seats, where I have placed the set of worksheets for Activity 2, and begin to work on them.

When everyone has completed the identification part of this experience, I interrupt the work on Activity 2 long enough to see if all have correctly labeled the columns in the four tables.

### EVALUATION

The answers students give on the Activity 2 worksheets provide a measure of how well they can use tables in solving problems.

## EXPERIENCE 3

## Matching Number Tables with Pairs

### OBJECTIVE

Out of six tables composed of number pairs, the student should be able to select the one table to which an additional number pair belongs.

### MATERIALS

6 magic markers
6 sheets of poster paper in light colors
15 tags (from Teaching Package)
Large envelope
Pins

### TEACHER STRATEGY

I like to have my students help prepare the materials for this experience. Six of them copy the first six tables from Experience 2 (A through F) on poster paper and place the tables along the front of the room. Others cut out the tags, reproduced from the Teaching Package, and place them in a large envelope. This does not take more than half the class period.

If more than fifteen students are to participate in this experience, supply enough additional tags, following the same principles of number pairs, so that each student will have one.

When the materials are ready, I have the students organize themselves into two teams, and I announce: "Today we are going to play a game called 'Pin the Tag on the Table.' Each member of each team will get a chance to pick one tag from the envelope and try to pin it at the bottom of the correct table. If he succeeds, his team receives five points. If he cannot place the tag correctly in two minutes, any member of the other team gets one chance to place it on the correct table. The team with the highest score wins."

I find it somewhat difficult to keep excited students from giving hints by cheers and groans, but an appeal to their sense of fairness usually suffices. This experience can be completed in one class period.

The proper placement of the tags is shown in figure 3.

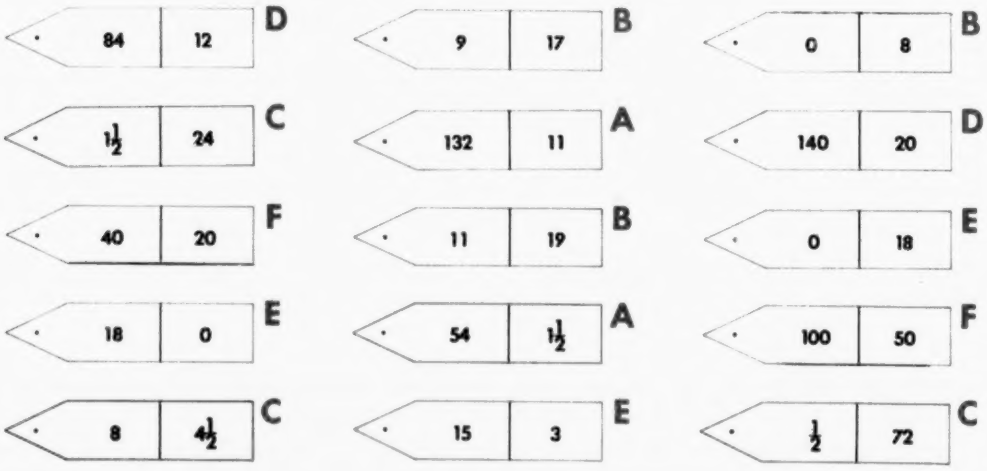


Fig. 3

## EVALUATION

If the children can match the tags and tables in somewhat less than two minutes each and enjoy preparing for the game and competing in it, I consider the experience well worth while.

# EXPERIENCE 4

## Collecting and Organizing Data in Tables

### OBJECTIVE

The student should be able to collect simple data and organize the material in tables.

### MATERIALS

3 6-ft. carpenter's rules or tape measures
2 stopwatches or watches with sweep-second hands
4 data-recording forms (from Teaching Package)
2 worksheets for each student

### TEACHER STRATEGY

I like to begin the experience by announcing that today we are going to be playing the roles of junior scientists or engineers. We are going to gather some facts and later examine what we find.

Since some students will be using the stopwatches and others will be using carpenter's rules, I take some time to demonstrate how these work to be sure the students can use them properly. Then I pass them around the room. The students should all get experience opening and closing the carpenter's rule. They also need practice in starting, stopping, and resetting the stopwatch.

Have the students group themselves in four committees, and have each committee select its own chairman.

*Committee 1* will need a stopwatch and will work in the classroom. The task of its members is to measure each other's heartbeats for a period of 30 seconds and record the information on a data-recording form. As far as possible let the students decide how they are going to perform their task. If they are floundering after several minutes of discussion, remind them of pulse points at the wrist and temple. However, remain in the background as much as possible. Give your students adequate time to build confidence in their ability to collect data first-hand.

*Committee 2* will need a stopwatch and a carpenter's rule. Its members will work in the hallway, in the gym, or on the sidewalk outside the building. Their task is to record the time it takes each person in the group to walk a distance of 52 feet, 10 inches (approximately one one-hundredth of a mile). Part of the task is laying off the prescribed distance, using the carpenter's rule. Let them decide among themselves how to do this. Results are to be recorded on a form.

*Committee 3* will need a carpenter's rule and will be working outside the school building. Its task is to measure the length of two separate blocks (curb to curb) in the neighborhood of the school and record the results on a form.

*Committee 4* will need a carpenter's rule. Its task is to measure the bumper to bumper lengths of five different models of standard cars and three different small (foreign) cars. Encourage its members to split up the work so that some are responsible for the regular cars while others are responsible for the small cars. The teachers' parking lot might provide the best place for this committee to work. Be sure to obtain permission to have students working there.

When the committees have completed their tasks and returned to the classroom, each chairman will report on the findings of his committee. Have these results placed on the board and discuss them. Get students thinking about how we might use the data. Prod them with questions. For example:

Ask Committee 1: "Ever wonder how many times your heart beats in an hour? In a day? In a year? Would my heartbeat (as an adult) be the same as yours?" Suggest that they might want to have your heartbeat measured and recorded.

Ask Committee 2: "How long do you think it might take you to walk a mile? Anyone here know how many feet are in a mile? There are 5,280. What part of a mile is the distance of 52 feet and 9.6 inches? It is one one-hundredth of a mile. How far do you think you could walk in an hour if you didn't stop along the way?"

Ask Committee 3: "If parallel parking is required, how many cars can park, do you think, on one side of the two blocks you measured? Are all blocks the same length? Do you think all blocks are square in shape?"

Ask Committee 4: "About how many cars would be in a line if the line is a block long? If the line is a mile long? Do three small cars equal about two regular cars in length?"

As the discussion proceeds try to see that all students have some chance to respond. Accept the wild answers some will give as well as the reasonable guesses. The aim in the lesson is to stimulate investigation of the data—do not require precise facts in tables or in answers to the questions.

At this point in the discussion, move to the board and say that we will now try to come to some agreements about the information they have found and put our facts in the form of tables.

Ask the chairman of Committee 1 to examine his form and tell you about how many times the heart of a sixth-grader (if this is your grade) beats every 30 seconds. Suggest starting a table and put on the board the beginning of table 1, shown in figure 4 in more complete form. Begin this table by entering the table identification, two column headings, and one listing of time.

Ask the chairman of Committee 2: "Looking at your information, can you tell us about how long it takes one of your committee members to walk a distance of 52 feet, 10 inches?" Again suggest putting the information in a table, and begin the one seen in figure 4 as table 2.

Ask the chairman of Committee 3: "What did your committee find for the approximate length of a block? Let's put this in a table also." Begin table 3 and enter a 1 under the first heading.

Ask the chairman of Committee 4: "What appears to you to be the most popular length of a standard car and of a small car? Let's record the information for both kinds."

[*Continued on page 60*]

TABLE 1

| Number of Heartbeats | Time |
|---|---|
| | 30 seconds |
| | 1 minute |
| | 1 hour |
| | 1 day |

TABLE 2

| Distance Walked | Time |
|---|---|
| 52'10" (1/100 mile) | |
| ½ mile | |
| 1 mile | |
| 5 miles | |

TABLE 3

| Blocks | Length |
|---|---|
| 1 | |
| 4 | |
| 10 | |
| 20 | |
| | 1 mile |

TABLE 4

| Car | Length |
|---|---|
| 1 standard | |
| 1 small | |
| 3 standard | |
| 3 small | |
| 6 standard | |

Fig. 4

COLLECTING AND ORGANIZING DATA IN TABLES UNIT 8, EXPERIENCE 4

Name ________________________________

Use the tables you have just made (tables 1–4) to help you answer these questions.

1. What is the average number of heartbeats in 1 minute for members of your committee? ______ How many beats in 3 minutes? ______ How many beats in 30 minutes? ______

2. How can you figure out how many times your heart beats in a week?

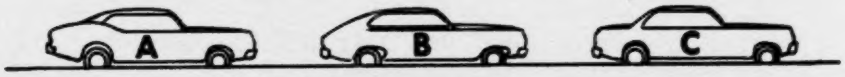
   ____________________________________________________________

   ____________________________________________________________

   See if you can now find the number of times your heart beats in a week.

   ____________________________________________________________

3. Here is a picture of 3 standard cars parked along one side of a block:



   If we allow 4 feet between Car A and Car B and 4 feet also between Car B and Car C, about how many feet is it from the front of Car A to the back of Car C? ______
   What would your answer be if these cars were all small cars? ______
   What would your answer be if Car A were a standard car and the other 2 were small cars? ______
   Would it make any difference in your answer if the standard car were parked between the 2 small cars? ______

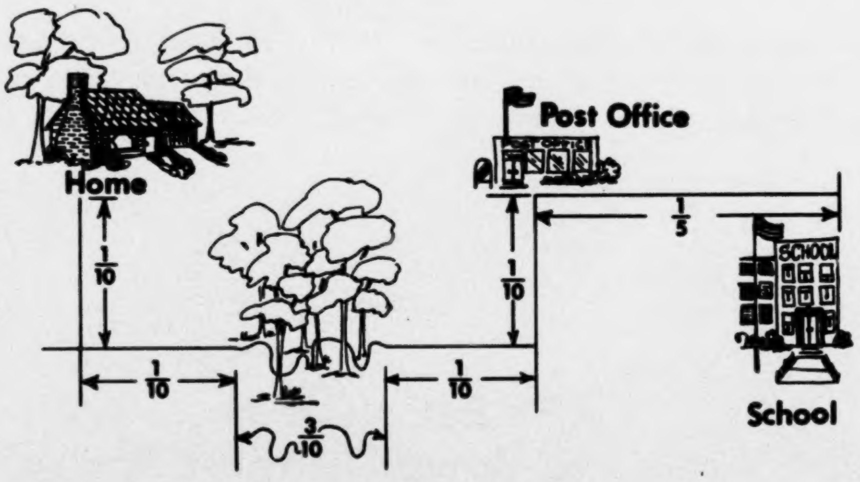
[*Continued*]

## COLLECTING AND ORGANIZING DATA IN TABLES UNIT 8, EXPERIENCE 4

*Continued* Name ______________________________

4. Use the number of feet in the length of 1 block from table 3. About how many standard cars can be parked along one side of the block if we allow 4 feet between cars? _____
   If police regulations require that we cannot park within 25 feet of a corner, how many standard-size cars can you park along 1 side of the block? _____

5. Here is a map of the walk Sam takes each day in going to and from school. All the measurements are in fractions of a mile.



Does Sam live more or less than a mile from school? _____

How long does it take the average committee member to walk a mile? _____

How long do you think it will take Sam to walk home from school? _____

When Sam starts from home how long will it take him to get from his house to the other side of the woods? _____

Direct the students to copy the beginnings of the tables from the board and add more information to them.

When the students have had ample time to complete these tables, conduct a discussion of what they have found. Explain that wrong answers should be changed, since they are going to use these tables to answer some questions later on and all will need the same data.

For the final part of this experience, give each student a copy of the worksheet and show the class how to use the tables to answer the questions. While the students are working, circulate among them to see that each gets started on the right track. When they have finished with the questions, collect the worksheets and see how each student did. When you return the papers, have the students with correct solutions explain orally to the rest of the group how they found their answers.

### EVALUATION

This experience should require the students to make some decisions about how they make these measurements and record them. Their success on the worksheet is some measure of the value of this experience.

## EXPERIENCE 5

## Using Tables to Solve Difficult Problems

### OBJECTIVE

By making and using suitable tables, the student should be able to solve three simple problems individually and, working as a member of a group, to help solve one difficult problem.

### MATERIALS

Pint jar
Lima beans
Large sheet of newspaper

Scissors
1-month sheet from calendar
Checkerboard
64 squares of paper
1 copy of each "stumper" (from Teaching Package)
3 worksheets for each student

## TEACHER STRATEGY

In the first part of this experience students work individually on three problems. They are encouraged to try to guess solutions, then told to check their guesses and if necessary to approach the problem more methodically and construct tables.

Distribute copies of the three worksheets to the students with the directions that they read Problem 1 carefully, ask questions if the problem is not entirely clear, and try to solve it. When most of the students have completed Problem 1, ask one of the students to put his solution on the chalkboard and explain it. Encourage the students to ask questions of each other and to volunteer different ways of solving this problem.

Proceed in this manner through the other problems. This part of the experience may take one class period.

In the second part of this experience divide the class into four groups. Have one member of each group reach into a grab bag for a copy of a "stumper" for his group. Each sheet lists the materials to be used and states a difficult problem to be solved. Each group is to decide for itself how to approach the problem, and the solution of the problem may take one class period.

When all groups have solved their problems and have written solutions on the sheets, call the groups together. One person from each group should state the problem and see if students in the other groups can suggest ways to solve the problem. The guesses of these students about how to solve the stumper may be recorded on the chalkboard. After a lively discussion, the group solution is placed on the chalkboard.

## EVALUATION

The manner in which the students attack and solve the problems and the stumpers should give you some insight into the value of the experience. Note particularly whether the earlier use of tables in solving problems resulted in the use of tables in solving stumpers.

## USING TABLES TO SOLVE DIFFICULT PROBLEMS UNIT 8, EXPERIENCE 5

Problem 1

Name ______________________

See if you can answer this question: "I am thinking of three consecutive even integers. The sum of the first and third of these integers is 28. What are the integers?"

Write down a few guesses. ______________________

Check your guesses to see if the sum of the first and third numbers is 28.

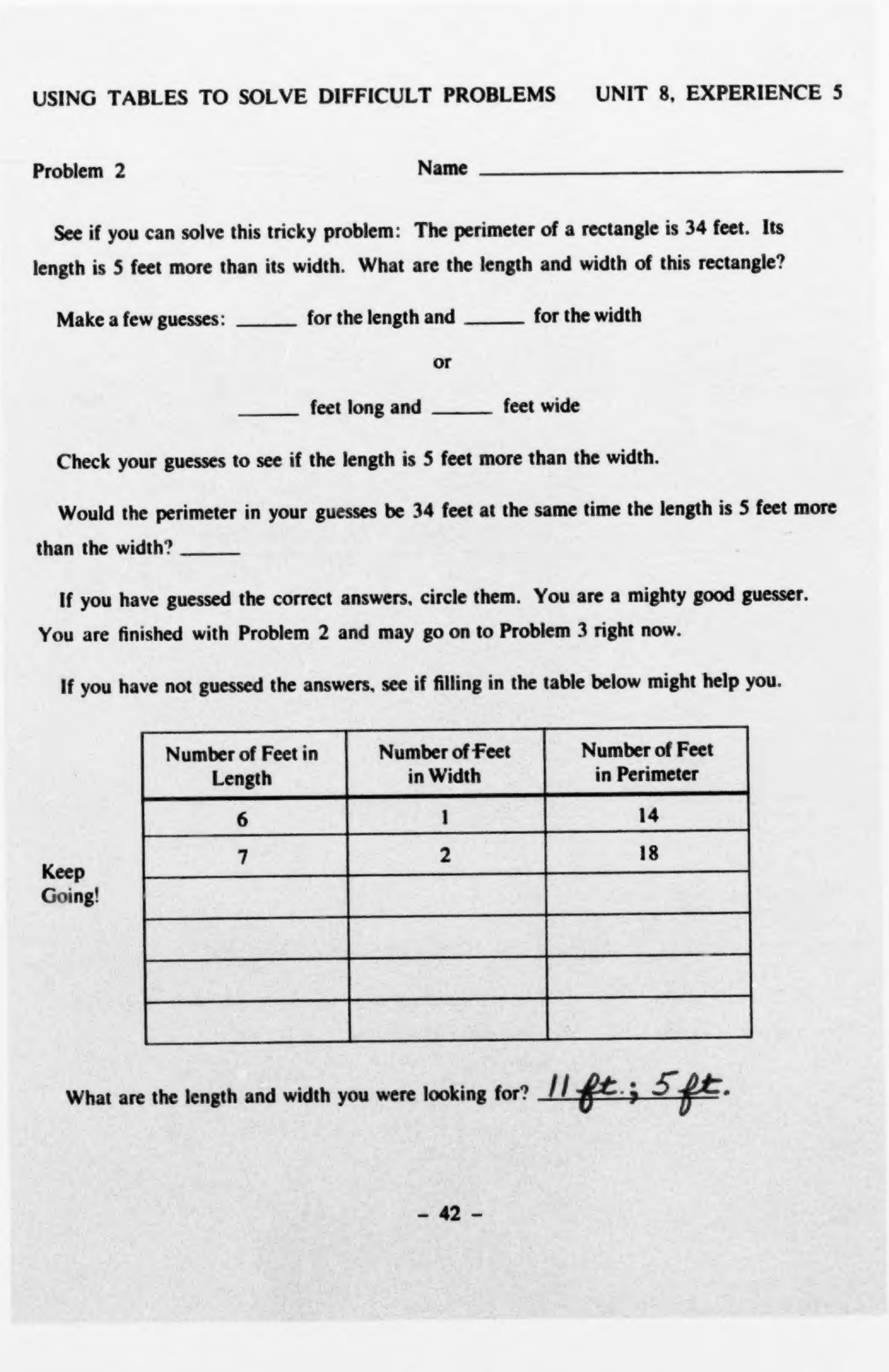
If you guessed the correct integers, circle the correct guess. Congratulations!

If your guesses didn't give the integers you were looking for, see if completing the table below will help you to find them.

| | 1st Number | 2nd Number | 3rd Number | Sum of 1st and 3rd |
|---|---|---|---|---|
| | 2 | 4 | 6 | 8 |
| Keep Going! | 4 | 6 | 8 | 12 |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

What are the three integers you were looking for? 12, 14, 16

You are finished with Problem 1. Go on to Problem 2 now.



USING TABLES TO SOLVE DIFFICULT PROBLEMS UNIT 8, EXPERIENCE 5

Problem 2 Name ______________________________

See if you can solve this tricky problem: The perimeter of a rectangle is 34 feet. Its length is 5 feet more than its width. What are the length and width of this rectangle?

Make a few guesses: ______ for the length and ______ for the width

or

______ feet long and ______ feet wide

Check your guesses to see if the length is 5 feet more than the width.

Would the perimeter in your guesses be 34 feet at the same time the length is 5 feet more than the width? ______

If you have guessed the correct answers, circle them. You are a mighty good guesser. You are finished with Problem 2 and may go on to Problem 3 right now.

If you have not guessed the answers, see if filling in the table below might help you.

Keep Going!

| Number of Feet in Length | Number of Feet in Width | Number of Feet in Perimeter |
|---|---|---|
| 6 | 1 | 14 |
| 7 | 2 | 18 |
| | | |
| | | |
| | | |
| | | |

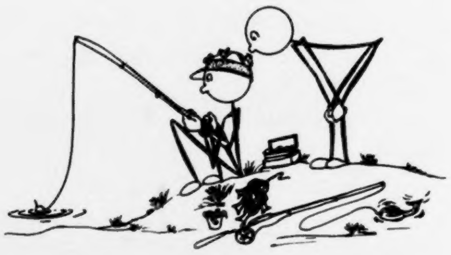
What are the length and width you were looking for? 11 ft.; 5 ft.

USING TABLES TO SOLVE DIFFICULT PROBLEMS UNIT 8, EXPERIENCE 5

Problem 3

Name ______________________

Suppose two boys, Tom and Sam, went fishing. Together they caught 6 fish. How many did Tom catch, and how many did Sam catch? See if this table, which has been started for you, helps to organize your answers.



Keep Going!

| Number of Fish Tom Caught | Number of Fish Sam Caught | Total Number of Fish Caught |
|---|---|---|
| 0 | 6 | 6 |
| 1 | 5 | 6 |
| 2 | 4 | 6 |
| 3 | 3 | 6 |
| 4 | 2 | 6 |
| 5 | 1 | 6 |
| 6 | 0 | 6 |

How many pairs of answers are correct? 7

Suppose Tom caught twice as many fish as Sam. How many fish did Tom catch? 4

Suppose Sam's mother said: "If Sam had caught 4 more fish, he would have caught the same number of fish as Tom." How many did Sam catch? 1

USING TABLES TO SOLVE DIFFICULT PROBLEMS UNIT 8, EXPERIENCE 5

Stumper 1 Name ______________________________

The equipment you may use is pencil and paper, a pint jar, and a bag of lima beans. Use any or all of it to help you solve this stumper:

The number of bugs in a bottle doubles every hour. The bottle is full of bugs at the end of one day. When was the bottle half full of bugs?

Solution: The jar is half full at the end of the 23rd hour, that is, the next to last hour.

## USING TABLES TO SOLVE DIFFICULT PROBLEMS UNIT 8, EXPERIENCE 5

**Stumper 2** Name ______________________________

The equipment you may use is pencil and paper, a large sheet of newspaper, and a pair of scissors. Use any or all of it to help you solve this stumper:

A sheet of newspaper is approximately .003 of an inch thick. Suppose it were possible to cut the sheet in half and place the 2 pieces together, one on top of the other, then cut them in half again and place the 4 pieces together in a pile, then cut them in half a third time and place the 8 pieces in a pile, and so on until the paper had been halved 25 times. How high would you guess the final pile would be? Check one of the following:

1 foot high _____ 1 mile high _____

100 feet high _____ More than 1 mile high _____

**Solution:**

Note to Teacher:

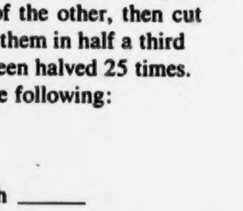
No student answer is given here because the answers vary greatly, but it should be based on the fact that each time the paper is halved the height of the pile is doubled. Thus after the 10th time the paper is halved the pile is

$$.003 \times 2 \times 2 \times . . . \text{ (10 times)},$$

or $.003 \times 2^{10}$, inches high. Note that $2^{10} = 1{,}024$, which is slightly over 1,000, so after the 10th time the pile is more than

$$.003 \times 1{,}000,$$

or 3, inches high. When the papers are halved 10 more times, the pile is over 3,000 inches, or 250 feet. When it is halved still 5 more times, the pile is $2^5$, or 32, times as high as 250 feet, which is more than 1 mile high.



## USING TABLES TO SOLVE DIFFICULT PROBLEMS UNIT 8, EXPERIENCE 5

Stumper 3 Name ______________________________

The equipment you may use is pencil and paper and a calendar page that shows one month. Use any or all of it to help you solve this stumper:

A woman was once told by her lawyer that she could pay her taxes in either of two ways. She could pay $100 a week for 3 weeks or she could pay 1¢ the first day, 2¢ the second day, 4¢ the third day, 8¢ the fourth day, and so on for 3 weeks, each day's taxes being twice that of the day before. Which way of paying her taxes should the woman choose?

Solution:

Note to Teacher:

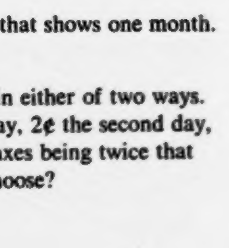
No student answer is given here because the answers vary greatly, but it should be based on the fact that

$$1 + 2 = 3, \quad 1 + 2 + 4 = 7, \quad 1 + 2 + 4 + 8 = 15,$$

and so on. Each time the sum is 1 less than twice the amount added. Thus on the 10th day $2^9$, or 512¢, was added, and so the sum is $10.23. On the 15th day we add

$$2^{14}¢ = (2^{10} \times 2^4)¢ = (1{,}024 \times 16)¢ = \$163.84,$$

and so the sum is $327.67, that is, over $300. Thus it is far less expensive to pay $100 per week for 3 weeks.



## USING TABLES TO SOLVE DIFFICULT PROBLEMS UNIT 8, EXPERIENCE 5

**Stumper 4** Name ______________________________

The equipment you may use is pencil and paper, a checkerboard, and 64 squares of paper. You may use any or all of this to help you solve this stumper:

Suppose a friend gives you this choice: "I will give you $1 for each square on the checkerboard, or I will give you 2¢ on the first square, 4¢ on the second square, 8¢ on the third square, 16¢ on the fourth square, and so on through the sixty-fourth square on the board." Which choice would you take?

Solution:

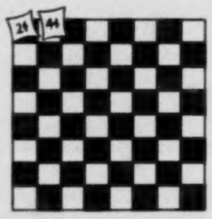
Note to Teacher:

No student answer is given here because the answers vary greatly, but it should be based on the fact that

$$2 + 4 = 6, \quad 2 + 4 + 8 = 14, \quad 2 + 4 + 8 + 16 = 30,$$

and so forth. Each sum is twice the corresponding sum in Stumper 3. Or each sum is 2 less than twice the last number added. Thus for the 10th square there will be $2^{10}$¢, or $10.24, or $20.46 for the first 10 squares. Trying a few more squares, we get a sum of $(2^{13} - 2)$¢, or $81.90, for the first 12 squares. Since this already exceeds $64.00, you would choose the second alternative. Just in case anyone asks,

$$(2^{65} - 2)¢ = 536{,}870{,}910¢ = \$5{,}368{,}709.10.$$



# 9

# EXPERIENCES WITH Ratio

The purpose of this unit is to provide students with some experiences related to the concept of ratio. While ratio provides a method for solving many types of problems, the emphasis in this unit is on its applications in drawing to scale. Scale drawings play an important role in many occupations. The carpenter, the plumber, the electrician, the sheet-metal worker, and the pattern-maker must all be able to read blueprints. The taxicab driver, the truck driver, and the service-station attendant must be able to utilize road maps, which are common examples of scale drawings.

In this unit extensive use is made of two-column tables in which a pair of numbers represents a ratio. The strategy also utilizes an overhead projector in connection with three of the experiences. The length of the object and the length of its image on the wall form a pair of numbers which illustrates the concept of ratio. This object-image theme is continued throughout the unit.

Several properties of ratio tables arise naturally from experiences in this unit. The following tables illustrate four different situations.

In one experience, with an overhead projector, the students notice that each image is exactly four times as long as the object. This is shown in table 1.

TABLE 1

| Object Length | Image Length | |
|---|---|---|
| 3 | 12 | (4 × 3) |
| 5 | 20 | (4 × 5) |
| 6 | 24 | (4 × 6) |

In another experience students discover that they can form new pairs in a table by adding the respective terms of existing pairs, as shown in table 2.

TABLE 2

| Object Length | | Image Length | |
|---|---|---|---|
| 2 | | 6 | |
| 4 | | 12 | |
| 6 | (2 + 4) | 18 | (6 + 12) |

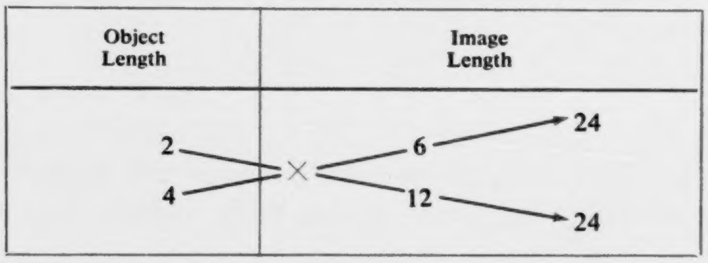
New pairs can also be formed by multiplying both members of a pair by the same number. See table 3.

TABLE 3

| Object Length | | Image Length | |
|---|---|---|---|
| 4 | | 8 | |
| 12 | (4 × 3) | 24 | (8 × 3) |

Another property of ratio which they may discover from these experiences is that the cross product of any two pairs is the same. See table 4.

TABLE 4



A laboratory approach is used in the experiences in this unit. Students are given specific tasks and expected to complete them with a minimum of teacher involvement. It is necessary that students have some primitive skill in measuring objects. They are allowed to complete the tasks at their own rates, and additional materials are provided to students who finish early.

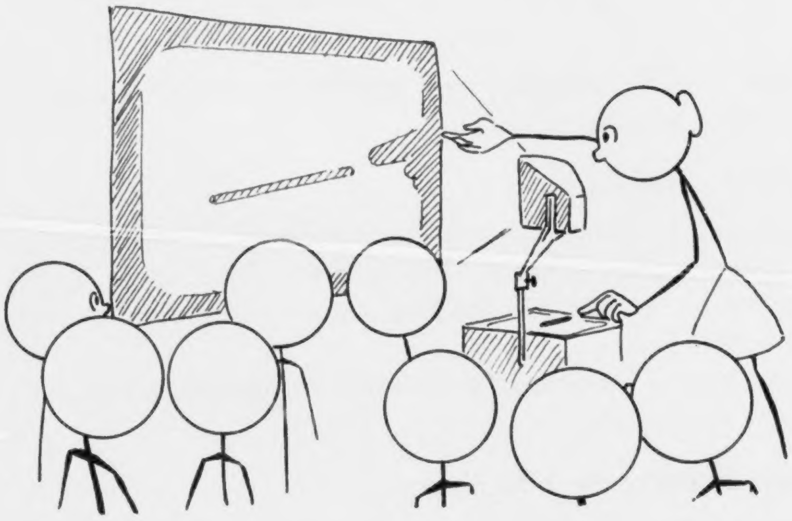
## OVERVIEW

Each of the six experiences in this unit includes a detailed section called "Teacher Strategy," which is usually one teacher's first-person account of a procedure that proved effective with his class.

The following brief summary will help you decide whether these experiences meet the needs of your own class and will also give you some indication of the amount of preparation involved. Student worksheets and other "handout" materials are provided in the Teaching Package. It is assumed that duplicating facilities are available to you.

### Experience 1: Find the Image

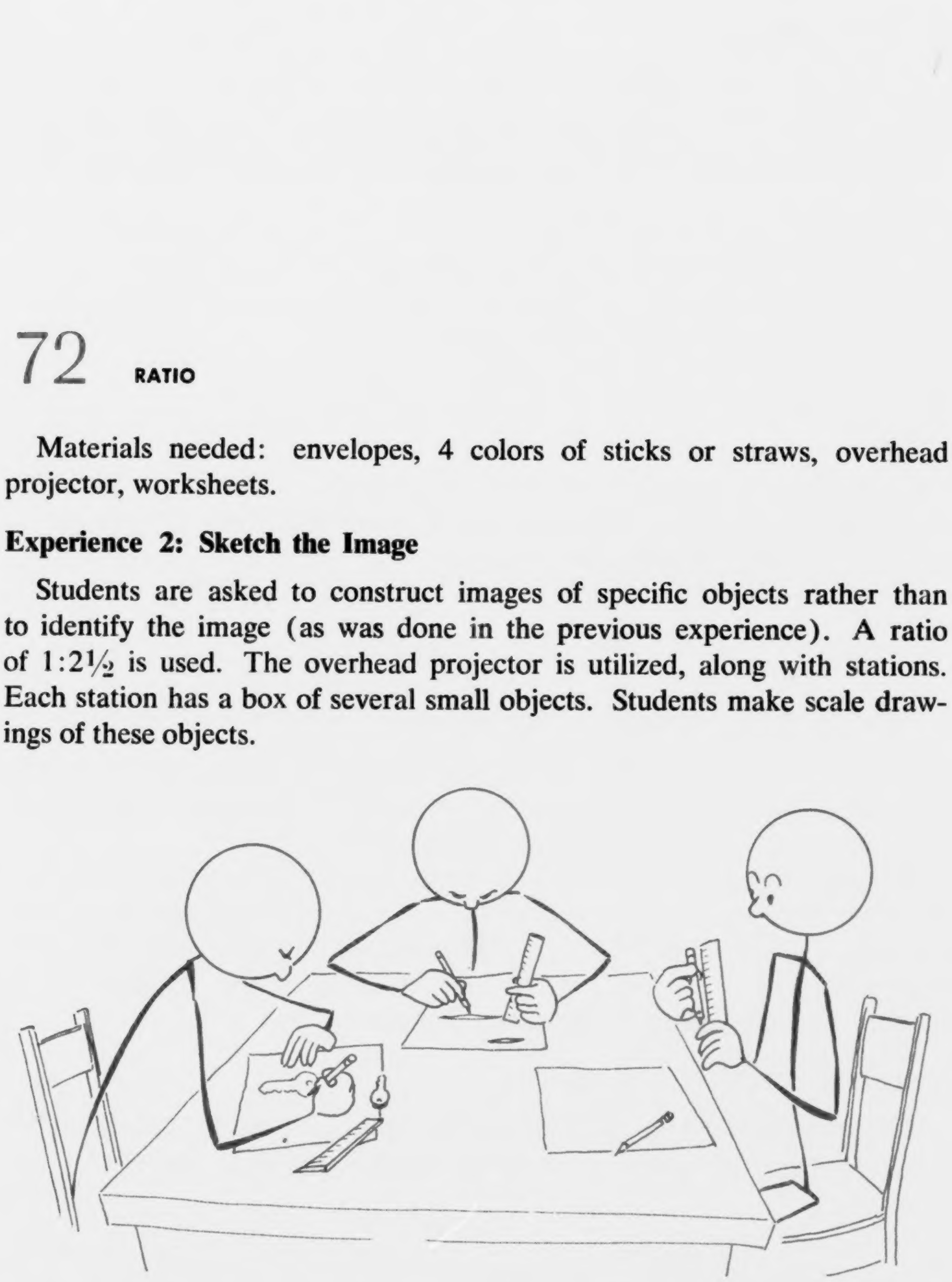
This experience serves to introduce the idea of a ratio by the use of an overhead projector. The object-image terminology used throughout the unit is developed here. Students are asked to match different colored sticks or straws with their images for a given ratio. Ratios of 1:2, 1:3, and 1:4 are used.



Materials needed: envelopes, 4 colors of sticks or straws, overhead projector, worksheets.

**Experience 2: Sketch the Image**

Students are asked to construct images of specific objects rather than to identify the image (as was done in the previous experience). A ratio of 1:2½ is used. The overhead projector is utilized, along with stations. Each station has a box of several small objects. Students make scale drawings of these objects.



Materials needed: unlined paper, small boxes, small objects, overhead projector, transparencies, string.

**Experience 3: Find the Scale**

Students use a centimeter ruler to measure line segments and their images. In some cases either the length of the object or the length of the image cannot be determined. Students record information in ratio tables and determine the ratio.

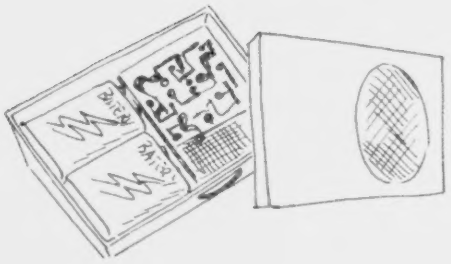
Materials needed: overhead projector, transparency, metersticks, centimeter rulers, worksheets.

**Experience 4: Recognize Like Shapes**

In this experience students work with regions rather than with one-dimensional segments. Six stations are established. Students are required to match objects with their images.

Materials needed: poster board, worksheets.

### Experience 5: Enlarge a Printed Circuit

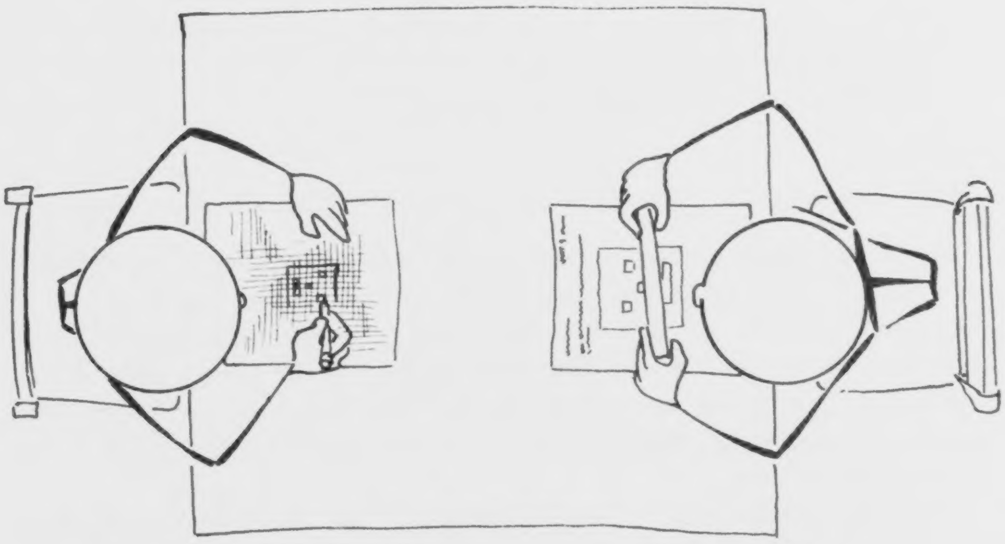


Students construct drawings, to scale, of printed circuits.

Materials needed: graph paper, worksheets.

### Experience 6: Shrink a Head

Students construct an image that is smaller than the object. A scale of 1 centimeter to ¼ inch is used.



Materials needed: graph paper, centimeter rulers, worksheets.

## EXPERIENCE 1
## Find the Image

### OBJECTIVE

For a given line segment the student should be able to identify its image when the object-image ratio is 1:2, 1:3, or 1:4.

## MATERIALS

9 envelopes
36 straws (red, blue, green, and yellow)
Overhead projector
3 worksheets for each student

**Stations.** Label 3 envelopes for each station as follows: Station A, Ratio 1:2; Station B, Ratio 1:3; Station C, Ratio 1:4. In each envelope place a set of 4 straws of different colors and lengths. Those to be used here are red, 1 inch; blue, 1½ inches; green, 2 inches; and yellow, 2½ inches. Place envelopes at the proper stations.

You may change the number of stations, envelopes, and straws to fit your class situation. If straws of different colors are not available, you may label the straws with numerals.

## TEACHER STRATEGY

You could begin by holding up a straw and announcing, "We call this the object." Then place the straw on the projector and call attention to the "image" on the chalkboard. Move the projector until your students agree that the image is about four times as long as the object, draw a line segment on the chalkboard which matches the length of the image, and check it by placing the straw at one end of the segment and flipping it end over end three times.

After the correct ratio of 1:4 is obtained, use a different convenient ratio such as 1:7. I devote about ten minutes to this demonstration.

After the demonstration I give the students worksheets, which they complete as they visit the three different stations. The student identifies the correct image on his worksheet for each of the straws for the ratio indicated at that station. He indicates this by writing the color of the straw (or the numeral on the straw) in the circular region nearest the correct image on the worksheet.

## EVALUATION

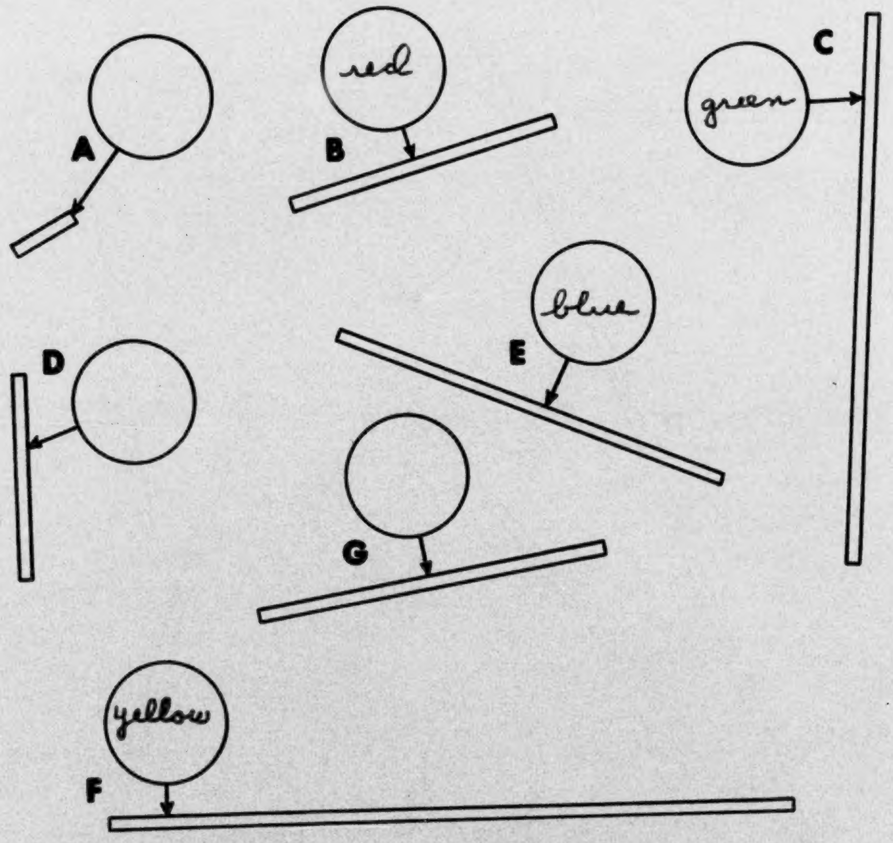
The activities in this experience are not too difficult for students. Most of the students should be able to correctly identify the images. Some of the line segments on the worksheets do not correspond to any of the objects. This is necessary to keep students from ordering the lengths of the

[*Continued on page 78*]

**FIND THE IMAGE** **UNIT 9, EXPERIENCE 1**

Station A Name ______________________________

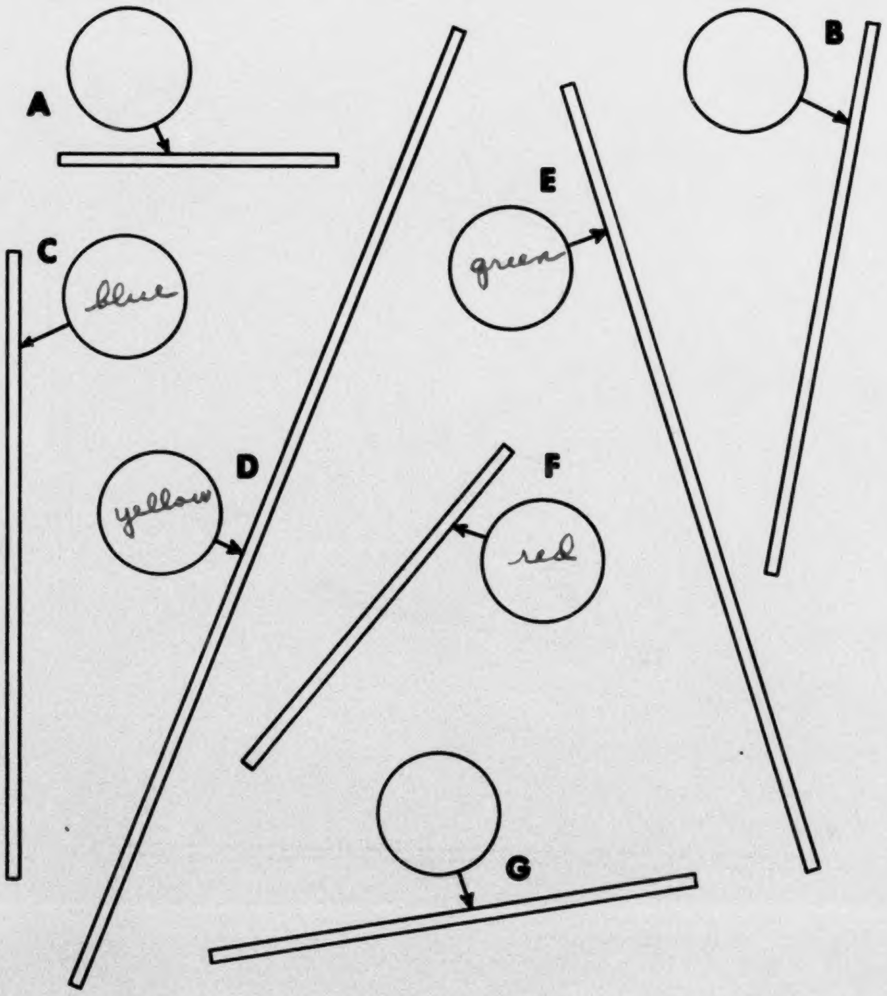
Each straw in one envelope has an image on this sheet if the object-image ratio is 1:2. Write the color of the straw or the numeral written on it in the circular region nearest the image.


A
red
B
green
C
blue
D
E
G
yellow
F

**FIND THE IMAGE** **UNIT 9, EXPERIENCE 1**

Station B Name ______________________________

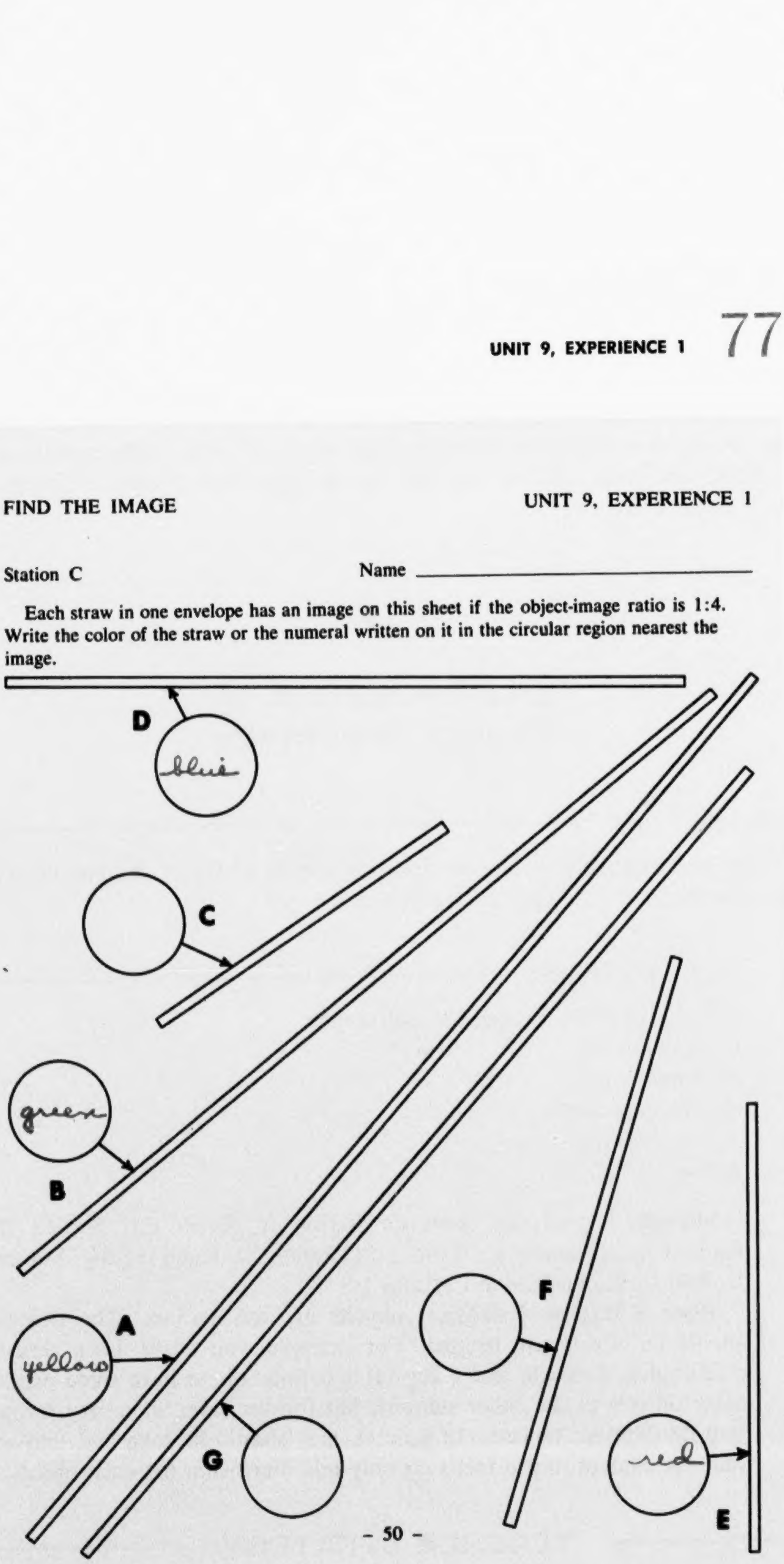
Each straw in one envelope has an image on this sheet if the object-image ratio is 1:3. Write the color of the straw or the numeral written on it in the circular region nearest the image.


A
B
C
blue
E
green
D
yellow
F
red
G

**FIND THE IMAGE** **UNIT 9, EXPERIENCE 1**

Station C Name ______________________

Each straw in one envelope has an image on this sheet if the object-image ratio is 1:4. Write the color of the straw or the numeral written on it in the circular region nearest the image.


D
blue
C
green
B
A
yellow
F
G
red
E

objects and of the images and setting up a correspondence. Students may use the flip-over technique to make comparisons between object and image.

Make sure that students are able to use the terms "object," "image," and "ratio" correctly.

## EXPERIENCE 2

## Sketch the Image

### OBJECTIVE

The student should be able to construct simple drawings of small objects using ratios of 1:1, 1:1½, 1:2½, 1:3, and 1:5.

### MATERIALS

6 sheets of unlined paper for each student
6 small boxes
24 small objects
Overhead projector
Transparencies
String

**Stations.** Label the boxes as Station A, Ratio 1:1; Station B, Ratio 1:1½; Station C, Ratio 1:2; Station D, Ratio 1:2½; Station E, Ratio 1:3; and Station F, Ratio 1:5.

Place a box of 4 different objects at each station. The objects should be of different lengths. For example, you might use a pencil, a toothpick, a match, and a key. It is permissible to have some of the same objects at the other stations, but for the sake of variety try to find 24 different objects. In general, they should be long and thin so that the student has to focus on only one dimension for each object.

### TEACHER STRATEGY

Place a small object, such as a pencil, on the stage of the overhead projector. Project the image on the chalkboard. Explain that you are going to push the projector back and forth until the length of the image is five and one-half times the length of the actual pencil (the object). When students agree that the image is about that long, draw on the chalkboard a line segment that is the length of the image. Now compare the length of the object and the length of the image. Move the projector until the correct ratio (1:5½) is achieved. Then, without moving the projector, place another object on the stage. If you use a six-inch ruler, the image should be thirty-three inches long.

Now move the projector closer to the chalkboard to form a different ratio, such as 1:2½. Use two or three different objects to illustrate the relationship between object and image for this setting.

Instruct the students to visit all six stations in any order and to draw the image of each object in the box on the unlined paper, using the ratio indicated on the box. They should label their drawings with the appropriate station letter and the name of the object. The images need not contain all the details of the objects.

I find this experience quite easy for most of the students. The 1:1½ and 1:2½ ratios are difficult for some students.

## EVALUATION

You may wish to cut several pieces of string, each of which has a length equal to the sum of the lengths of all the correct images for the objects in a particular box and hence for a particular ratio. Students who finish early can check to see whether the sum of the lengths of their images is the same as the length of the string. This is a partial check on their accuracy. You may also ask them to find the sum of the lengths of the objects and the ratio of this sum to the corresponding sum of the lengths of the images. They may discover the concept illustrated in table 2 in the introduction to this unit.

I make several transparencies with correct drawings of the images. Students place these on top of their drawings as a further check of their work.

# EXPERIENCE 3
## Find the Scale

### OBJECTIVE

The student should be able to determine the missing numbers in a ratio table and find the ratio of lengths of pairs of line segments.

### MATERIALS

Overhead projector
Transparency (from Teaching Package)
2 metersticks
1 centimeter ruler for each student (from the Teaching Package)
5 worksheets for each student

### TEACHER STRATEGY

Before the class begins, set up the overhead projector to produce on the chalkboard an object-image ratio of 1:5. Place the transparency on the projector with all but the first line segment masked by a piece of paper. Since the first segment is ten centimeters long, the image on the chalkboard will be fifty centimeters long.

If the students do not already have centimeter rulers, you can make these from the pattern in the Teaching Package; cut them out and paste them on cardboard strips.

Ask a student to measure the object on the overhead with a centimeter ruler. Have another student measure the image on the chalkboard with a meterstick. Record the results on the chalkboard in a ratio table.

Pull the mask down to reveal the second line segment and ask another student to measure the object. Before the image is measured, ask students to guess its length. After the guesses are made, students should measure the length of the image and compare it with their guesses.

Continue revealing more line segments and have either the object or the image measured first. After the students record their guesses about the corresponding image or object, have the measurement made.

Ask the students to explain how they arrived at their predictions. Typical responses follow:

Since 10 gives 50, then 30 would give 50 + 50 + 50.
Everything is multiplied by 5.
Five 30s make 150.

If you feel that more explanation is needed, use the same set of line segments as images for a different ratio, such as 1:6, by moving the projector back the required distance.

The completed tables on the chalkboard might look like tables 5 and 6.

TABLE 5

| Object (cm) | Image (cm) |
|---|---|
| 10 | 50 |
| 6 | 30 |
| 13 | 65 |
| 3 | 15 |
| 15 | 75 |
| Ratio 1:5 | |

TABLE 6

| Object (cm) | Image (cm) |
|---|---|
| 10 | 60 |
| 6 | 36 |
| 13 | 78 |
| 3 | 18 |
| 15 | 90 |
| Ratio 1:6 | |

After this demonstration, hand out one or more of the worksheets. Explain that on each worksheet there are four pairs of line segments. Students are to measure the length of each object (the solid-line segment) in centimeters and the length of its image (the dashed-line segment) and record the pairs of measurements in the table. Then they are to determine the object-image ratio and write it in the table. However, to make the experience more challenging, we pretend that one of the students dropped a blob of modeling clay on each worksheet. These blobs cover parts of some objects and of some images. The students are to determine the correct length of any segments that are partially covered and draw them on the worksheets.

The students work individually. You may need to give additional instruction and practice in measuring lengths of segments with a centimeter ruler.

## EVALUATION

The objective of this experience is not related to measurement but rather to the concept of a ratio. You must determine the extent to which students are able to supply missing numbers in a ratio table.

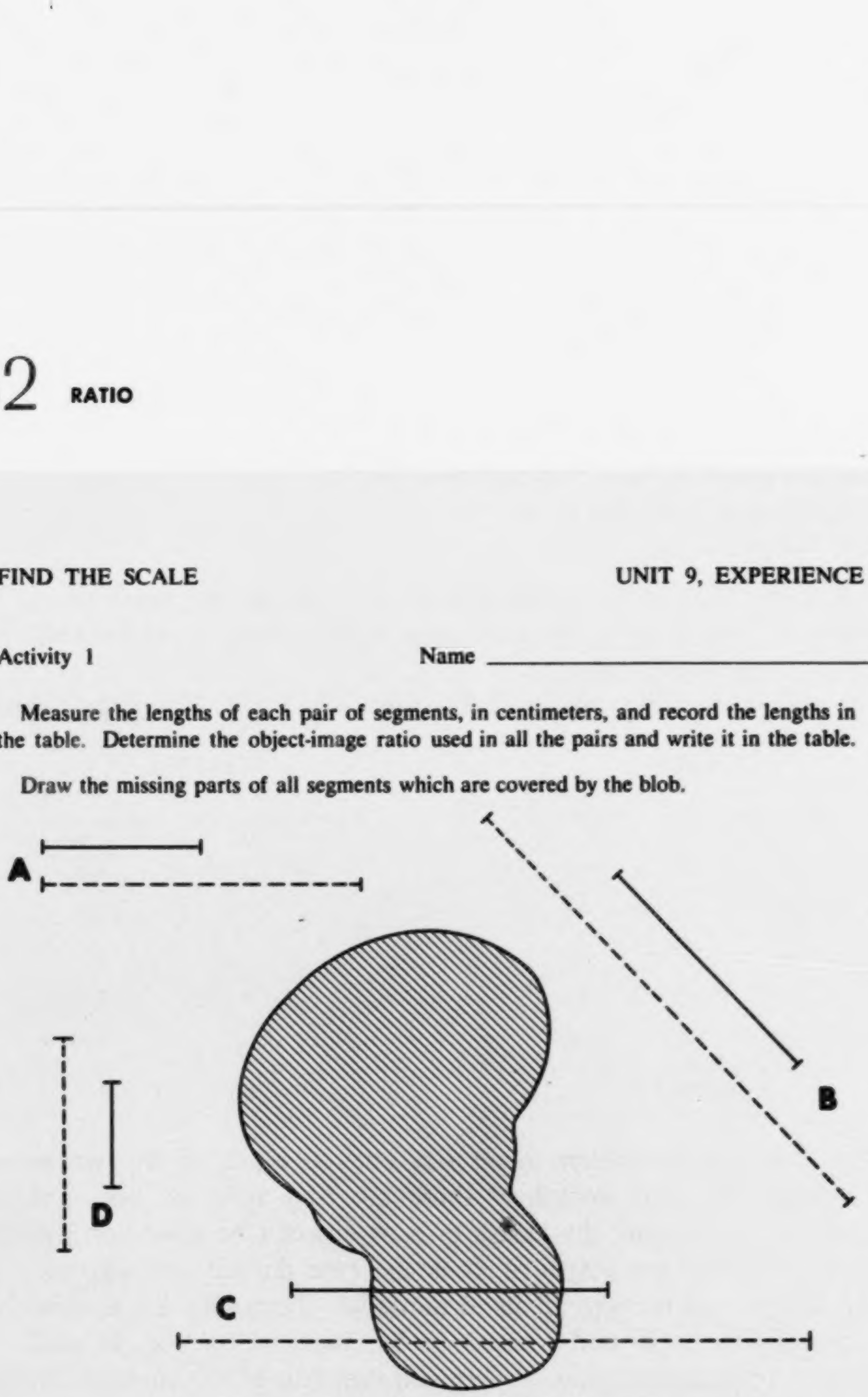
You may wish to evaluate the students' ability to fill in the blanks in a ratio table by giving a short paper-pencil test. Such a test would contain ratio tables that have some numbers supplied and some missing.

FIND THE SCALE UNIT 9, EXPERIENCE 3

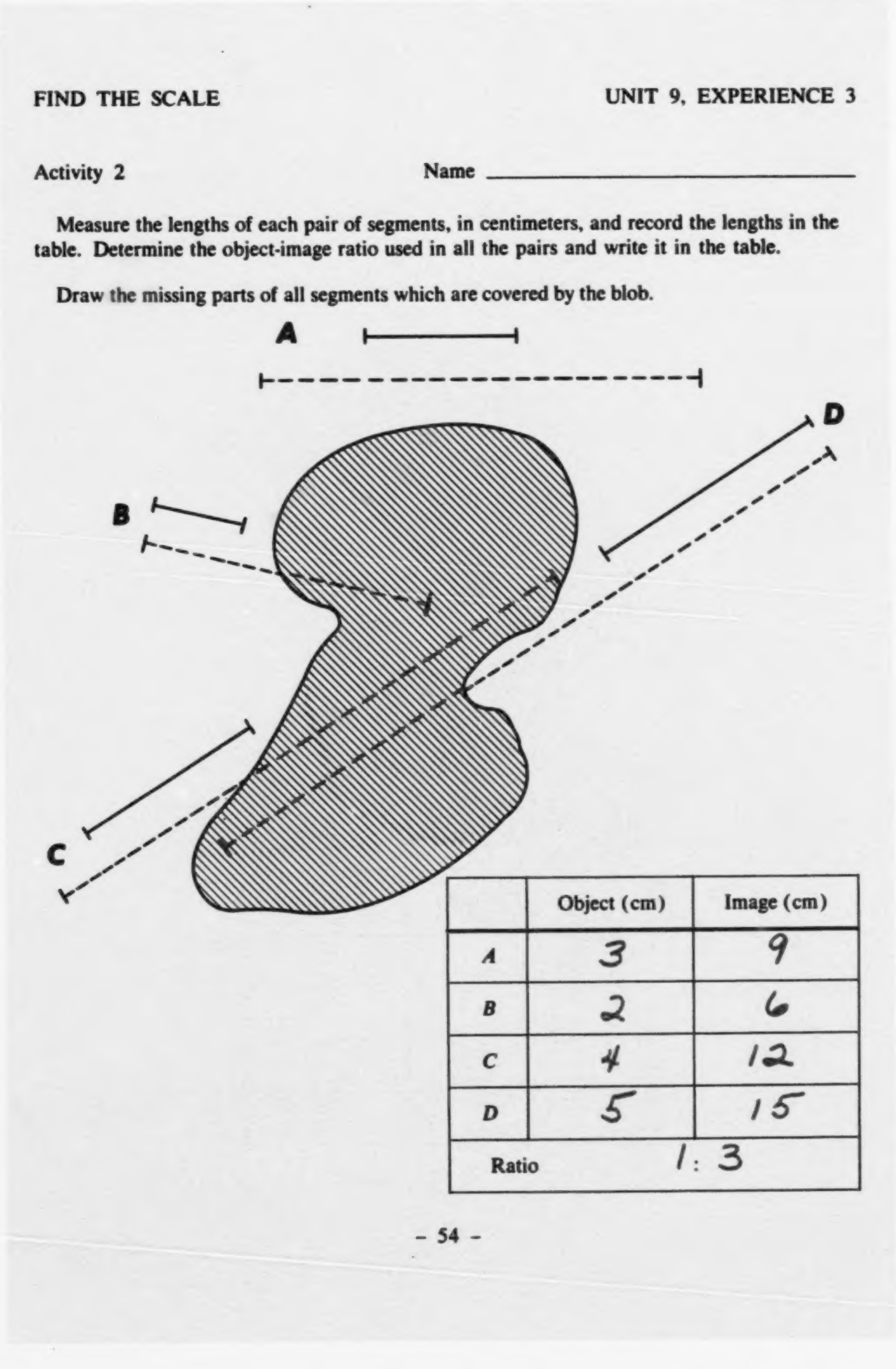
Activity 1 Name ______________________

Measure the lengths of each pair of segments, in centimeters, and record the lengths in the table. Determine the object-image ratio used in all the pairs and write it in the table.

Draw the missing parts of all segments which are covered by the blob.



| | Object (cm) | Image (cm) |
|---|---|---|
| *A* | 3 | 6 |
| *B* | 5 | 10 |
| *C* | 6 | 12 |
| *D* | 2 | 4 |
| Ratio | 1 : 2 | |



**FIND THE SCALE** **UNIT 9, EXPERIENCE 3**

**Activity 2** **Name** ______________________________

**Measure the lengths of each pair of segments, in centimeters, and record the lengths in the table. Determine the object-image ratio used in all the pairs and write it in the table.**

**Draw the missing parts of all segments which are covered by the blob.**

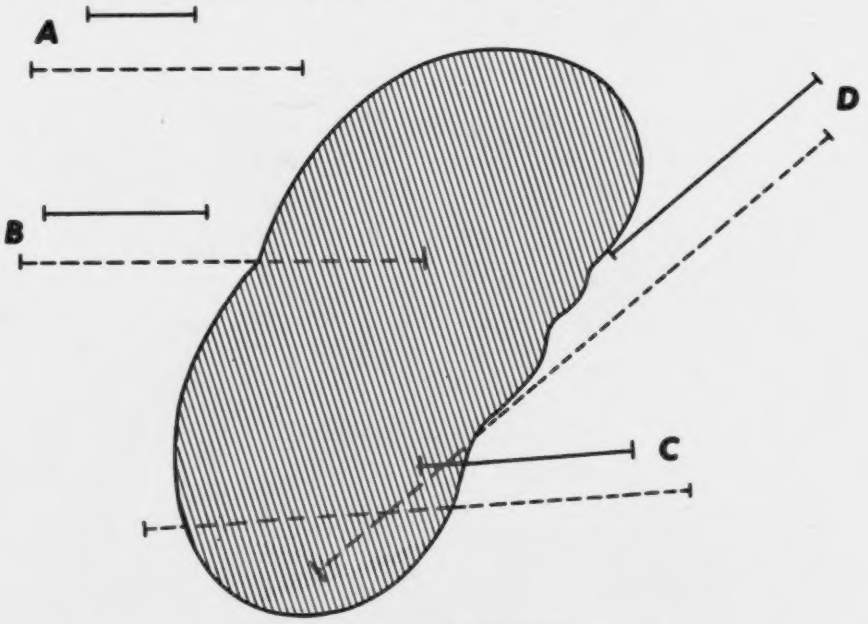
| | Object (cm) | Image (cm) |
|---|---|---|
| A | 3 | 9 |
| B | 2 | 6 |
| C | 4 | 12 |
| D | 5 | 15 |
| Ratio | 1 : 3 | |

**FIND THE SCALE** **UNIT 9, EXPERIENCE 3**

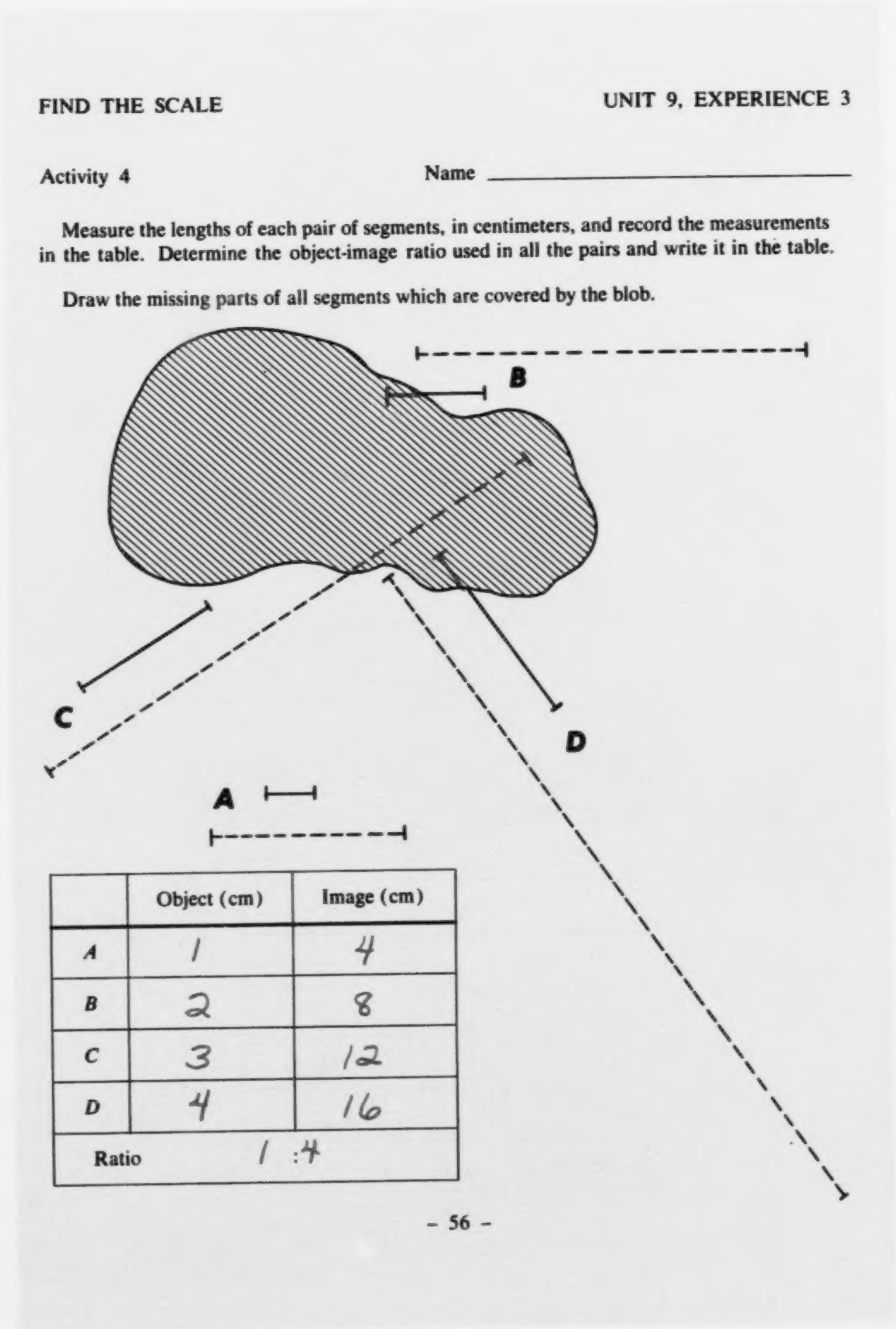
Activity 3 Name ____________________

**Measure the lengths of each pair of segments, in centimeters, and record the lengths in the table. Determine the object-image ratio used in all the pairs and write it in the table.**

**Draw the missing parts of all segments which are covered by the blob.**



| | Object (cm) | Image (cm) |
|---|---|---|
| A | 2 | 5 |
| B | 3 | $7\frac{1}{2}$ |
| C | 4 | 10 |
| D | 5 | $12\frac{1}{2}$ |
| Ratio | $1 : 2\frac{1}{2}$ | |



FIND THE SCALE

UNIT 9, EXPERIENCE 3

Activity 4

Name ______________________________

Measure the lengths of each pair of segments, in centimeters, and record the measurements in the table. Determine the object-image ratio used in all the pairs and write it in the table.

Draw the missing parts of all segments which are covered by the blob.

| | Object (cm) | Image (cm) |
|---|---|---|
| *A* | 1 | 4 |
| *B* | 2 | 8 |
| *C* | 3 | 12 |
| *D* | 4 | 16 |
| Ratio | 1 :4 | |

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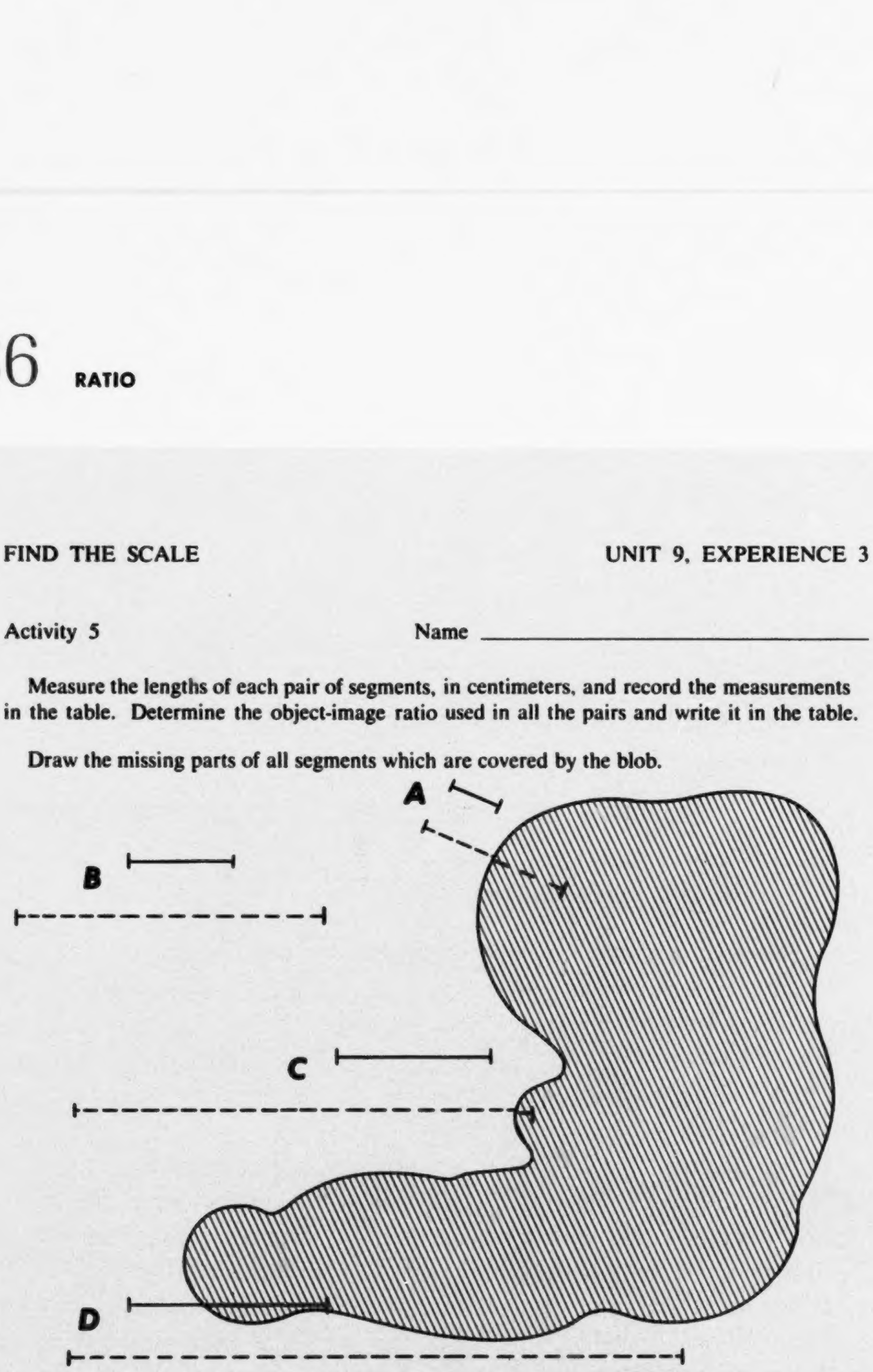
**FIND THE SCALE** **UNIT 9, EXPERIENCE 3**

**Activity 5** **Name** ______________________________

**Measure the lengths of each pair of segments, in centimeters, and record the measurements in the table. Determine the object-image ratio used in all the pairs and write it in the table.**

**Draw the missing parts of all segments which are covered by the blob.**



| | Object (cm) | Image (cm) |
|---|---|---|
| A | 1 | 3 |
| B | 2 | 6 |
| C | 3 | 9 |
| D | 4 | 12 |
| Ratio | 1 : 3 | |

# EXPERIENCE 4

## Recognize Like Shapes

### OBJECTIVE

The student should be able to recognize pairs of similar figures.

### MATERIALS

Several large poster boards
1 worksheet for each student

**Stations.** Six stations are suggested for this experience. Attach a large region cut from poster board to the wall at each station. (Suggested shapes for these regions are given in the Teaching Package.) You may wish to have one of the regions attached to the ceiling. Each such region, called an image, should be approximately 4 feet across.

For each of the first three stations cut from poster board ten small regions, some of which are similar in shape to the large region. The small regions are called objects. It is important that the intended similar parts be accurately constructed so that students will not be confused. Sample shapes, shown in the Teaching Package, should be followed on a scale such that the longest side of each region is between 2 inches and 12 inches. At each station the regions are labeled 1 through 10.

At each of the last three stations the small object regions should be glued on a large poster board in such a way that the student has to mentally rotate the figures in order to determine which ones match the large image in shape.

### TEACHER STRATEGY

In this experience you extend the idea of object-image ratio from segments to regions. I introduce the concept of like shapes (similar figures) by showing two cardboard cutouts of scalene triangular regions. The smaller one has dimensions of four, five, and six inches, while the larger one has dimensions of sixteen, twenty, and twenty-four inches. By placing the object (the smaller cutout) in front of the image, I get students to volunteer the information that they are the same shape. By placing the corresponding vertices together the students see that "the corners fit." Holding the image at arm's length and the object one-fourth that distance from my eyes and closing one eye, I point out that the regions appear to coincide. This suggests to the students that if the object is placed suitably on the overhead projector the image on the chalkboard will fit the image cutout.

I encourage the suggestion of a student that I have the class measure the lengths of corresponding edges of the regions to discover the object-image ratio of 1:4. (It should be noted the ratio of the areas is not 1:4 but 1:16.)

You could also demonstrate the ratio 1:4 of the lengths of corresponding sides by holding a piece of string along one side of the image and then folding it by putting the ends together twice. The resulting folded string should have the same length as the corresponding side of the object.

The students visit each station and indicate on their worksheet which objects have the same shape as the large image at that station. They should visit Stations A, B, and C before visiting the others.

## EVALUATION

When most of the students have finished, you may read the correct answers and discuss the problems encountered in identifying similar figures.

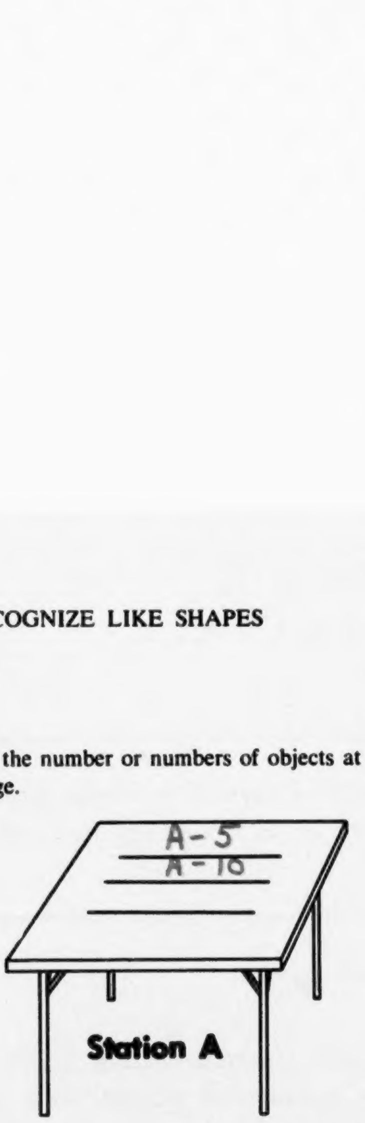
Try to determine to what extent students recognize the fact that in two similar figures all of the corresponding parts have been enlarged by the same ratio.

RECOGNIZE LIKE SHAPES

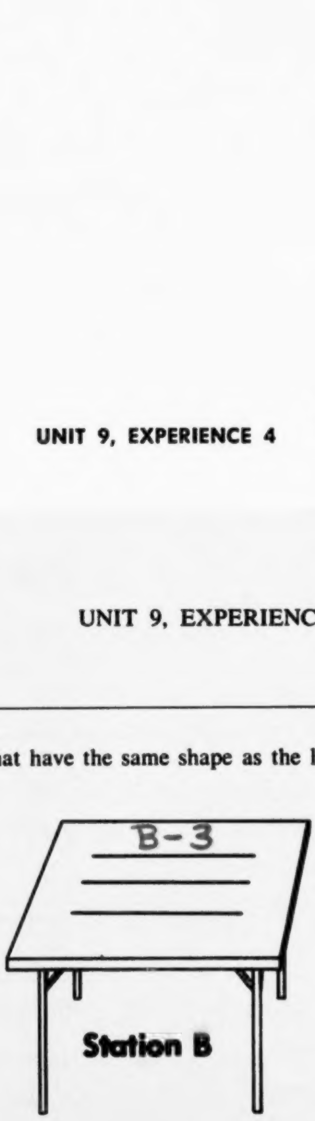
UNIT 9, EXPERIENCE 4

Name ______________________________

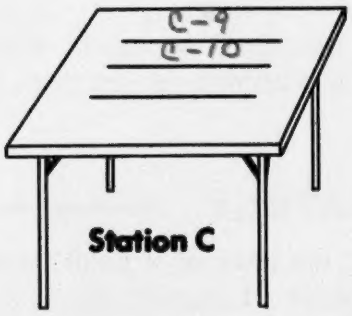
List the number or numbers of objects at each station that have the same shape as the large image.



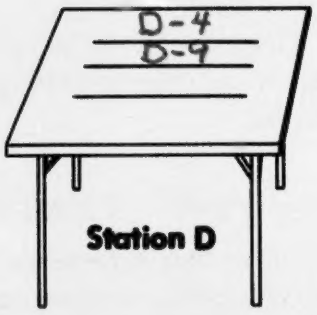
A-5
A-10



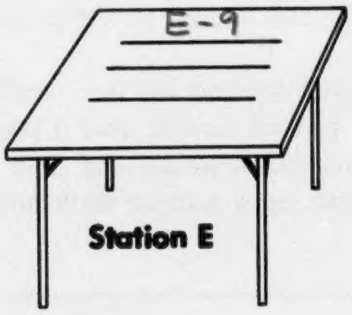
B-3



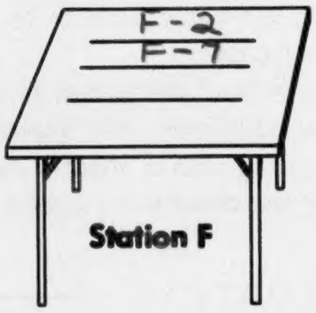
C-9
C-10



D-4
D-9



E-9



F-2
F-7

## EXPERIENCE 5
## Enlarge a Printed Circuit

### OBJECTIVE

The student should be able to construct the image of a simple geometric figure that is drawn on graph paper, using an object-image ratio of 1:3.

### MATERIALS

2 sheets of ¼-in. graph paper for each student
2 worksheets for each student

Graph paper is sometimes described as cross-section paper with squares of a certain size. For this experience quarter-inch paper should be used because the figures on the worksheets, to be enlarged, are placed on a background of quarter-inch squares.

Graph paper is inexpensive when purchased in reams. You will find many other uses for graph paper in your mathematics and science classes.

### TEACHER STRATEGY

I introduce this experience by removing the back of a small transistor radio and showing students the printed circuit. I explain that a certain electronic manufacturer desired a display model for training employees. The display would have to be made to scale and should be three times as large as the object.

After a brief discussion I distribute to each student the first worksheet for this experience, one showing a simple printed circuit, and a piece of graph paper. As a student finishes, I check his drawing and then have him try the more complicated drawing shown on a second worksheet.

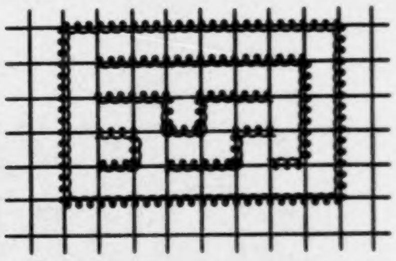
### EVALUATION

When most of the students finish making both drawings, I ask them to explain how they determined the lengths of the various segments in the first image. We also discuss how they drew the slanted line segments and the curves of the second.

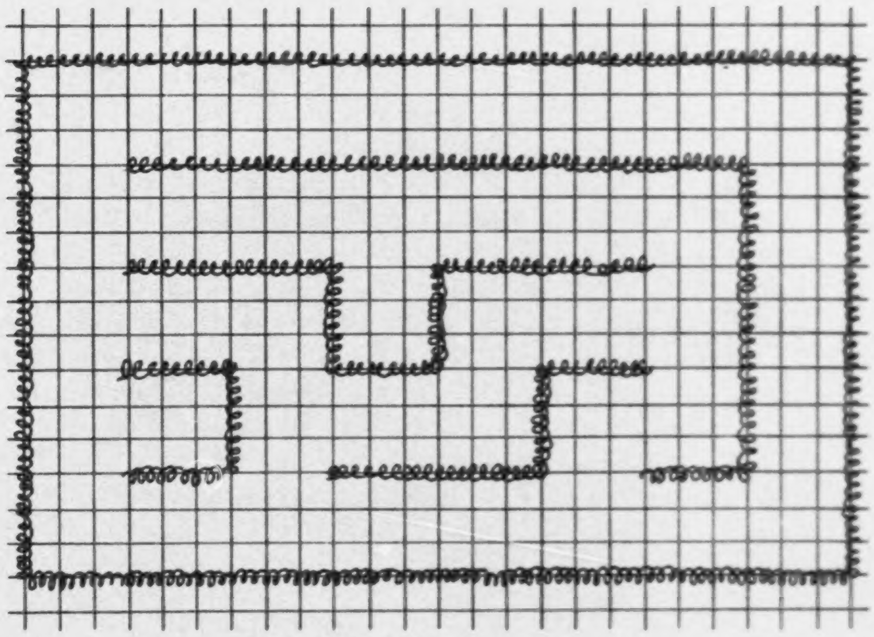
**ENLARGE A PRINTED CIRCUIT** **UNIT 9, EXPERIENCE 5**

**Activity 1** **Name** ______________________________

**Using graph paper, make an image of this object which has line segments 3 times as long.**



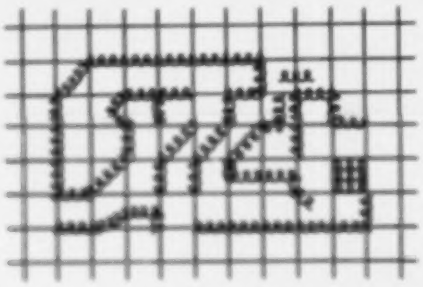
**Place your image in the space below.**



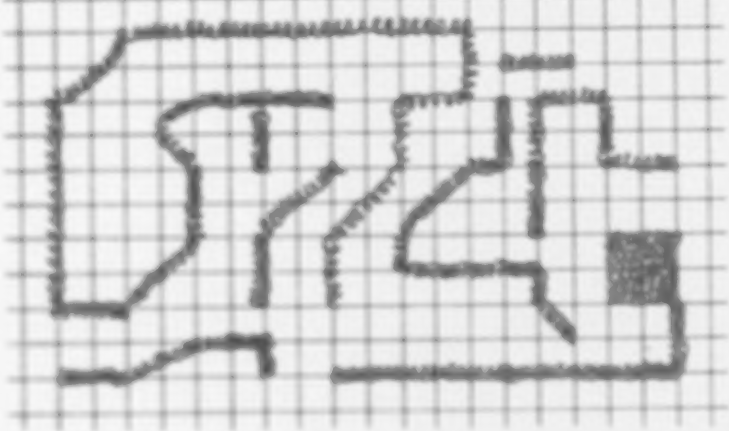
**ENLARGE A PRINTED CIRCUIT** **UNIT 9, EXPERIENCE 5**

Activity 2 Name ____________________

Using graph paper, make an image of this object in which line segments and curves are 2 times as long as they are here.



Place the image in the space below.



# EXPERIENCE 6
## Shrink a Head

## OBJECTIVE

The student should be able to make a scale drawing on quarter-inch graph paper, using a 1 cm:¼ in. ratio, of a region that has been drawn on unlined paper.

## MATERIALS

1 sheet of ¼-in. graph paper for each student
1 centimeter ruler for each student
1 worksheet for each student

## TEACHER STRATEGY

In this experience your students will make a scale drawing in which the image is smaller than the object. A line segment one centimeter long on the object is to be represented by a line segment one-fourth of an inch long on the image.

In Experience 5 the printed circuit that is to be enlarged is presented on graph paper. In this experience the object is presented on unlined paper, and the image is constructed on graph paper. It is necessary for the student to measure the length of the line segments using a centimeter ruler. The students should be encouraged to make a ratio table to make it easier to determine the length of the line segments in the image. Such a table might look like table 7.

TABLE 7

| Object (cm) | Image (in.) |
|---|---|
| 1 | ¼ |
| 2 | ½ |
| 3 | ¾ |
| 4 | 1 |
| 8 | 2 |

I introduce this experience by discussing the ancient practice of head shrinking. If a real head that is eight inches high is shrunk to one-eighth its length, then the shrunken head or image will be only one inch high. If the distance between the eyes is three inches on the object, then it will be three-eighths of an inch on the image.

You may find it interesting to discuss the distortion mirrors in fun houses, where the image is stretched in one direction and shrunk in the other direction.

Distribute the worksheet, a piece of graph paper, and a centimeter ruler to each of the students in the class. The worksheet has a picture of a "head" which is to be reduced.

It may be necessary to get the students started with the construction by suggesting that everyone construct the image of the line segment at the top of the head. You may also help the students get the figure centered on the paper. If further help is needed, you may place on the board the ratio table for this experience developed earlier (table 7) and have students fill in several additional entries.

## EVALUATION

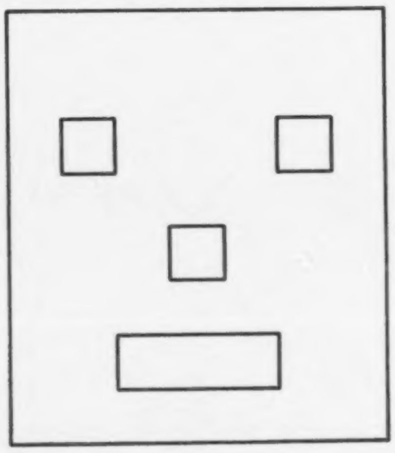
Students who finish the scale drawing of the head should be encouraged to make a drawing where the horizontal direction is stretched and the vertical direction is shrunk.

When the majority of the students have finished the original construction, place several pairs of numbers (with dimensions) on the board and have students indicate which pairs belong to the ratio table for the head shrinking.

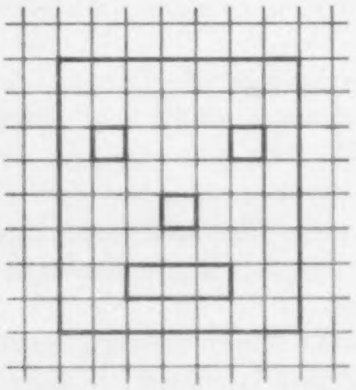
**SHRINK A HEAD** **UNIT 9, EXPERIENCE 6**

Name ______________________________

Construct the figure below on graph paper, using the ratio 1 centimeter: $\frac{1}{4}$ inch.



Place your answer in the space below.



# 10

# EXPERIENCES WITH Graphs

The purpose of this unit is to acquaint students with the usefulness of graphs to describe physical situations and relationships between two sets of numbers. To achieve this purpose, several situations have been devised to be investigated by students or teams of students at various situations in the classroom.

The uses and understandings of graphs to summarize and describe numerical data or experimental results have become more and more important in today's world. This unit will help slow learners to see how physical situations that produce numerical data can be summarized with simple geometric concepts. This linkage of concrete situations to sets of numbers to geometric concepts is a fundamental characteristic of mathematics and its applications.

The situations all involve laboratory-type, activity-oriented experiences where your role as a teacher is reduced to three major functions: one, introducing the experience and explaining the use of the materials at the stations; two, supervising the activities at each station to see that the students understand what they are supposed to do; three, summarizing the main ideas of each experience. Ideally, an outside observer would think you were doing very little. Most of the work lies in planning the experience and preparing materials. Evaluation of each experience is generated by your observations of the students' activities at the stations, by the summary period of each experience, and by the answers on the worksheets.

Graph paper with five rulings per inch is suggested for use in this unit, although quarter-inch graph paper could also be used. Graph paper is relatively inexpensive when purchased by the ream. You will find many uses throughout the year for graph paper in your science and mathematics classes.

There are four experiences in this unit; each experience will take at least one class period. The time will vary according to the background of your students.

## OVERVIEW

Each of the experiences includes a detailed section called "Teacher Strategy," which is one teacher's first-person account of a procedure that proved effective with his class.

The following brief summary will help you decide whether these experiences meet the needs of your own class and will also give you some indication of the amount of preparation involved. Student worksheets and other "handout" materials are provided in the Teaching Package. It is assumed that duplicating facilities are available to you.

### Experience 1: Making Straight-Line Graphs of Physical Data

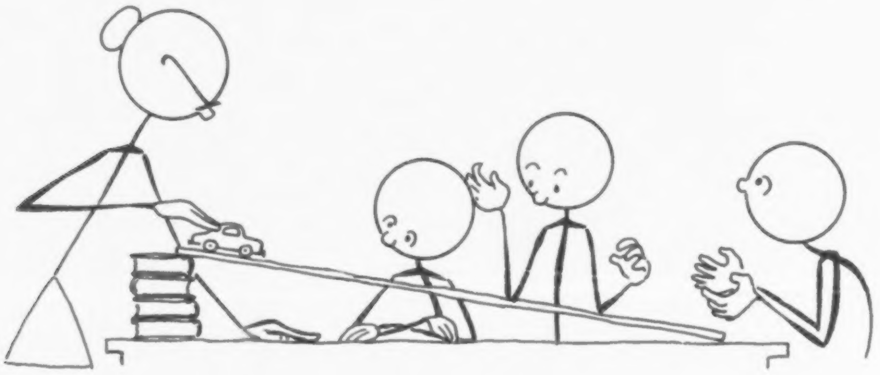
In this experience the students learn to collect data from physical experiments and to make a graph of the data. At one of the four stations they

measure in a tall cylindrical jar the height of material for the number of measures poured in and graph height against number of measures. At another station students graph the distance rolled by a cylindrical can in terms of the number of revolutions. At a third they graph the number of revolutions of a bicycle wheel against the number of revolutions of the pedals. At the last station they balance different numbers of paper clips with the appropriate number of pennies. The data at each station lead to straight-line graphs, and the students draw these.

Materials needed: graph paper, stopwatch, pendulum, straightedges, coffee measure, box of rice, olive jar, ruler, cylindrical can, chalk, bicycle, balance, paper clips, pennies, worksheets.

**Experience 2: Making Curved-Line Graphs of Physical Data**

This experience is similar to Experience 1 except that the data lead to curved-line graphs. One such graph results from a teacher demonstration of the time it takes for a ball or toy car to roll down the length of an inclined plane for the different heights to which one end is raised. There are four stations, and at the first students count the number of pieces into which a newspaper sheet is cut, in terms of the number of cuts. They then graph the resulting (exponential) graph. At another station the students time twenty swings of a pendulum for various lengths of the pendulum and graph the data. At another they determine the area of square regions in terms of the length of the diagonal and graph the data. At the last station the students measure the distance from a variable weight to the fulcrum of a balance, using a fixed number of pennies in a cup at one end of the balance and an increasing number of pennies in a cup at the other end. They graph the data. None of the graphs, of course, is a straight-line graph.



Materials needed: board, set of books of equal thickness, ball or toy car, stopwatch, newspaper sheets, scissors, pendulum, colored squares in different sizes, ruler, balance, pennies, worksheets.

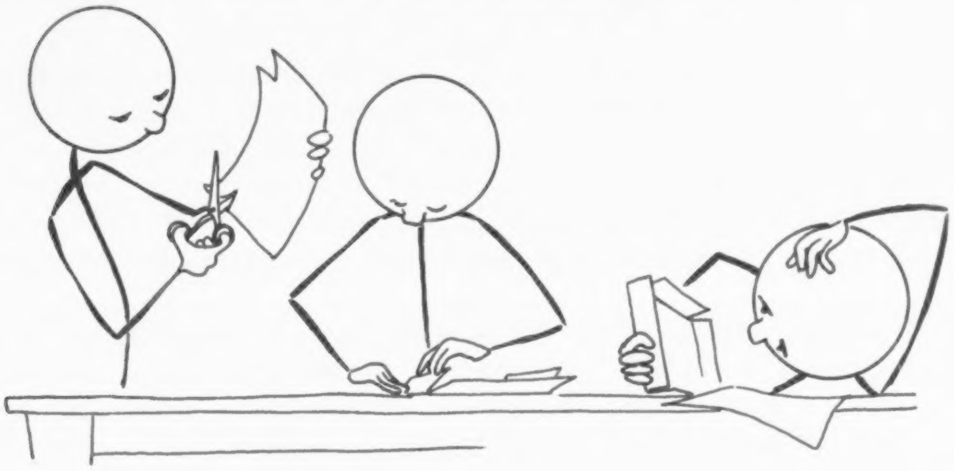
### Experience 3: Using Graphs to Determine Additional Results in Experiments

In this experience the students use graphs made in Experiences 1 and 2 to interpolate, predicting results for measures between those actually found in the experiments, and they also use graphs to extrapolate, predicting results for measures beyond those found in an experiment.

Materials needed: graphs previously made, worksheets.

### Experience 4: Choosing the Graph to Fit Data for a Physical Situation

In this experience the students carry out three simple experiments and then are asked to select the graph that fits the observed data. In some cases they have to graph the results of the experiment before making their selection. In one experiment the students remove four one-inch squares from the corners of a sheet of paper, fold up the sides, and calculate the volume of the resulting box. They repeat this, removing larger squares, and choose the one of four accompanying graphs which fits the data. At another station the students determine the time to make an eight-hundred-mile trip for various average rates of travel and select the appropriate graph. At the third station the students calculate the area of circular regions of different radii and select the graph that best fits the data.



Materials needed: paper, rulers, scissors, circular objects of varying size, worksheets and accompanying graphs.

# EXPERIENCE 1

## Making Straight-Line Graphs of Physical Data

### OBJECTIVE

The student should be able to plot points for pairs of numbers produced by physical experiments and to connect these points with a straight-line graph. He should be able to choose suitable scales and to label the graphs correctly.

### MATERIALS

Supply of graph paper
Stopwatch
Pendulum
1 straightedge for each student
4 worksheets for each student

You can make a pendulum by tying 3 or 4 feet of light string or heavy thread to a bob such as a heavy machine nut. You may hold the pendulum or suspend it from a hook.

**Stations.** Some additional materials are needed for the stations. A box of rice, a tall narrow jar such as an olive jar, a coffee measure, and a ruler are needed for Station A. A cylindrical can with a vertical line marked on it is needed for Station B, together with chalk to mark on the floor or a long roll of paper that can be laid out on the floor and marked. A bicycle is needed for Station C. Station D requires a balance, 20 pennies, and 2 boxes of paper clips. The balance may be made by tacking or taping paper cups at each end of a meterstick or yardstick and placing a fulcrum at the midpoint of the stick. A fulcrum can be made by folding and taping a piece of cardboard in the shape of a triangular prism.

## TEACHER STRATEGY

I like to introduce this experience by swinging the pendulum and asking the students to guess how long it would take for twenty swings, a complete swing being the motion from one end of the swing until it returns to about the same position. I show them how the stopwatch can be set at zero, started, and stopped. Then I time five swings and record the time on the chalkboard. I reset the stopwatch, time ten swings, and record this result on the board. I repeat this for fifteen and for twenty swings. If the length of the string from the hook to the middle of the heavy bob is about a meter, the time for a complete swing is theoretically two seconds. The results would be as shown in table 1.

TABLE 1

| Number of Swings | Time in Seconds |
|---|---|
| 5 | 10 |
| 10 | 20 |
| 15 | 30 |
| 20 | 40 |

Then I suggest that we draw a picture or graph that contains the result in the table. I prefer to work on a transparency ruled like graph paper and use an overhead projector to show the details of the graphing on the chalkboard. I draw a vertical and a horizontal line for axes and label them with the headings from the table, usually the left-hand heading on the horizontal axis and the right-hand heading on the vertical one. I show them how to plot (5, 10) on the graph and then plot the other three points. The points seem to lie on a straight line. By laying on a straightedge and drawing a line segment through the four points, we confirm the fact that they do.

Since the emphasis here is on using a pendulum of fixed length, I delay until later (a discussion of the worksheet for Experience 3) any questions about the time of swing for a different *length* of pendulum. The time of the swing depends very little upon how large a swing is made or upon the weight of the bob used.

For some classes I repeat the graph with a different choice of scales so students see that a straight-line graph still results. The graph might look like figure 1 on the following page.

Then I go to each station, point out the materials that will be used, and

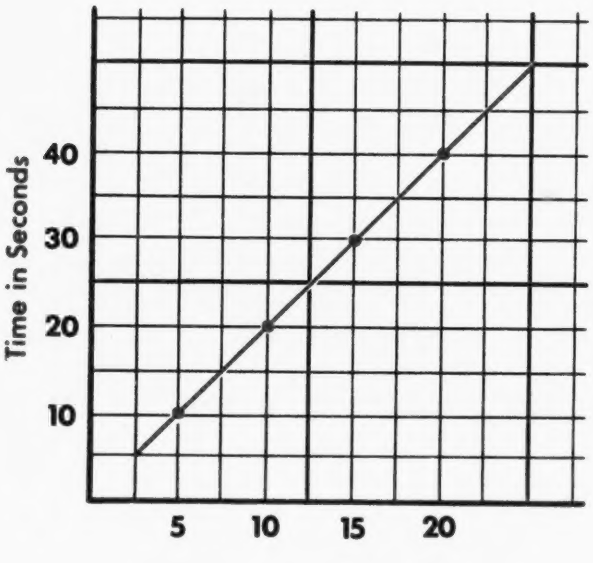


Fig. 1

explain what the students are to do. I pass out the worksheets so the students can follow the directions. I suggest that at each station two different students be chosen to perform the experiment and that everyone should record the results.

I divide the class into four teams and assign them to their starting stations. I visit the stations, giving assistance and answering questions that are raised.

After all students have completed the four graphs, I have them discuss and compare their results. I point out that all of these graphs have straight lines, but the lines on other graphs may be curved. I also point out that some straight-line graphs slope downhill instead of uphill.

These graphs are saved to be used in a later experience, so I make certain that each student has four correct graphs.

## EVALUATION

Correct tables and graphs are a good indication that the students have met the objective of this experience.

## MAKING STRAIGHT-LINE GRAPHS OF PHYSICAL DATA

UNIT 10, EXPERIENCE 1

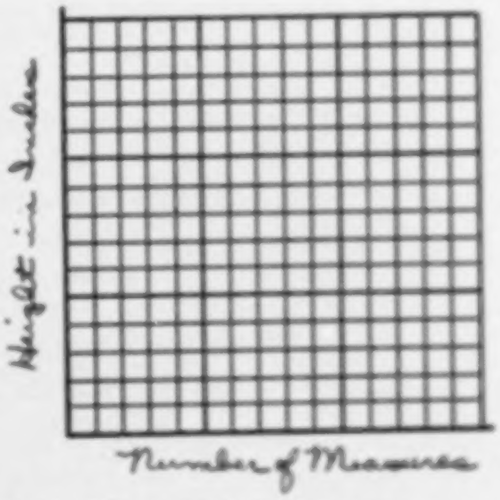
**Station A**

Name ______________________________

At this station you will find a coffee measure, a box of rice, a tall jar, and a ruler.

1. Pour 3 level measures of rice in the jar.
2. Measure the height of the rice in the jar to the nearest $\frac{1}{8}$ inch.
3. Enter the height in the table opposite the "3."
4. Repeat the above three steps for 5 measures, 7 measures, and 9 measures.
5. Label the axes of your graph, choose and mark the scale used on each axis, and plot the four points for the number pairs in the table.
6. Connect the points, using a straightedge. If they do not lie on a straight line, repeat the steps and check the measurements.
7. If you have not already visited four stations, go on to Station B.

| Measures | Inches |
|---|---|
| 3 | |
| 5 | |
| 7 | |
| 9 | |



No answers can be given here. The graph should be a straight-line segment.

MAKING STRAIGHT-LINE GRAPHS OF PHYSICAL DATA

UNIT 10, EXPERIENCE 1

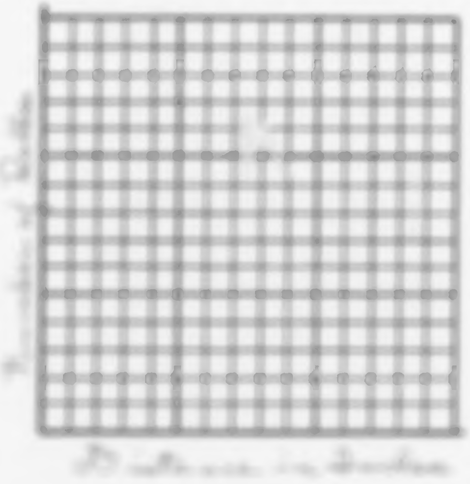
Station B

Name ______________________________

At this station you will find a can with a mark on it and a line marked on the floor.

1. Place the can so the mark on it is directly on the mark on the floor.
2. Roll the can 1 roll (1 revolution) on the floor until the mark on the can touches the floor. Mark the floor at this spot.
3. Measure the distance to the nearest $\frac{1}{8}$ inch between the two marks on the floor.
4. Enter the distance in the table opposite the "1."
5. Repeat the above steps for 2 rolls, 3 rolls, and 4 rolls.
6. Label the axes of your graph, indicate the scales, and plot the points on the graph below. Use a straightedge to connect the points.
7. If you have not already visited four stations, go on to Station C.

| Rolls | Inches |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |



No answers can be given here. The graph should be a straight-line segment.

## MAKING STRAIGHT-LINE GRAPHS OF PHYSICAL DATA

UNIT 10, EXPERIENCE 1

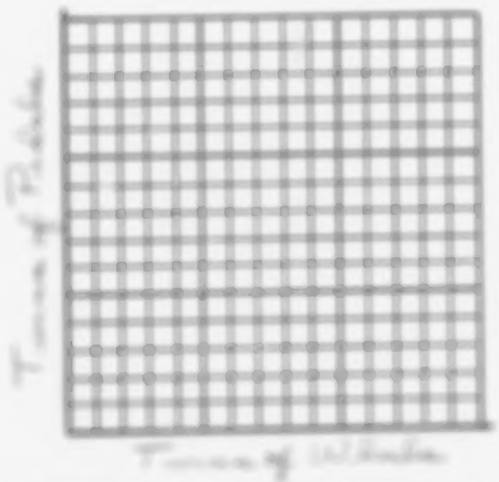
Station C

Name ______________________________

At this station you will find a bicycle placed upside down on its seat and handlebars.

1. Put the pedals in straight up-and-down position and mark the rear tire at the point directly opposite the bike's rear fork. Move the pedals slowly for safety.
2. Turn the pedals slowly 1 complete turn and count how many turns are made by the rear wheel. The mark on the rear wheel will help in counting, and fractions of a turn of the rear wheel can be counted by counting the spokes. For example, if there are 24 spokes, each one is $\frac{1}{24}$ of a turn.
3. Enter the number of turns of the wheel in the table opposite the "1."
4. Repeat the steps for 3 turns, 5 turns, and 7 turns of the pedals.
5. Label the axes of your graph, indicate the scales, and plot the points. Connect them with a straight-line segment.
6. If you have not already visited four stations, go on to Station D.

| Pedals | Wheel |
|---|---|
| 1 | |
| 3 | |
| 5 | |
| 7 | |



No answers can be given here. The graph should be a straight-line segment.

**MAKING STRAIGHT-LINE GRAPHS OF PHYSICAL DATA**

**UNIT 10, EXPERIENCE 1**

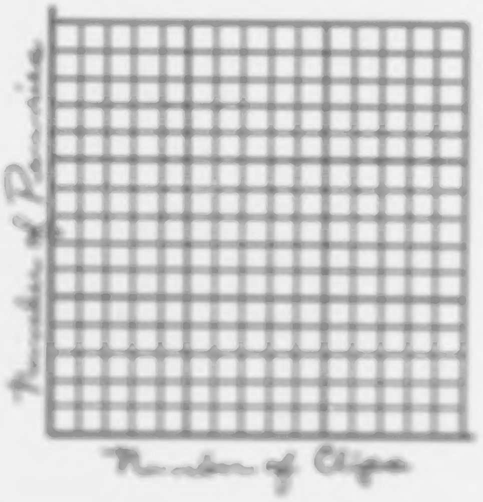
Station D

Name ______________________

At this station you will find a balance, a box of 20 pennies, and a supply of paper clips.

1. After balancing the stick place 5 pennies in one cup and balance by putting paper clips in the other cup. Count how many paper clips are used.
2. Enter the number of paper clips in the table opposite the "5."
3. Repeat the steps for 10 pennies, 15 pennies, and 20 pennies.
4. Label the axes of your graph, indicate the scale, plot the points, and connect them with a straight-line segment.
5. If you have not already visited four stations, go on to Station A.

| Pennies | Clips |
|---|---|
| 5 | |
| 10 | |
| 15 | |
| 20 | |



No answers can be given here. The graph should be a straight-line segment.

# EXPERIENCE 2

## Making Curved-Line Graphs of Physical Data

### OBJECTIVE

The student should be able to plot points for pairs of numbers produced by physical experiments and connect the points by a smooth curve.

### MATERIALS

5-ft. board
16 books of equal thickness
Ball or toy car
Stopwatch
4 worksheets for each student

**Stations.** Some additional material is needed at the stations: Station A, several large sheets of newspaper and a pair of scissors; Station B, the pendulum used in Experience 1; Station C, ruler and colored art paper cut in squares with diagonals of 3, 5, 7, and 9 inches; Station D, the balance used in Experience 1 and 40 pennies.

### TEACHER STRATEGY

I like to introduce this experience by measuring the time for a ball or toy car to roll down an inclined plane for various heights of one of its ends, making a table and a graph of the time of descent for various heights. To begin with, I pile up four books of equal thickness to put under one end of the board and measure with the stopwatch the time for the ball to roll down the board. For the second pair of numbers I double the number of books under the end of the board and measure the time again. It is less because of the greater steepness of the inclined plane. Two more measurements are made—the end of the plank is raised to three times and four times its original height. As in Experience 1, I usually use an overhead projector to show how the pairs of numbers are plotted to get a graph, somewhat as shown in figure 2 on the following page.

Obviously no straight-line segment will pass through the four points,

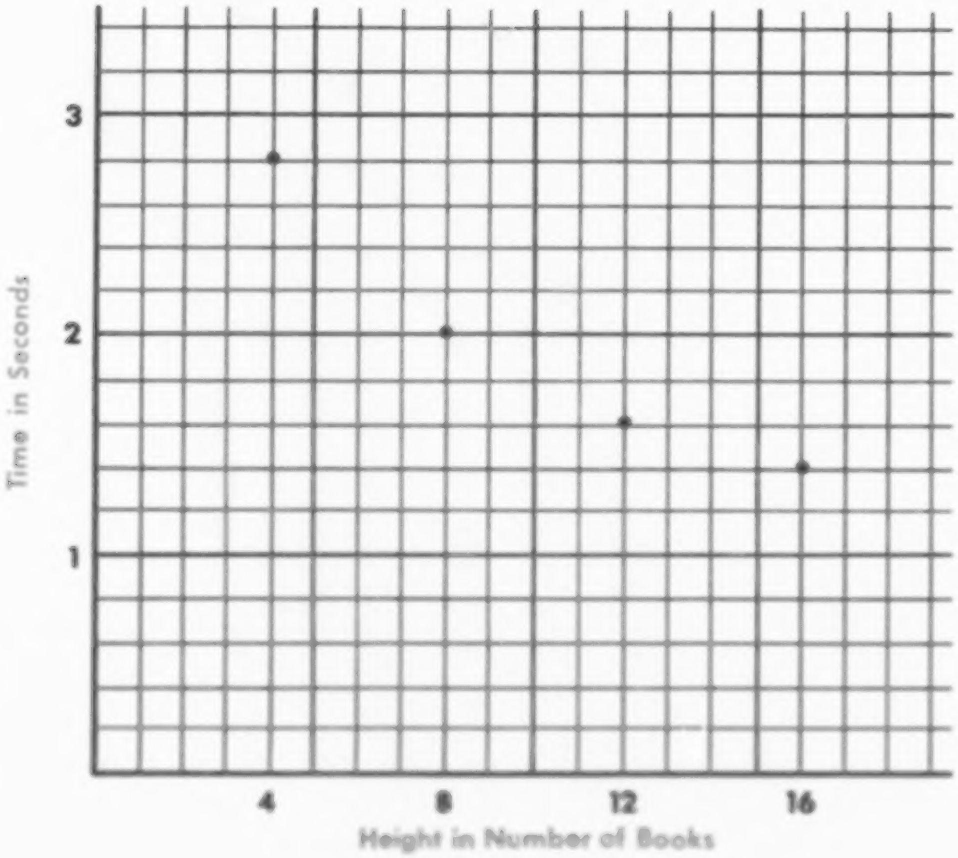


Fig. 2

so I draw a smooth curve through the points (extending it slightly beyond the plotted points in anticipation of its use in the next experience). The completed graph is shown in figure 3.

At this point I hand out copies of the four worksheets to each student and, while students follow the directions on the worksheets, describe the experiments to be performed at the stations. I then divide the class into four groups and assign them to stations to carry out the experiments and to record and graph the data. The order in which the groups visit the stations is immaterial.

A final short discussion helps assure that all students have correct graphs. These graphs will be used in the next experience.

## EVALUATION

Correct tables and graphs are a good indication that the students have met the objective.

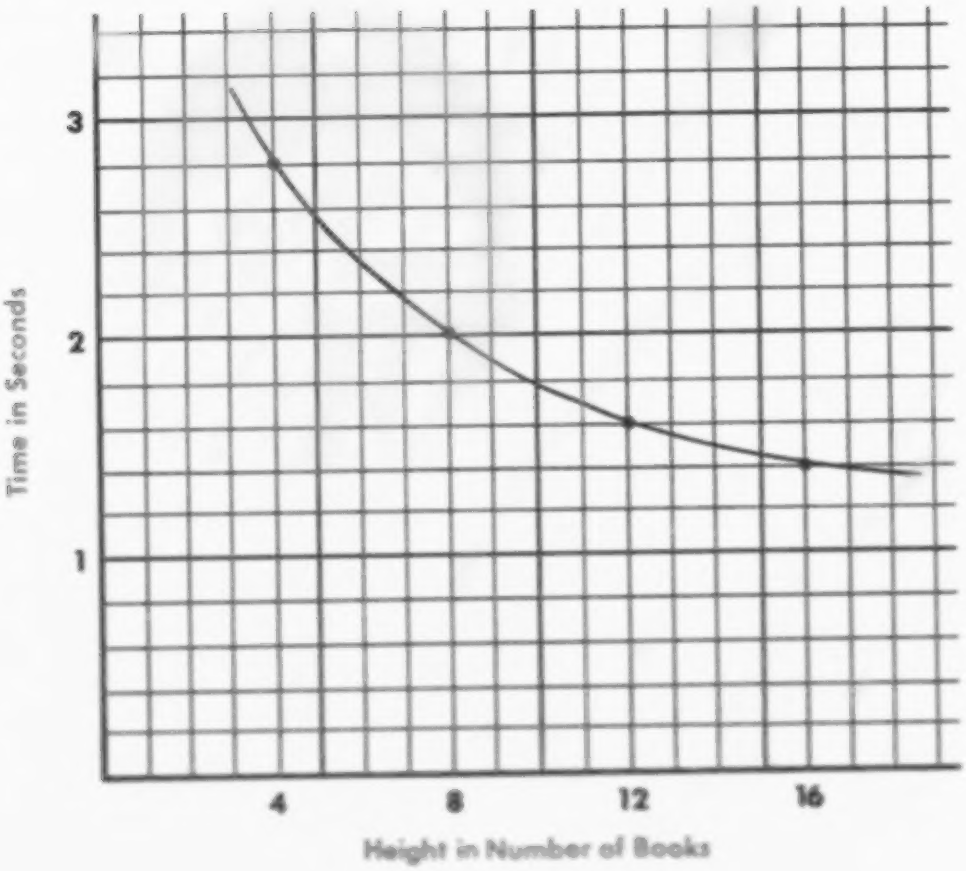


Fig. 3

MAKING CURVED-LINE GRAPHS OF PHYSICAL DATA

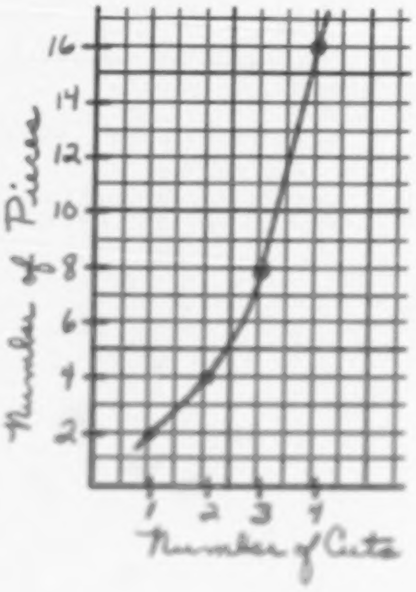
UNIT 10, EXPERIENCE 2

Station A

Name ________________________

At this station you will find some sheets of newspaper and a pair of scissors. Select one of your group to carry out the steps of the experiment, and all record the data on your sheets.

1. Take a sheet of newspaper and cut it in half. Count the number of times it was cut (1) and the number of pieces there are (2).
2. Record the number pair in the table below.
3. On your graph label the horizontal axis "Number of Cuts" and the vertical axis "Number of Pieces."
4. Mark on the graph the point that represents 1 cut and 2 pieces.
5. Holding the 2 pieces together, cut through both pieces. You have now made 2 cuts and have 4 pieces. Record the number pair in the table and mark the point on the graph.
6. Repeat for 3 cuts through the 4 pieces from Step 5.
7. Repeat for 4 cuts through the pieces from Step 6.
8. Connect the dots with a smooth curve.
9. If you have not already visited four stations, go to Station B.



| Cuts | Pieces |
|---|---|
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |

## MAKING CURVED-LINE GRAPHS OF PHYSICAL DATA

UNIT 10, EXPERIENCE 2

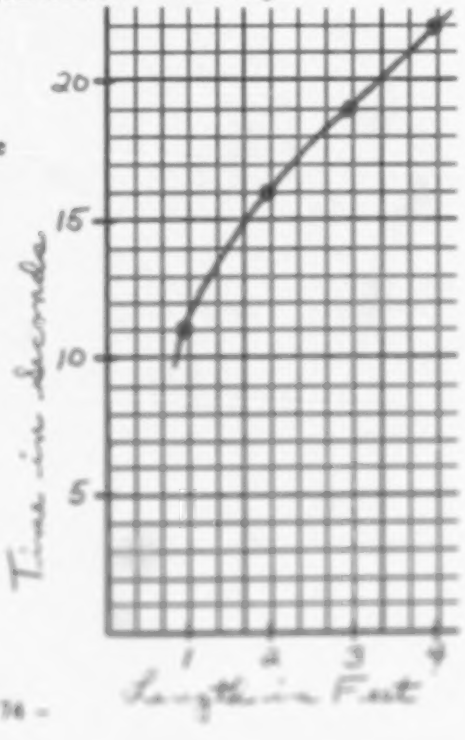
Station B

Name ______________________________

At this station you will find a pendulum device and a stopwatch. Select one of your group to carry out the steps of the experiment, and all record the data on your sheets.

1. Hold or tie the pendulum so it is exactly 1 foot long from the top to the center of the weight.
2. Swing the bob for 10 complete swings and count the time to the nearest second. Remember that back and forth counts as 1 complete swing.
3. Enter the time in the table below opposite the "1."
4. On your graph label the horizontal axis "Length in Feet" and the vertical axis "Time in Seconds."
5. Place a dot on the graph which represents your result for the 1-foot length.
6. Now make the pendulum 2 feet long and again time 10 swings. Record and graph your result.
7. Repeat for a pendulum 3 feet long and one that is 4 feet long.
8. Connect the dots with a smooth curve.
9. If you have not already visited four stations, go on to Station C.

| Feet | Seconds |
|---|---|
| 1 | 11 |
| 2 | 16 |
| 3 | 19 |
| 4 | 22 |



**MAKING CURVED-LINE GRAPHS OF PHYSICAL DATA** **UNIT 10, EXPERIENCE 2**

Station C Name ______________________________

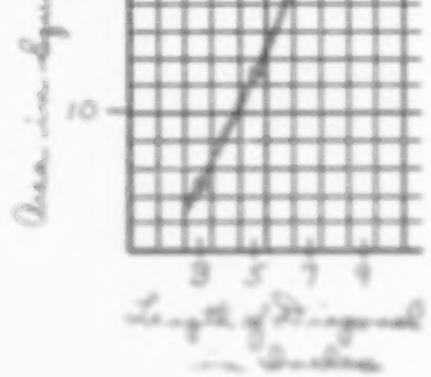
At this station you will find four squares of paper and a ruler. Select one of your group to carry out the steps of the experiment, and all record the data on your sheets.

1. Measure the length of the diagonal of the smallest square. Compute the area of this square from this measurement, using the formula

$$A=\frac{1}{2}d^2.$$

2. Enter the length of the diagonal and the corresponding area of the square in the table.
3. On your graph label the horizontal axis "Length of Diagonal in Inches" and the vertical axis "Area in Square Inches."
4. Place the point on the graph which shows the result for Step 1.
5. Repeat Steps 1, 2, and 4 for the other three squares.
6. Connect the dots with a smooth curve.
7. If you have not already visited four stations, go on to Station D.

| Diagonal in Inches | Square Inches |
|---|---|
| 3 | 4½ |
| 5 | 12½ |
| 7 | 24½ |
| 9 | 40½ |



MAKING CURVED-LINE GRAPHS OF PHYSICAL DATA

UNIT 10, EXPERIENCE 2

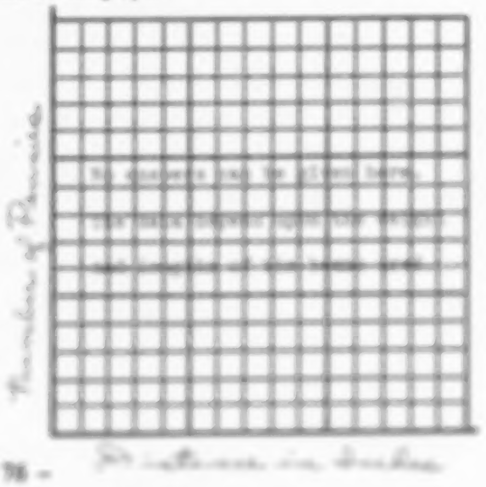
Station D

Name ______________________________

At this station you will find a balancing device and a supply of 40 pennies. Select one of your group to carry out the steps of the experiment, and all record the data on your sheets.

1. Place 10 pennies in one cup and 15 pennies in the other cup. Adjust the position of the stick on the fulcrum until the device is in balance again.
2. Measure the distance from the fulcrum to the heavier cup.
3. Enter the distance in the table below, opposite "15."
4. On your graph label the horizontal axis "Number of Pennies" and the vertical axis "Distance in Inches."
5. Place a point on the graph which indicates the distance that 15 pennies must be from the fulcrum to balance 10 pennies.
6. Place 5 more pennies in the heavier cup (for a total of 20), and adjust until the device balances. Measure the distance from the fulcrum to the heavier cup. Enter this in the table.
7. Place a point on the graph which indicates the distance that 20 pennies must be from the fulcrum to balance 10.
8. Repeat with totals of 25 and 30 pennies in the heavier cup. Make the appropriate entries in the table and place the appropriate points on the graph.
9. Connect the dots with a smooth curve.
10. If you have not already visited four stations, go on to Station A.

| Pennies | Distance |
|---|---|
| 15 | |
| 20 | |
| 25 | |
| 30 | |



# EXPERIENCE 3

## Using Graphs to Determine Additional Results in Experiments

### OBJECTIVE

The student should be able to use his graphs to determine additional data by determining pairs of numbers on the graph both between the plotted points and beyond the plotted points.

### MATERIALS

Rice, jar, measure, and ruler, from Experience 1
Graphs from Experience 1, Stations A and D, and Experience 2, Stations A, B, and C, for each student
1 worksheet for each student

### TEACHER STRATEGY

I like to introduce this experience by projecting on the chalkboard the graph from Experience 1, Station A. I ask the students to look at the projection and their own graph of this experiment (rice in the jar). I ask how far the rice would come up in the jar if four measures were used. After giving enough time for some of them to discover the answer from the graph, I show how the graph gives the answer. I explain that the answer is found by finding the point marked 4 on the horizontal axis, moving straight up to the line of the graph, then moving across to the vertical axis and reading the correct height there. Figure 4 shows the sequence of steps.

Repeat for six measures and eight measures until you are certain all know how to proceed. Emphasize that it was not necessary to do the experiment but doing the experiment can check results. To check the prediction of height for four measures, ask two students to go to the station, put in four measures of rice, and measure the height. If necessary, verify for the other predictions.

Now ask the question, "How many measures are needed to bring the rice to a height of six inches?" This time the answer is found by locating the correct number of inches on the vertical axis, going horizontally to the line of the graph, then moving straight down to the horizontal axis, where the correct number of measures is read. Figure 5 shows the sequence of steps.

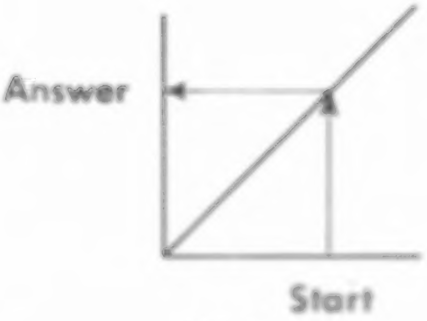


Fig. 4

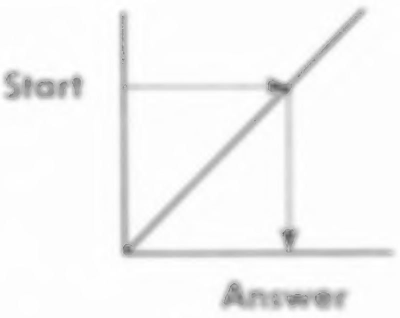


Fig. 5

To verify, fill the jar to the required number of inches and then empty it again, keeping careful count of how many measures of rice it contained (a fraction of a measure will have to be estimated).

In both of these prediction situations be sure to include questions where fractions of a measure are involved.

The questions asked here involve "interpolation," an important mathematical technique by which predictions or guesses are made *in between* the results shown by the points on a graph.

Now demonstrate that it is not necessary to stay between the plotted points on the graph. Have the students use their graphs, extending the line segment if necessary, to answer the following two questions: What would the height of the rice in the jar be if *two* measures were used? How many measures would fill the jar to ten inches? The technique involved here is called "extrapolation."

You may now distribute worksheets to each student with the instructions that the students read the directions and answer the questions individually. While they are completing these worksheets, move about the room giving the individual help that is needed.

Several of the problems on this worksheet are worth discussing in a group when most students have finished.

Be sure that the students understand the uncertainty in extrapolation when it is used with any graphs that are not straight-line graphs.

## EVALUATION

Answers on the worksheets will furnish an excellent evaluation of this experience.

## USING GRAPHS TO DETERMINE ADDITIONAL RESULTS IN EXPERIMENTS

UNIT 10, EXPERIENCE 3

Name ______________________________

Use your graphs from Experiences 1 and 2, as indicated, to answer the questions below.

**Experience 1, Station D** (No answers can be given here.)

1. How many pennies would it take to balance 120 paper clips? _____
2. If 30 pennies were placed in one end, how many paper clips would it take to balance them? _____
3. How many paper clips does 1 penny weigh? _____
4. How many pennies does 1 paper clip weigh? _____

**Experience 2, Station B**

5. Suppose the pendulum is 2 feet, 3 inches long. How long will 10 complete swings take? 34 sec.
6. How long will 10 swings take if the pendulum is $3\frac{1}{2}$ feet long? 42 sec.
7. What length pendulum will swing 10 times in 25 seconds? 1 ft. 3 in.
8. What length pendulum would make a good clock and make 1 complete swing every second? 8 ft. or 9¾ in.

**Experience 2, Station C**

9. What area does a square have if its diagonal is 4 inches long? 8 sq. in.
10. What is the area if the diagonal is $6\frac{1}{2}$ inches long? 21.1 sq. in.
11. What is the diagonal of a square whose area is 30 square inches? 7¾ in.
12. What is the diagonal of a square whose area is 10 square inches? 4½ in.

**Experience 2, Station D**

13. How many pieces of paper would there be if you made 6 cuts? 64
14. What if you made 10 cuts? If you want to have some fun, try doing this with a fresh sheet of newspaper. Don't be surprised if you have some trouble. 1,024
15. How many cuts do you think it would take to make 256 pieces? 8

# EXPERIENCE 4

## Choosing the Graph to Fit Data for a Physical Situation

### OBJECTIVE

The student should be able to select from four graphs the one that best represents the data for a given physical situation.

### MATERIALS

Scratch paper
3 worksheets and accompanying graphs for each student

**Stations.** At Station A there should be 4 sheets of 8½-by-11-in. paper for each student, rulers, and scissors. Station C requires 6 circular objects of varying diameters, such as records (45 rpm and 33⅓ rpm), lids of varying sizes, paper plates, and wheels.

### TEACHER STRATEGY

I explain to my students that in this experience they will divide into three groups to visit the stations but work individually at each station, and I tell them that their success in this experience is a result of all they learned earlier in the unit.

I pass out the three worksheets with their accompanying graphs and describe what the students are to do at each station.

When most of the students have completed their worksheets I have a group discussion on how the students recognize the correct graphs.

### EVALUATION

As in earlier experiences, the students' individual answers on the worksheets give a measure of the extent to which the objective was met.

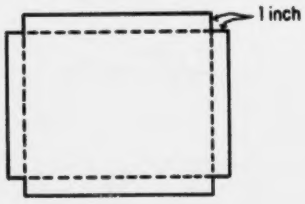
## CHOOSING THE GRAPH TO FIT DATA FOR A PHYSICAL SITUATION

**UNIT 10, EXPERIENCE 4**

**Station A** **Name** ______________________________

At this station you will find a supply of $8\frac{1}{2}$-by-11-inch paper, rulers, and scissors. You will also have four graphs to choose from.

1. Cut a 1-inch square from each corner of a piece of paper, like this:



2. Fold on the dotted lines so that a box is formed.
3. Compute the volume of the box by measuring length, height, and width and multiplying according to the formula below:

$$V = l \times w \times h.$$

4. Enter the volume in the table below, opposite the "1."
5. Repeat these four steps for $1\frac{1}{2}$-inch, 2-inch, and 3-inch squares.

| Side of Square | Volume |
|---|---|
| 1 | 58.5 |
| $1\frac{1}{2}$ | 66 |
| 2 | 63 |
| 3 | 37.5 |

**Show your calculations here.**

$s = 1$, $l = 9$, $w = 6.5$, $h = 1$ $V = 58.5$
$s = 1\frac{1}{2}$, $l = 8$, $w = 5.5$, $h = 1.5$ $V = 66$
$s = 2$, $l = 7$, $w = 4.5$, $h = 2$ $V = 63$
$s = 3$, $l = 5$, $w = 2.5$, $h = 3$ $V = 37.5$

6. Pick the graph that best represents the various situations after removing squares of 1, $1\frac{1}{2}$, 2, and 3 inches.

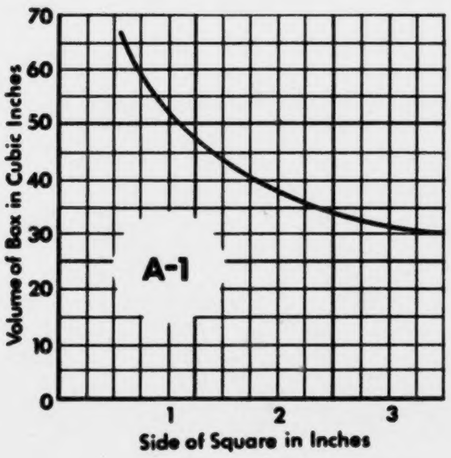
The graph that best represents the data is Graph A-2.

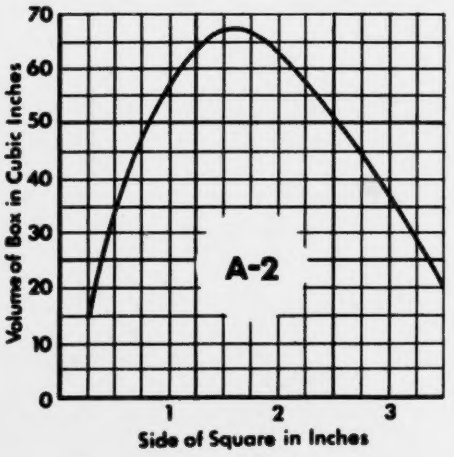
CHOOSING THE GRAPH TO FIT
DATA FOR A PHYSICAL SITUATION

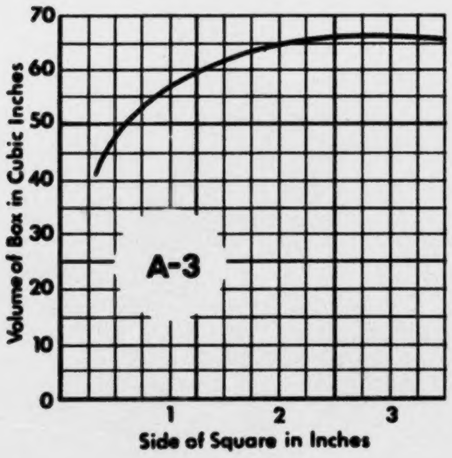
UNIT 10, EXPERIENCE 4

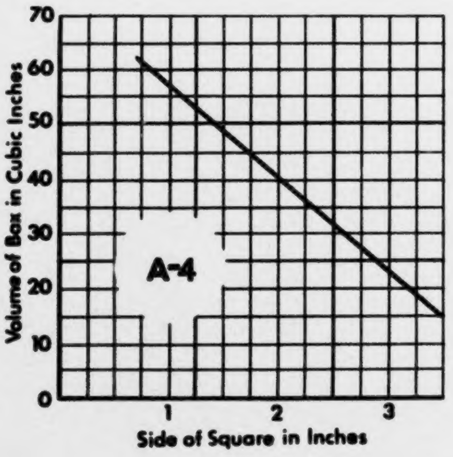
Graphs for Station A

Name ________________________________









## CHOOSING THE GRAPH TO FIT DATA FOR A PHYSICAL SITUATION

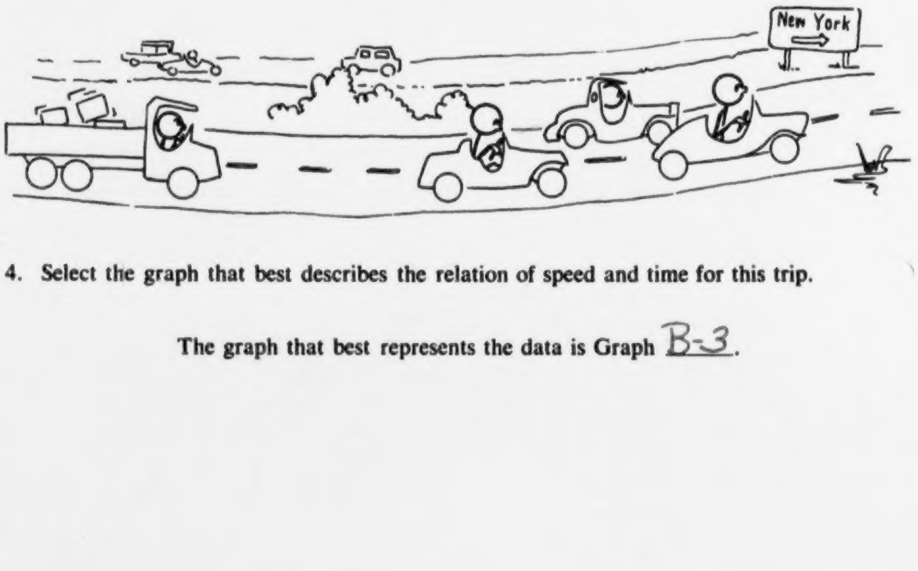
**UNIT 10, EXPERIENCE 4**

**Station B** **Name** ____________________

1. Suppose you drive from Chicago to New York by a route 800 miles long. If your average speed is 30 miles per hour, how many hours will the trip take? 26 2/3

2. Supposing your average speed is 40 miles per hour, how long will the trip take in hours? 20

3. Repeat for 50 miles per hour. 16 For 60 miles per hour. 13 1/3



4. Select the graph that best describes the relation of speed and time for this trip.

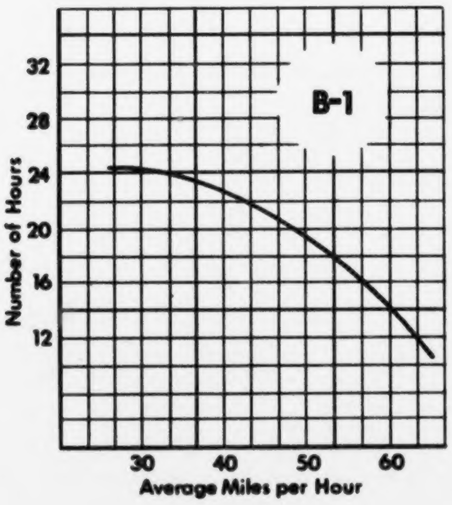
The graph that best represents the data is Graph B-3.

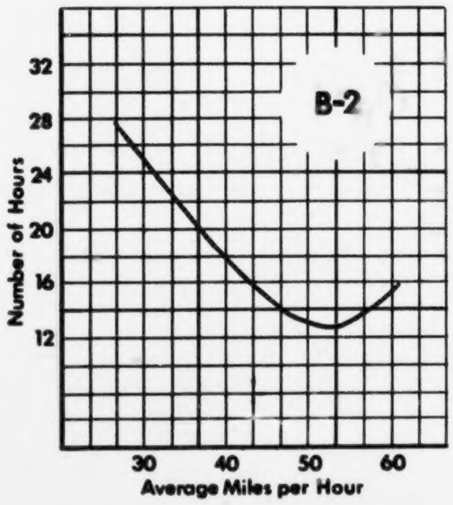
CHOOSING THE GRAPH TO FIT
DATA FOR A PHYSICAL SITUATION

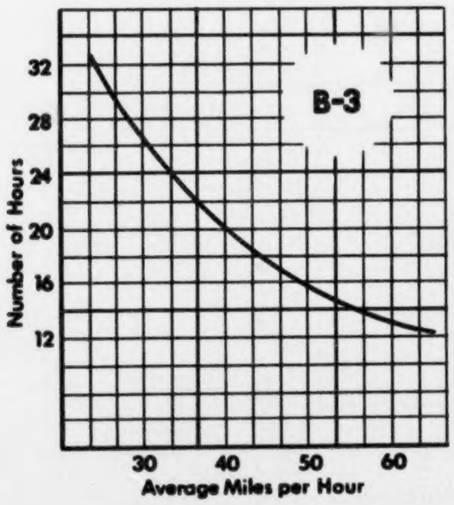
UNIT 10, EXPERIENCE 4

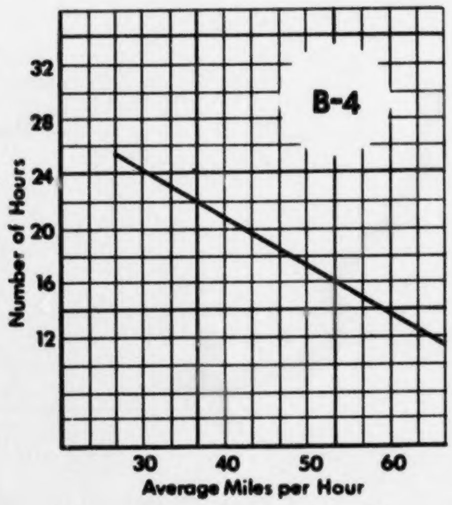
Graphs for Station B

Name ______________________________









## CHOOSING THE GRAPH TO FIT DATA FOR A PHYSICAL SITUATION

UNIT 10, EXPERIENCE 4

Station C

Name ______________________________

At this station you will find several circular objects.

1. Measure the diameter of the smallest circular object. Remember that the radius is one-half of the diameter.
2. Compute the area, using the formula that area is equal to $\pi$ times the square of the radius and remembering that $\pi$ is approximately $3\frac{1}{7}$:

$$A = 3\frac{1}{7}(r^2).$$

3. Repeat this for the other objects. Fill in the table below.

| Diameter in Inches | Square Inches |
|---|---|
| | |
| No answers can be | |
| given here. | |
| | |
| | |
| | |

4. Pick the graph that best shows the relationship between diameter and area for circles.

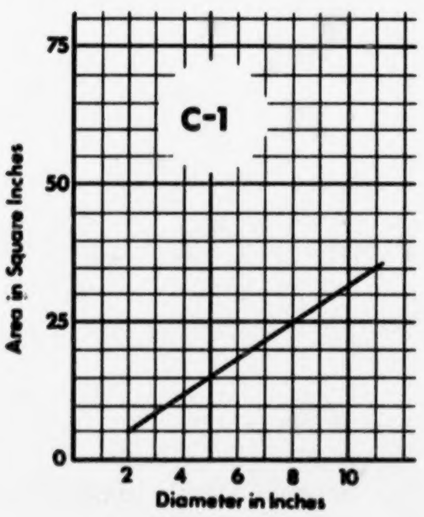
The graph that best represents the data is Graph C-2

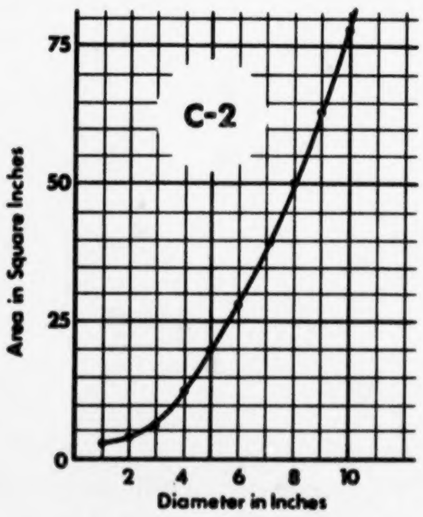
CHOOSING THE GRAPH TO FIT
DATA FOR A PHYSICAL SITUATION

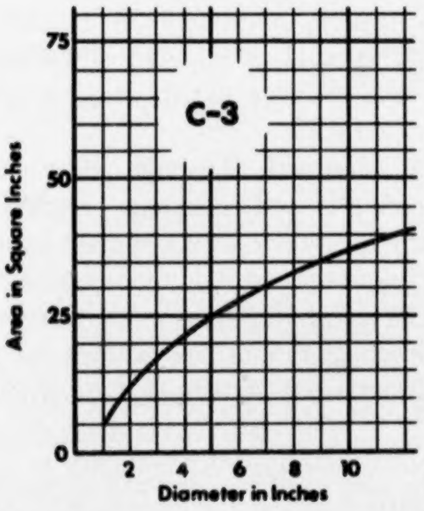
UNIT 10, EXPERIENCE 4

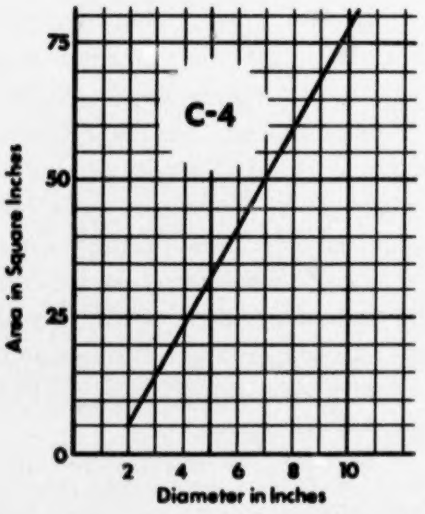
Graphs for Station C

Name ______________________________









# 11

## EXPERIENCES WITH

# Organizing Data

The purpose of this unit is to help students develop a basic understanding of the nature of descriptive statistics by collecting and organizing data into tables and graphs, then describing the results. Since one can scarcely read a newspaper or magazine without being confronted with statistical information, it seems important that students become aware at an early age of the methods used in presenting the statistics.

Some of these experiences are laboratory-oriented. Through these experiences the students will work both individually and in groups to gather data and analyze them on the basis of information they have obtained from introductory discussions. It will often be necessary to call them together at the end of the class session to give them an opportunity to share their results. Comparing and discussing the different data and descriptions from the same activities will help them understand what statistical data really represent.

## OVERVIEW

Each of the five experiences in this unit includes a detailed section called "Teacher Strategy," which is usually one teacher's first-person account of a procedure that proved effective with his class.

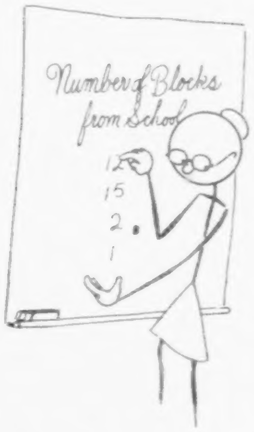
The following brief summary will help you decide whether these experiences meet the needs of your own class and will also give you some indication of the amount of preparation involved.

Student worksheets and other "handout" materials are provided in the Teaching Package. It is assumed that duplicating facilities are available to you.

### Experience 1: Data from the Class

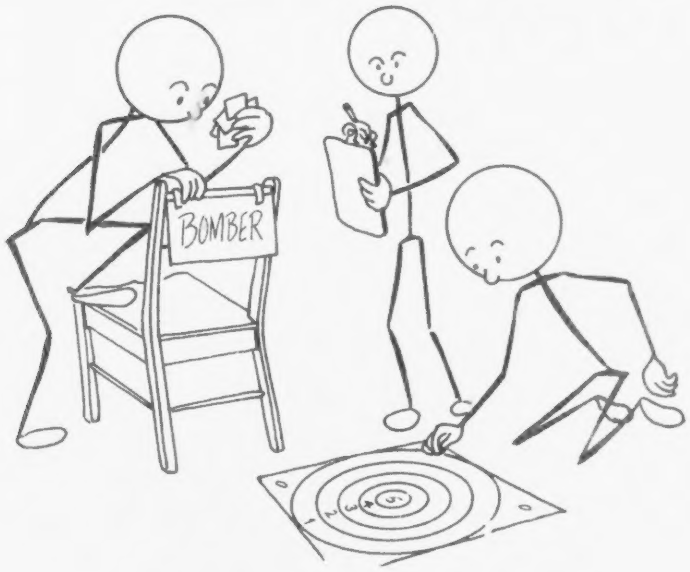
Students focus attention on everyday situations from which data can be collected. They are led to see a need for organizing these data in order to get some meaning from what might otherwise be "just a bunch of numbers." They are then given the opportunity to organize some prepared data on their own.

Materials needed: worksheets.



### Experience 2: Collecting and Organizing Your Own Data

Students are given the opportunity to use the principles learned in Experience 1 to collect and organize their own data. These data are obtained from gamelike experimental activities. In some of these activities they work as teams, and in others they work individually.



Materials needed: posters, markers, target, paper squares, masking tape, buttons, yardstick, envelopes with paragraphs of prose, dice, group data sheets, worksheets.

### Experience 3: Graphing and Interpreting Data

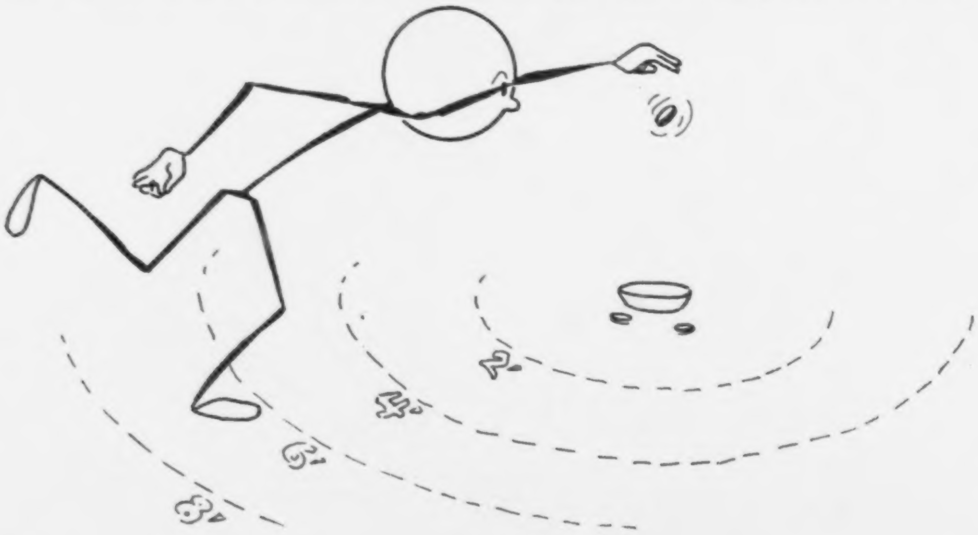
Letter Count
Frequency
1
2,3
2
6,7
3
15,12
4
10,9
5
9,12
6
3,
7
4,
8
1,
9 or more
3,

Gamelike activities again serve as a means for the students to collect their own data. This time they work independently. The students are also introduced to bar-graph representations of data, using information they collected in Experience 2. Finally, they construct tables and graphs to aid them in describing the data they collected.

Materials needed: class data from Experience 2, projector transparencies, magazine clippings, construction paper, spinner faces, gummed reinforcements, Popsicle sticks, wire, rulers, worksheets.

### Experience 4: More on Interpreting Data

Students investigate relationships between pairs of numbers, the first member from one set and the second member from another set. The introduction is a consideration of the relation between hand lengths and the number of dots counted as a part of Experience 3. They are then shown



instances where there is a distinct relation between pairs of numbers and others where there is no evident relation, so that they will know what to look for in their graphs.

Materials needed: Experience 2 Button Toss materials; Experience 3 worksheets, group report, spinners, and paragraphs; grids; transparency; worksheets.

### Experience 5: Culminating Experience

This experience serves as a summary. The students are given a data sheet, some blank grids and tables, and a list of questions. In order for them to answer the questions it is necessary to organize the data into graphs and tables. Thus they will show their competence at using these tools to solve problems.

Materials needed: data sheet, worksheets.

# EXPERIENCE 1

## Data from the Class

### OBJECTIVE

The student should be able to see the need for organizing data and able to record a set of data in a table in an organized manner, using tallies.

### MATERIALS

2 worksheets for each student

### TEACHER STRATEGY

This experience has a twofold purpose. First, we want the students to see a need for organizing data; secondly, we want them to see how to organize the data. In order to achieve the first goal it is important that data obtained from the students be placed on the board in a random fashion—then they will see that it will be necessary to organize the information that has been collected in order to describe it well.

I like to begin this experience by asking, "Who thinks he lives the farthest from school?" When I receive an answer I ask, "How many blocks away from school do you live?" and write the number given in the response on the board under the title "Number of Blocks from School."

"Does anyone live farther away than that? How far?" I put these numbers on the board.

"Now who thinks he lives closest to school?" I record that number.

"Does anyone live closer? Does anyone live a distance from school that is not listed on the board?" I add these data to the list, then question students in turn until everyone is represented by some number on the board. I make no attempt to enter the numbers systematically; rather, I enter them in order as they are determined.

Now I go back over the list and find out how many students live each of the given numbers of blocks from school. I use tally marks to record this information.

I tell the students that we have gathered the information for a frequency distribution table that tells about the distances the students live from school. I ask them if there is some way we can organize the information in the table so we can readily see the shortest distance, the longest distance, the number of students living more than five blocks away, and so forth. The students suggest listing the distances in order from shortest to longest. After rearranging the data in a table as they suggest, I ask questions like the following to see if the students know how to use the table: "Which distances are the most frequent? How many students walk more than six blocks to school?"

To give the students additional experience in recording data I ask, "How many of you remember what a polygon is?" and have one of the students describe one. If no one remembers, I give them a couple of examples like triangles, squares, or pentagons. Then I ask the students to draw three different polygons and count the number of sides. I ask them to tell me the number of sides of their polygons and make a list on the board like 9, 3, 6, 4, 3, 8, . . . until all the numbers are recorded.

I ask the students for suggestions on organizing the data. They should suggest making a list like the one shown in table 1, which lists an increasing order, using tallies to note the frequency and then showing the totals. From such a table it is easy for the students to decide which polygon was drawn most frequently.

TABLE 1

| Number of Sides | Frequency Tallies | Total |
|---|---|---|
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |
| 9 | | |
| 10 | | |

Next I ask each student to write down a number between 1 and 10, inclusive, and ask the class to suggest ways to show the data in a table. One like table 2 (p. 130) will probably be suggested. It should be completed.

Then I ask, "Which number was written most frequently? Least frequently? Are there any numbers that weren't written at all? If we did this

TABLE 2

| Number | Frequency Tallies | Total |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |
| 9 | | |
| 10 | | |

again, do you think the distribution would be the same?" If it seems necessary, I repeat this activity.

With this discussion as background, I pass out copies of the first worksheet and read through the directions with the students. I leave on the board the tables that have been constructed in class, since the students may need some guidance in beginning to make the first tables. As each student completes the first worksheet, I give him a copy of the second one.

When all have completed their worksheets, I use the questions at the end of the sheets as a basis for discussion. Such questions help us to describe the data we collect.

It is important to check to make sure that the students complete their tables correctly.

## EVALUATION

The students' responses to the questions should make it easy to see whether they are able to understand what their tables represent. Students may wish to compare their own class, about the numbers of brothers and sisters, with the one studied. Encourage them to do so, and let them decide how to get the information needed.

DATA FROM THE CLASS UNIT 11, EXPERIENCE 1

Activity 1 Name ______________________

The students in Classroom 204 turned in a record of their birthdays. Here they are:

| | | | | |
|---|---|---|---|---|
| Jan. 20 | Oct. 20 | Feb. 14 | Aug. 16 | Feb. 12 |
| Aug. 15 | Nov. 12 | Aug. 18 | Jan. 2 | Dec. 20 |
| Apr. 12 | Apr. 15 | Mar. 20 | Aug. 14 | Jan. 15 |
| Sept. 10 | Jan. 9 | Oct. 10 | June 9 | Oct. 14 |
| Feb. 15 | Sept. 22 | June 9 | Nov. 11 | Jan. 20 |

Use this information to complete the frequency tables below. Then answer the questions.

| Month | Tally | Frequency |
|---|---|---|
| Jan. | 𝍸 | 5 |
| Feb. | \|\|\| | 3 |
| Mar. | \| | 1 |
| Apr. | \|\| | 2 |
| May | | 0 |
| June | \|\| | 2 |
| July | | 0 |
| Aug. | \|\|\|\| | 4 |
| Sept. | \|\| | 2 |
| Oct. | \|\|\| | 3 |
| Nov. | \|\| | 2 |
| Dec. | \| | 1 |

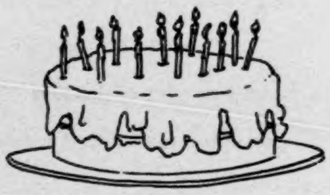
| Day | Tally | Frequency | Day | Tally | Frequency | Day | Tally | Frequency |
|---|---|---|---|---|---|---|---|---|
| 1 | | 0 | 11 | \| | 1 | 21 | | 0 |
| 2 | \| | 1 | 12 | \|\|\| | 3 | 22 | \| | 1 |
| 3 | | 0 | 13 | | 0 | 23 | | 0 |
| 4 | | 0 | 14 | \|\|\| | 3 | 24 | | 0 |
| 5 | | 0 | 15 | \|\|\|\| | 4 | 25 | | 0 |
| 6 | | 0 | 16 | \| | 1 | 26 | | 0 |
| 7 | | 0 | 17 | | 0 | 27 | | 0 |
| 8 | | 0 | 18 | \| | 1 | 28 | | 0 |
| 9 | \|\|\| | 3 | 19 | | 0 | 29 | | 0 |
| 10 | \|\| | 2 | 20 | 𝍸 | 5 | 30 | | 0 |

What month has the most births? Jan. How many? 5

Do any months have no births? Yes If so, which one(s)? May, July

How many months have at least 3 births? 4

How many have less than 2? 4



On what day of the month did most births occur? 15

On what days were there no births? 1, 3, 4, 5, 6, 7, 8, 13, 17, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30

DATA FROM THE CLASS UNIT 11, EXPERIENCE 1

Activity 2 Name ______________________________

The data below came from a survey of the number of brothers and sisters of students in a certain class. "B" means brothers and "S" means sisters, so that 2B 3S means 2 brothers and 3 sisters.

| | | | |
|---|---|---|---|
| 2B 3S | 2B 3S | 3B 1S | 1B 3S |
| 3B 2S | 0B 0S | 3B 3S | 1B 4S |
| 1B 0S | 2B 0S | 2B 2S | 3B 4S |
| 4B 2S | 0B 2S | 0B 0S | 1B 4S |
| 1B 1S | 2B 5S | 3B 1S | 2B 3S |
| 3B 1S | 4B 0S | 4B 1S | 0B 4S |
| 2B 2S | 0B 2S | 2B 1S | 6B 1S |
| 1B 3S | 2B 0S | 2B 2S | 1B 2S |

Complete the frequency tables below and then answer the questions that follow.

| Number of Brothers | Tally | Frequency |
|---|---|---|
| 0 | 𝍸 | 5 |
| 1 | 𝍸 \|\| | 7 |
| 2 | 𝍸𝍸 | 10 |
| 3 | 𝍸 \| | 6 |
| 4 | \|\|\| | 3 |
| 5 | | 0 |
| 6 | \| | 1 |
| | | |

| Number of Sisters | Tally | Frequency |
|---|---|---|
| 0 | 𝍸 \| | 6 |
| 1 | 𝍸 \|\| | 7 |
| 2 | 𝍸 \|\|\| | 8 |
| 3 | 𝍸 \| | 6 |
| 4 | \|\|\|\| | 4 |
| 5 | \| | 1 |
| | | |
| | | |

| Total Number of Both | Tally | Frequency |
|---|---|---|
| 0 | \|\| | 2 |
| 1 | \| | 1 |
| 2 | 𝍸 | 5 |
| 3 | \|\| | 2 |
| 4 | 𝍸𝍸 | 10 |
| 5 | 𝍸 \|\| | 7 |
| 6 | \|\| | 2 |
| 7 | \|\|\| | 3 |

The greatest number of brothers anyone has is 6.

The greatest number of sisters is 5.

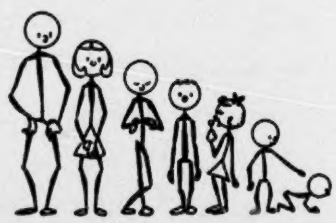
The least number of brothers is 0.

The least number of sisters is 0.

What is the most frequent number of brothers? 2

Of sisters? 2

Of brothers and sisters? 4



# EXPERIENCE 2

## Collecting and Organizing Your Own Data

### OBJECTIVE

The student should be able to collect data from experimental situations and organize them in a meaningful fashion in the tables provided.

### MATERIALS

4 posters with class data sheets
Markers
Station and activity materials as listed below

About one-fourth of the students in the class will be working on each activity at one time. They work in groups of three in the first two activities and individually in the last two. Table 3 indicates the number of sets of materials (apart from worksheets) needed for various class sizes.

TABLE 3

| Size | 1-4 | 5-8 | 9-12 | 13-16 | 17-20 | 21-24 |
|---|---|---|---|---|---|---|
| Activity 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| Activity 2 | 1 | 1 | 1 | 2 | 2 | 2 |
| Activity 3 | 1 | 2 | 3 | 4 | 5 | 6 |
| Activity 4 | 1 | 2 | 3 | 4 | 5 | 6 |

**Activity 1, Station A—"Bomber over Target"**

Target
2-in. squares of construction paper
1 individual worksheet for each student
1 worksheet for each group

The target is made from a large square of cardboard or paper with 5 concentric circles that have radii of 2, 6, 10, 14, and 18 inches as shown in figure 1 on the following page. It should be taped to the

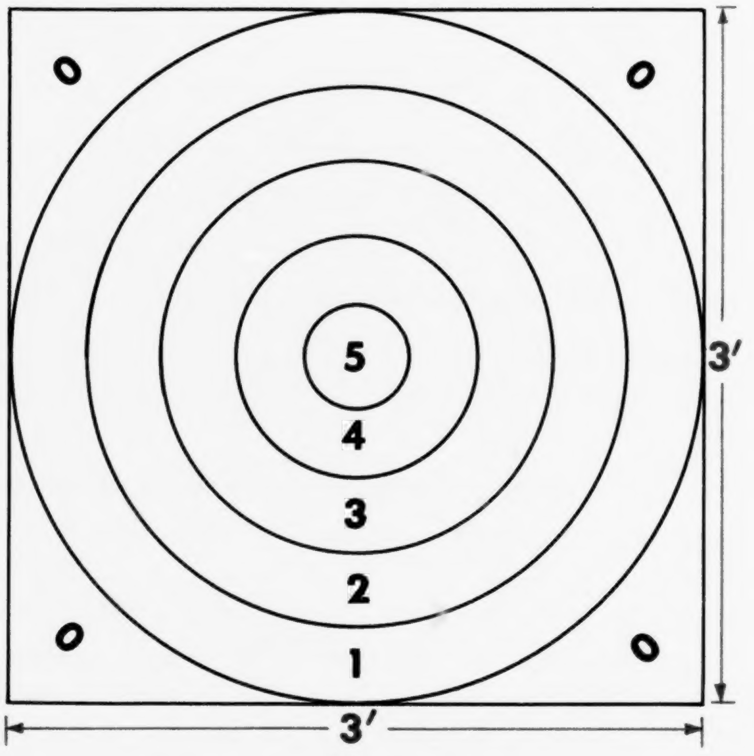


Fig. 1

floor if necessary to keep it flat and in place. Directions for playing the game and recording results are given under Teacher Strategy.

**Activity 2, Station B—"Button Toss"**

Circular cardboard disc (1 in. diameter) or other marker
Masking tape
5 matching buttons
Yardstick (or substitute)
1 individual worksheet for each student
1 worksheet for each group

Tape the disc to the floor or make a mark on the floor. With chalk, string, or masking tape make a line about 10 feet from the target.

**Activity 3—"Vowel Count"**

1 envelope for each student, containing a paragraph clipped from a magazine or newspaper
1 worksheet for each student

The magazine or newspaper clippings may be mounted on construction paper for greater durability.

**Activity 4—"Horseshoe Game"**

1 button or other small object as a marker for each student
1 die for each student
2 worksheets for each student

## TEACHER STRATEGY

In this experience the students collect and organize their own data, both individually and in groups of three.

Divide the class into four main groups, then divide each of these groups into teams of three. Have each team select one chairman for Activity 1 and another for Activity 2. It is the responsibility of the chairman to record the data for the team activity.

It will be necessary for you to control the time spent in the four activities so that there is no conflict about the time spent on Bomber over Target and Button Toss; the times spent on Vowel Count and Horseshoe Game can vary, since they are done individually, so long as the total of the two is the same as for the two team activities. A fifteen-minute warning is suggested so that students will have time to complete their work and the chairmen for Activities 1 and 2 will have time to report the team results.

In Bomber over Target, Activity 1, each of the three players takes three turns at each position on the team. Team Member 1 takes five of the squares, stands on a chair next to the target, and drops the squares over the target, one at a time, from shoulder height. Team Member 2 announces the spaces where the squares land, counting a square as landing in the zone with the higher number if it lands equally in two target zones. Team Member 3 enters tallies in the Individual Data Sheet for Team Member 1. After three turns apiece, each player finds his individual score for each of the three trials. Together, the players figure out the remaining data for the team data sheet; then the chairman enters the scores for his team on the poster for the class. The team data sheet is retained for future class discussion.

Button Toss, Activity 2, is conducted in much the same way. Each player has a turn at each position on the team—or two turns, if time allows. Team Member 1 tosses five buttons from behind a line placed about ten feet from the target. Team Member 2 measures the distance each button lands from the target. Team Member 3 records this information on the data sheets. At the warning signal, the players complete the individual and team data sheets, and the chairman enters the information on the poster for the class.

The results of these activities are more interesting to discuss if the students are able to *see* the compilation of data obtained from all the students. An easy way to show this compilation is to have large pieces of poster paper taped around the room for a summary of the reports, as shown in figure 2 for the team scoring data reports of the Button Toss game. Team captains and individuals record their information, as already mentioned, at the completion of each activity. These reports will be used as material for class discussion and as a kickoff for the use of graphs, in the next experience, to represent data.

When the students have completed all the activities ask them to describe their results to the class. Some questions like the following will help them make this description.

What was the least number of tosses required in any horseshoe game? What was the greatest number? Which number of tosses appeared most frequently?

What vowel was used most frequently in your sentences? What vowel was used the least?

Do your results agree with those compiled by the entire class?

The class data sheets should be saved, since some of the data are used in Experience 3.

[*Continued on page 144*]

*BUTTON TOSS*
Class Data Sheet

| Scores | Frequency | | | | | | | | Total Frequency |
|---|---|---|---|---|---|---|---|---|---|
| | Team 1 | Team 2 | Team 3 | Team 4 | Team 5 | Team 6 | Team 7 | Team 8 | |
| 0–4 | | | | | | | | | |
| 5–9 | | | | | | | | | |
| 10–14 | | | | | | | | | |
| 15–19 | | | | | | | | | |
| 20–24 | | | | | | | | | |
| 25–29 | | | | | | | | | |
| 30–34 | | | | | | | | | |
| 35–39 | | | | | | | | | |
| 40–44 | | | | | | | | | |
| 45–49 | | | | | | | | | |
| 50 | | | | | | | | | |

Fig. 2

COLLECTING AND ORGANIZING YOUR OWN DATA

UNIT 11, EXPERIENCE 2

Activity 1, Station A

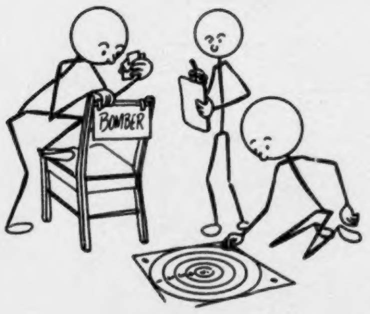
Name ____________________

*BOMBER OVER TARGET*

Individual Data Sheet

| Target Area | Trial 1 | | Trial 2 | | Trial 3 | |
|---|---|---|---|---|---|---|
| | Tally | Score | Tally | Score | Tally | Score |
| 5 | | | | | | |
| 4 | | | | | | |
| 3 | | | | | | |
| 2 | | | | | | |
| 1 | | | | | | |
| 0 | | | | | | |
| Score Totals | | | | | | |

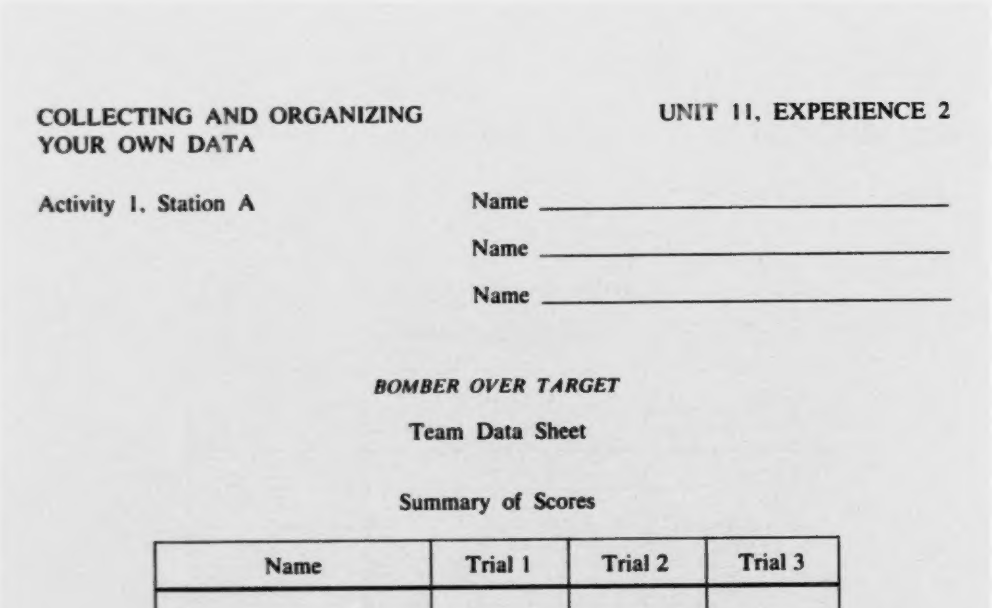
To obtain the score, multiply the number of the target area by the number of buttons that landed there. For example, if the frequency for Target Area 5 is 2, the score to be entered for that area would be 10.



Score for Trial 1 _____

Score for Trial 2 _____

Score for Trial 3 _____



**COLLECTING AND ORGANIZING YOUR OWN DATA**

**UNIT 11, EXPERIENCE 2**

**Activity 1, Station A**

Name ______________________
Name ______________________
Name ______________________

***BOMBER OVER TARGET***

**Team Data Sheet**

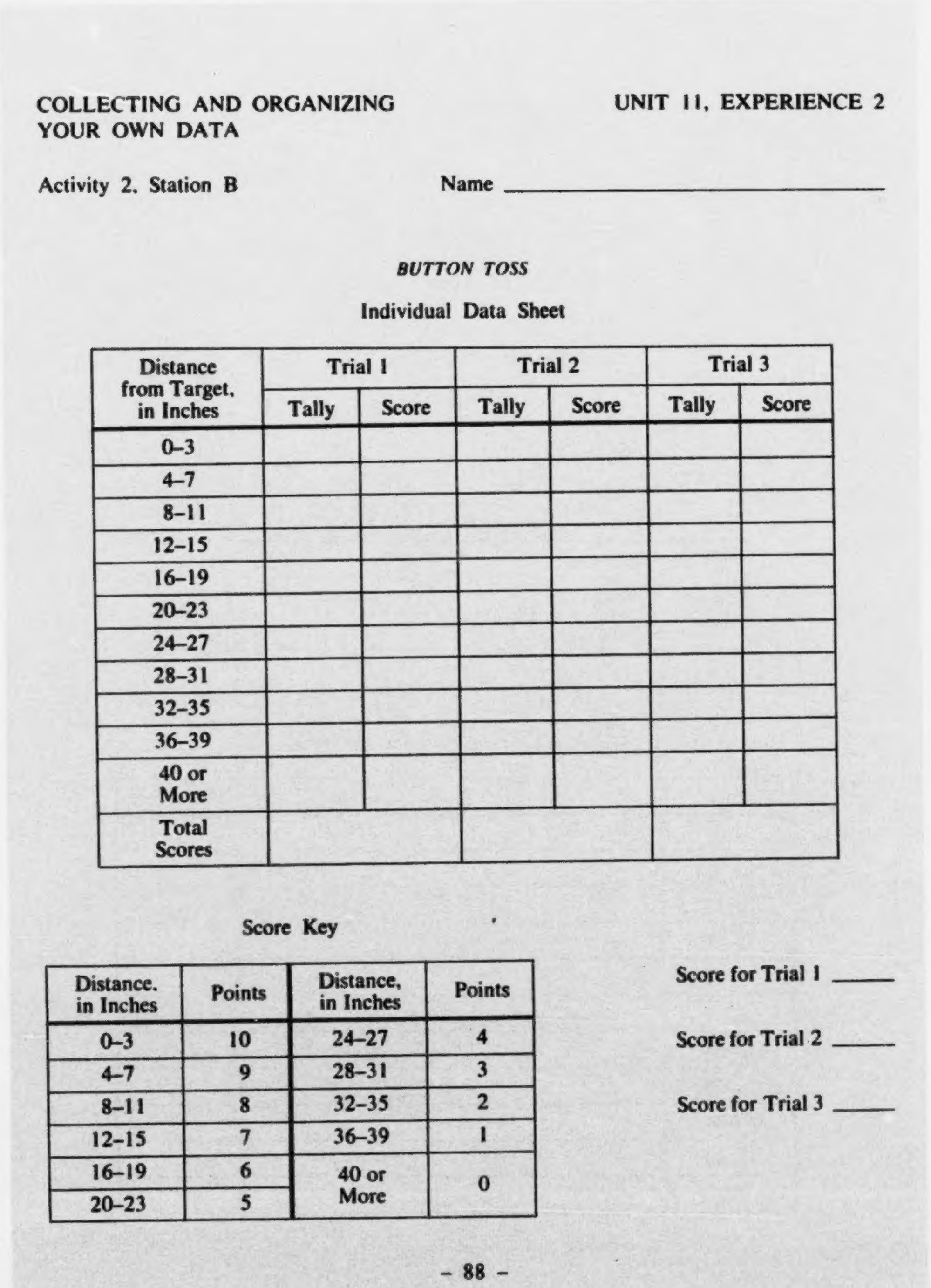
**Summary of Scores**

| Name | Trial 1 | Trial 2 | Trial 3 |
|---|---|---|---|
| | | | |
| | | | |
| | | | |

**Distribution of Scores for the Team**

| Scores | Tally | Frequency |
|---|---|---|
| 0–4 | | |
| 5–9 | | |
| 10–14 | | |
| 15–19 | | |
| 21–25 | | |

– 87 –



COLLECTING AND ORGANIZING YOUR OWN DATA

UNIT 11, EXPERIENCE 2

Activity 2. Station B

Name ______________________________

### *BUTTON TOSS*

Individual Data Sheet

| Distance from Target, in Inches | Trial 1 | | Trial 2 | | Trial 3 | |
|---|---|---|---|---|---|---|
| | Tally | Score | Tally | Score | Tally | Score |
| 0–3 | | | | | | |
| 4–7 | | | | | | |
| 8–11 | | | | | | |
| 12–15 | | | | | | |
| 16–19 | | | | | | |
| 20–23 | | | | | | |
| 24–27 | | | | | | |
| 28–31 | | | | | | |
| 32–35 | | | | | | |
| 36–39 | | | | | | |
| 40 or More | | | | | | |
| Total Scores | | | | | | |

Score Key

| Distance, in Inches | Points | Distance, in Inches | Points |
|---|---|---|---|
| 0–3 | 10 | 24–27 | 4 |
| 4–7 | 9 | 28–31 | 3 |
| 8–11 | 8 | 32–35 | 2 |
| 12–15 | 7 | 36–39 | 1 |
| 16–19 | 6 | 40 or More | 0 |
| 20–23 | 5 | | |

Score for Trial 1 _____

Score for Trial 2 _____

Score for Trial 3 _____

**COLLECTING AND ORGANIZING YOUR OWN DATA** **UNIT 11, EXPERIENCE 2**

Activity 2. Station B

Name ______________________

Name ______________________

Name ______________________

*BUTTON TOSS*

Team Data Sheet

Summary of Scores

| Team | Trial 1 | Trial 2 | Trial 3 |
|---|---|---|---|
| Team Member 1 | | | |
| Team Member 2 | | | |
| Team Member 3 | | | |

Distribution of Scores for the Team

| Scores | Tally | Frequency |
|---|---|---|
| 0–4 | | |
| 5–9 | | |
| 10–14 | | |
| 15–19 | | |
| 20–24 | | |
| 25–29 | | |
| 30–34 | | |
| 35–39 | | |
| 40–44 | | |
| 45–49 | | |
| 50 | | |

COLLECTING AND ORGANIZING
YOUR OWN DATA

UNIT 11, EXPERIENCE 2

Activity 3

Name ____________________

### *VOWEL COUNT*

From your English classes, you are already familiar with the vowels *a, e, i, o,* and *u.* I'll bet you never expected to use them in math. Actually, all we're trying to do is find out which ones are used the most.

Take the clipping out of your "Vowel Count" envelope.

Look at the first word. Suppose it is "today." For that word you would put a tally in the *o* column and another tally in the *a* column. Do this for every word in the sentences you have. When you have finished, find the total number of tallies for each vowel and place that number in the last column. Then place your results in the proper place on the sheet for the class report.

| Vowel | Tally | Frequency |
|---|---|---|
| *a* | | |
| *e* | | |
| *i* | | |
| *o* | | |
| *u* | | |

The quick brown fox jumped ...
Vowel Count Envelope

COLLECTING AND ORGANIZING
YOUR OWN DATA

UNIT 11, EXPERIENCE 2

Activity 4

Name ______________________________

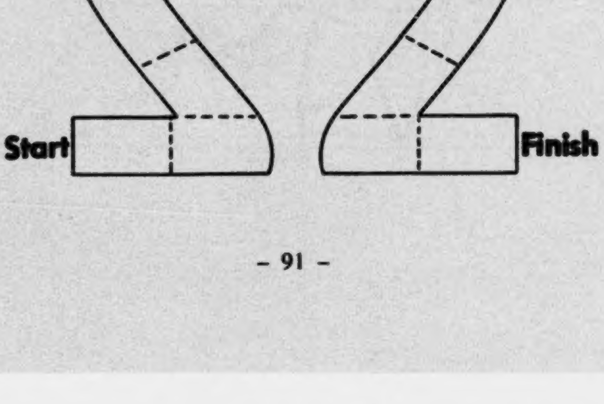
### *HORSESHOE GAME*

The object of this game is to move your marker from start to finish in as few moves as possible.

Place your marker at the starting line. Toss the die. Move the marker forward along the horseshoe as many spaces as the top of the die indicates, and record a tally mark in the Tally of Tosses column on your Individual Data Sheet for this activity. Continue tossing the die, moving forward the indicated number of spaces, and recording a tally mark for each toss.

You must cross the finish line on an exact count. If your die toss results in a number more than the number of spaces that remain you must keep your marker where it is, record a tally for the toss, and toss again (with a tally each time) until you get a number that you can use.

You may continue to play the game until the warning signal is given by your teacher. Then complete the frequency table on the Individual Data Sheet and record your data on the table for the class.



COLLECTING AND ORGANIZING
YOUR OWN DATA

UNIT 11, EXPERIENCE 2

Activity 4

Name ______________________________

***HORSESHOE GAME***

**Individual Data Sheet**

| Trial | Tally of Tosses | Frequency |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |
| 9 | | |
| 10 | | |
| 11 | | |
| 12 | | |
| 13 | | |
| 14 | | |
| 15 | | |
| 16 | | |
| 17 | | |
| 18 | | |
| 19 | | |
| 20 | | |

Summary of Trials

| Number of Tosses | Tally | Frequency |
|---|---|---|
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |
| 9 | | |
| 10 | | |
| 11 | | |
| 12 | | |
| 13 | | |
| 14 or More | | |

## EVALUATION

The discussion and the completed worksheets will indicate how well the students can collect and organize data.

# EXPERIENCE 3

## Graphing and Interpreting Data

## OBJECTIVE

The student should be able to make a representation in bar-graph form of the data he has collected and make meaningful statements to describe the data, indicating an understanding of range and average.

### MATERIALS

Class data from Experience 2
Projector transparency showing bar graph from Button Toss, Experience 2
Station materials (see below)
9 worksheets for each student

Duplicate stations will probably be needed. The number depends on the size of your class.

**Station A, Letter Count.** A short paragraph from a magazine or newspaper, enclosed in an envelope, is needed for each student expected to work at the station at any one time. The clipping may be mounted on construction paper for reinforcement. Extra envelopes might be supplied so that a student who wishes to analyze a second paragraph and collect additional data would be able to do so. All clippings at any one station should be from the same source.

**Station B, Spinner Sums.** For this activity you should prepare enough spinners to provide one for each student working on this activity at any one time.

### Spinners

Mount the faces, duplicated from the Teaching Package, on medium-weight cardboard. Make a small hole in the center of each spinner and add about 10 gummed reinforcements around the hole to serve as a washer. Using a compass point, punch a hole in the midpoint of a Popsicle stick, then attach it to the disc with a thumbtack. Identify the pointing end of the Popsicle stick in some way—with poster paint, crayon, or nail polish, for example.

Figure 3 pictures the completed spinner.

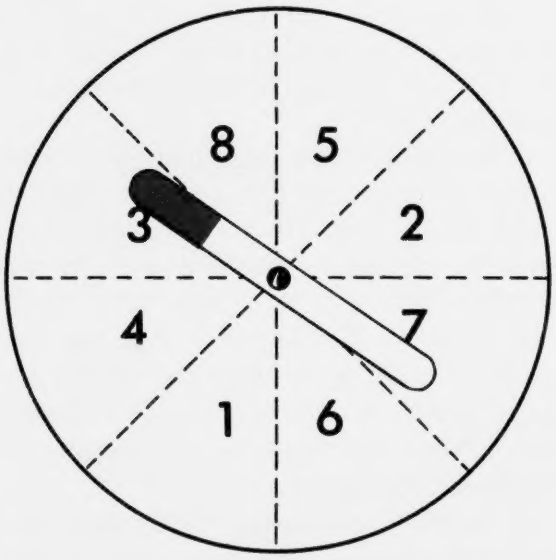


Fig. 3

**Station C, Dot Count.** Only rulers are needed for this station, in addition to the worksheet. There should be a ruler for each student.

## TEACHER STRATEGY

I like to begin this experience by considering the class results for the Button Toss game of Experience 2. I have at hand a record of the total frequencies of the scores and an overhead projector transparency of a bar-graph representation of this information. (If no projector is available, a large-scale graph on manila paper could be substituted for the transparency.) I show the graph to the students along with the frequency table and try to elicit the observation from students that graphs are useful because they picture the numerical facts in a frequency table and help us to summarize the data relationships clearly and quickly.

I ask students to tell me how to construct a bar graph such as that shown on the transparency, using the data in a table. In the discussion that follows I emphasize that it is important to have a column for each entry to that table and add that columns should be of equal width, each vertical space should represent the same number of units, and so forth. Referring to the transparency, I ask questions such as "What total score was achieved most often?" and "What were the lowest and highest scores?"

I also ask them, "From the graph, what appears to be the 'average' score?" After they have answered this question, we discuss adding up all scores and dividing; then the students use this method to calculate the average for this activity. Since we want whole number scores, we round the average to the nearest whole number and round up in case of an average involving one-half. The average is usually quite close to that previously chosen by the students.

This demonstration should show them that by a careful examination of a graph of data they can come up with a close "average" for the data.

Now I show the students the class data sheet from the Bomber over Target game, showing the scores, together with frequencies. I give the students the worksheet for Activity 1 and direct them to construct the graphs for this set of data. We discuss a scale for the vertical columns and determine a scale according to the amount of data collected. After they have graphed the data in the first column (frequency of landings in the 0 target area), I have them continue individually. They should have little trouble completing the graph.

When the graph has been completed, I have them describe their results in a way that answers questions like those they answered in connection with Button Toss. The descriptions should indicate the range, most frequent scores, average score, and so on.

For the next three activities I divide the class into three groups, so that group data reports can be made; but the students work individually on the worksheets for these activities and for Activity 5.

The group data report forms should be posted in three sections of the room. They may be made from large pieces of manila paper or poster paper, and they should be fashioned like the one indicated for Spinner Sums in figure 4.

You will need to allow time during the day for students to obtain the information from the group data sheets. You can either allow time in class for students to complete the graphs and descriptions or assign this to be done as homework. Perhaps it would be better to have this done during class time, for then you would know it was their work and you could better evaluate their understanding.

*SPINNER SUMS*

| Sum | Tally | | | Total Frequency |
|---|---|---|---|---|
| | Group 1 | Group 2 | Group 3 | |
| 2 | | | | |
| .<br>.<br>. | | | | |
| 16 | | | | |

Fig. 4

After the students have had an opportunity to complete all the worksheets, discuss the questions they were to answer. Have someone from each group put a sample graph on the board so students can compare results.

In talking about the Letter Count activity, let the students compare the numbers of letters in words from different sources. Ask why there are so many words of four and five letters in their paragraphs. Have them think of the words they use. You might have a second-grade reader and a college textbook to show the students, letting them see how the lengths of the words compare.

In connection with the Spinner Sum activity, discuss some questions like the following:

Can you think of a reason why certain sums hardly ever occurred?
Is it just by chance?
Why do some appear so often?
What are the ways we can get certain sums—for instance, 8 or 9 and 2 or 16?

Save the Dot Count data for Experience 4.

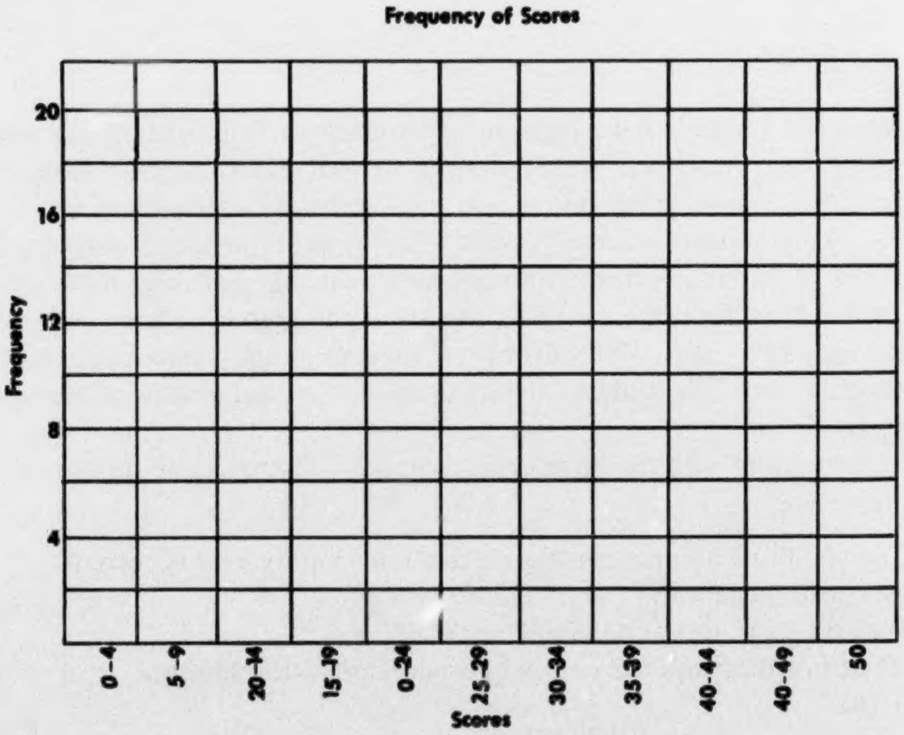
## EVALUATION

The responses of the students to the discussion questions and their work on the worksheets should give you a very good idea of their competence in describing data they have collected.

GRAPHING AND INTERPRETING DATA UNIT 11, EXPERIENCE 3

Activity 1 Name ______________________________

Follow the directions given in class to construct a bar graph to represent the class data from the Bomber over Target game.



Activity 2, Station A Name ______________________________

### *LETTER COUNT*

Take a paragraph out of one of the Letter Count envelopes.

Look at the first word. Suppose it is the word *when*. *When* is a 4-letter word, so you should put a tally mark after the 4 in the Tally column of the table below. Use hyphenated words as one word.

When you have tallied the number of letters for each word in your paragraph, find the total number of tallies for each number of letters and put that number in the last column.

Individual Data Sheet

| Number of Letters in Word | Tally | Frequency |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |
| 9 or More | | |

Enter these results in your group data report, then go on to the next page.

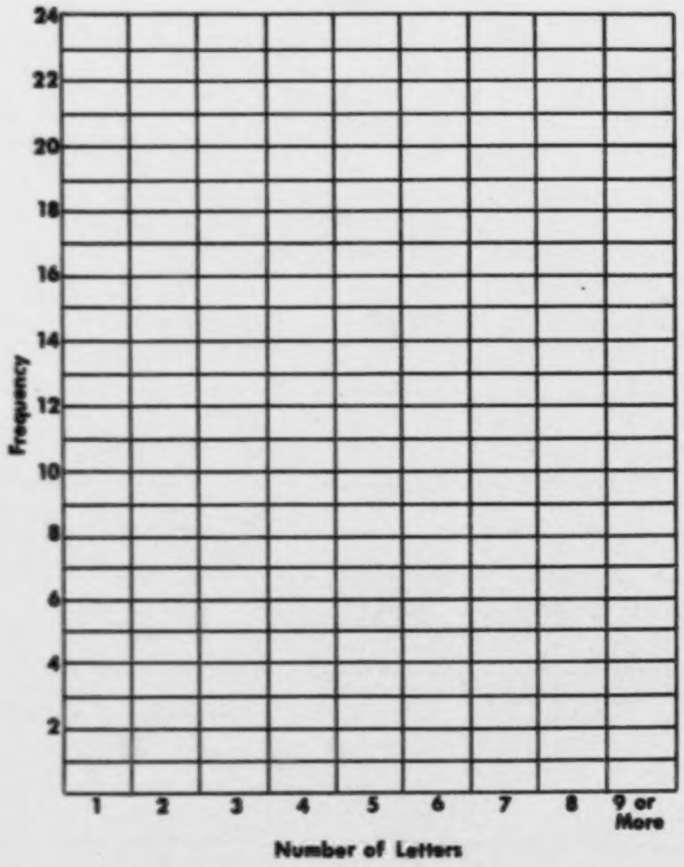
*[Continued]*

GRAPHING AND INTERPRETING DATA UNIT 11, EXPERIENCE 3

Activity 2–*Continued* Name ______________________________

Make a graph of the information in your Individual Data Sheet.

**Individual Data**



When your group data sheet has been completed, make a graph of the information. No form is provided.

GRAPHING AND INTERPRETING DATA UNIT 11, EXPERIENCE 3

Activity 3, Station B Name ______________________________

### SPINNER SUMS

Pick up a spinner from Station B and take it to your desk. Keep it flat on your desk and spin the pointer once. Look at the number that the pointer indicates. If a pointer stops on a line, spin it again. Then spin it again to get a second number.

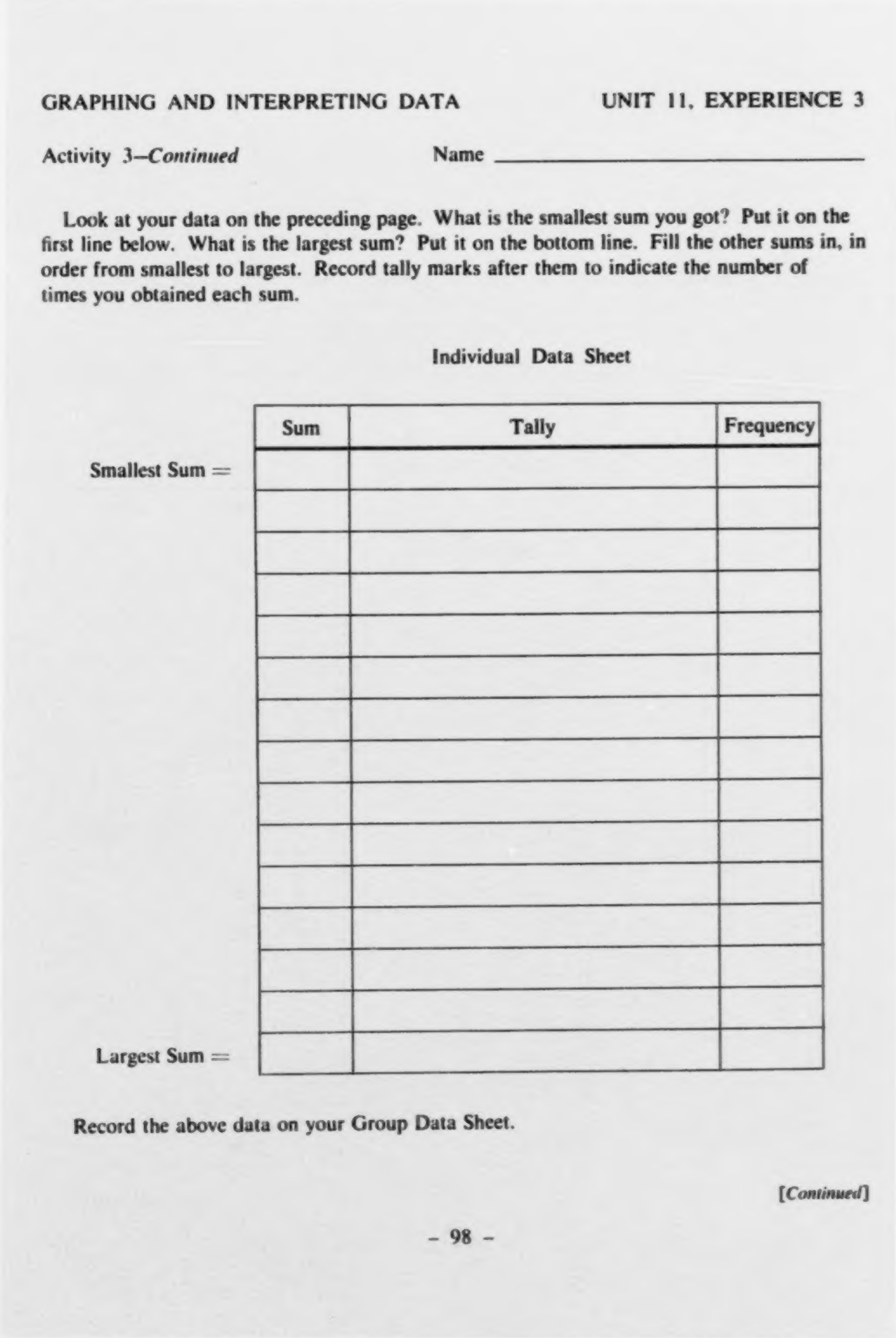
Find the sum of the numbers. Write the sum after Trial 1, below. Continue spinning the pointer and finding the sums until you have filled in all the spaces in the charts below.

| Trial | Sum |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| 13 | |
| 14 | |
| 15 | |
| 16 | |

| Trial | Sum |
|---|---|
| 17 | |
| 18 | |
| 19 | |
| 20 | |
| 21 | |
| 22 | |
| 23 | |
| 24 | |
| 25 | |
| 26 | |
| 27 | |
| 28 | |
| 29 | |
| 30 | |
| 31 | |
| 32 | |

| Trial | Sum |
|---|---|
| 33 | |
| 34 | |
| 35 | |
| 36 | |
| 37 | |
| 38 | |
| 39 | |
| 40 | |
| 41 | |
| 42 | |
| 43 | |
| 44 | |
| 45 | |
| 46 | |
| 47 | |
| 48 | |

[*Continued*]



GRAPHING AND INTERPRETING DATA UNIT 11, EXPERIENCE 3

Activity 3—*Continued* Name ______________________________

Look at your data on the preceding page. What is the smallest sum you got? Put it on the first line below. What is the largest sum? Put it on the bottom line. Fill the other sums in, in order from smallest to largest. Record tally marks after them to indicate the number of times you obtained each sum.

**Individual Data Sheet**

| | Sum | Tally | Frequency |
|---|---|---|---|
| Smallest Sum = | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| Largest Sum = | | | |

Record the above data on your Group Data Sheet.

[*Continued*]

GRAPHING AND INTERPRETING DATA UNIT 11, EXPERIENCE 3

Activity 3–*Continued* Name ______________________________

Complete this chart just as you did when you organized your own data except that this time you should use the data reported by your group on the group data report form. When you have completed the chart, use the information to construct a graph in the grid provided on the next page.

Group Data Sheet

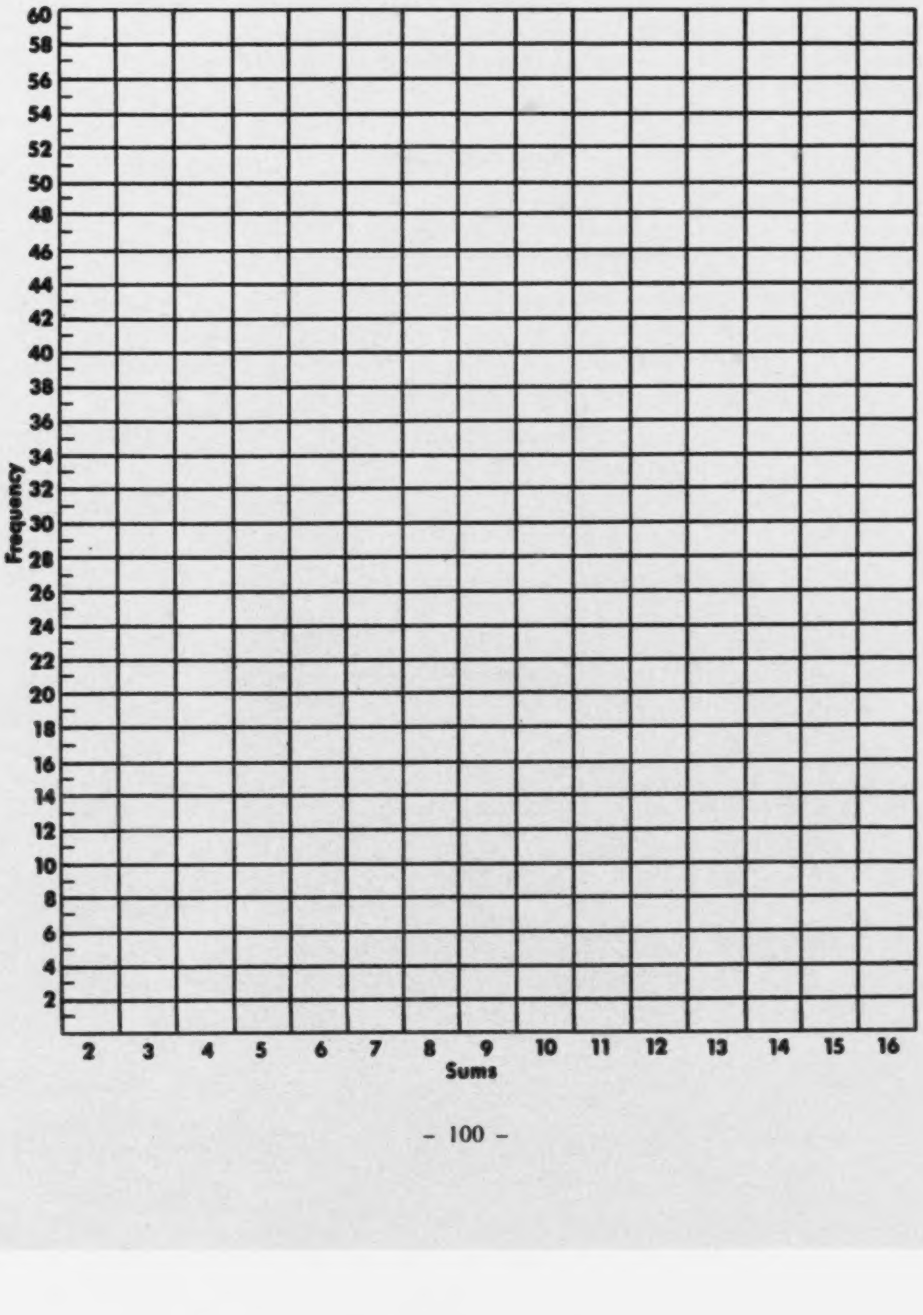
| Sum | Frequency |
|---|---|
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| 13 | |
| 14 | |
| 15 | |
| 16 | |

[*Continued*]

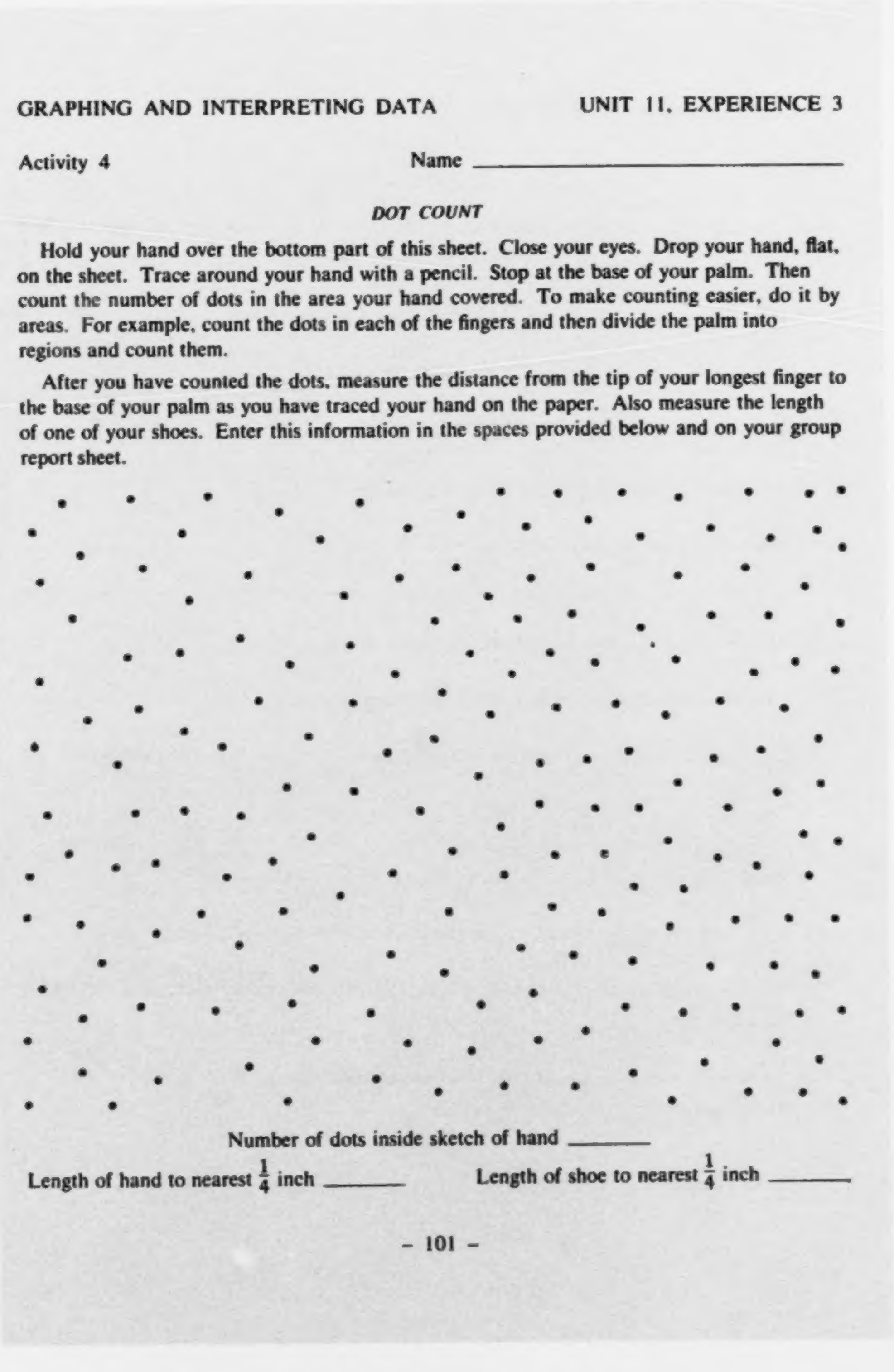
GRAPHING AND INTERPRETING DATA UNIT 11, EXPERIENCE 3

Activity 3—*Continued* Name ______________________________

Graph of Group Data



- 100 -



GRAPHING AND INTERPRETING DATA UNIT 11. EXPERIENCE 3

Activity 4 Name ______________________

## DOT COUNT

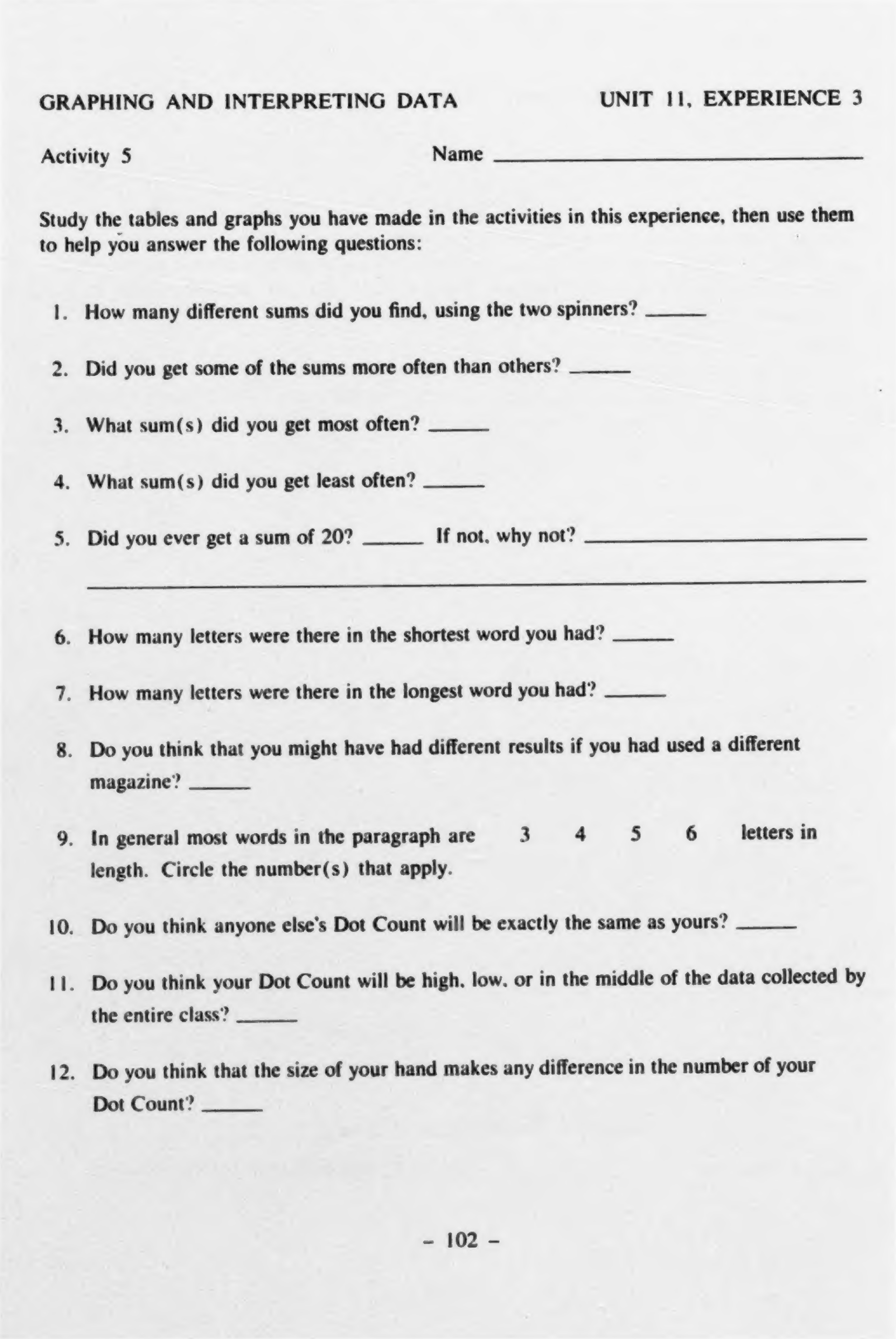
Hold your hand over the bottom part of this sheet. Close your eyes. Drop your hand, flat, on the sheet. Trace around your hand with a pencil. Stop at the base of your palm. Then count the number of dots in the area your hand covered. To make counting easier, do it by areas. For example, count the dots in each of the fingers and then divide the palm into regions and count them.

After you have counted the dots, measure the distance from the tip of your longest finger to the base of your palm as you have traced your hand on the paper. Also measure the length of one of your shoes. Enter this information in the spaces provided below and on your group report sheet.

Number of dots inside sketch of hand ______

Length of hand to nearest $\frac{1}{4}$ inch ______ Length of shoe to nearest $\frac{1}{4}$ inch ______

- 101 -



**GRAPHING AND INTERPRETING DATA** **UNIT 11, EXPERIENCE 3**

Activity 5 Name ______________________________

Study the tables and graphs you have made in the activities in this experience, then use them to help you answer the following questions:

1. How many different sums did you find, using the two spinners? _____
2. Did you get some of the sums more often than others? _____
3. What sum(s) did you get most often? _____
4. What sum(s) did you get least often? _____
5. Did you ever get a sum of 20? _____ If not, why not? ______________________
______________________________________________________________
6. How many letters were there in the shortest word you had? _____
7. How many letters were there in the longest word you had? _____
8. Do you think that you might have had different results if you had used a different magazine? _____
9. In general most words in the paragraph are 3 4 5 6 letters in length. Circle the number(s) that apply.
10. Do you think anyone else's Dot Count will be exactly the same as yours? _____
11. Do you think your Dot Count will be high, low, or in the middle of the data collected by the entire class? _____
12. Do you think that the size of your hand makes any difference in the number of your Dot Count? _____

# EXPERIENCE 4
## More on Interpreting Data

### OBJECTIVE

The student should be able to determine whether there is any apparent relation between different types of data, using graphical methods.

### MATERIALS

Experience 2 Button Toss materials
Experience 3 worksheets and group reports
Experience 3 spinners and paragraphs
3 grids
1 transparency (from Teaching Package) showing 3 graphs
8 worksheets for each student

### TEACHER STRATEGY

I like to introduce this experience by pointing out that there are other ways of representing and describing data in addition to the frequency tables and graphs we have been using. One way is to construct a graph containing two different types of data, like the one in the Teaching Package comparing heights and weights of the boys in a classroom, and look for a pattern. I project this completed graph, explain how the data were placed on the grid, and ask the students if they notice any pattern in this set of points. Someone should see that they form a path in an upward motion from bottom left to upper right. Then see if anyone can explain the meaning of the pattern. The students may suggest that as the height increases, so does the weight; or they may say that the points lie near a line that is higher on the right.

Then I project another part of the same transparency. This one pairs the height of each student in a mathematics class with his grade.

I ask the students to describe any pattern that they see. When they observe that there seems to be no pattern and that the dots are quite well scattered over the entire grid, I tell them that there is no simple relation between the heights of students in this class and their grades.

Now I remind the students that they collected some interesting data in Experience 3, ask what kind of graph they would get if they plotted the Dot Count data, and project the grid for this purpose. Several may guess that the number of dots will increase with the length of the hand, giving an upward movement as in the graph for heights and weights. I ask each student to report his Dot Count data and plot the dots on the transparency. When all the points have been plotted I ask the students to describe the relationship, if any. (There may be one, but my own experience has been that because of the random scattering of the dots there is no clear pattern.)

At this point I have the students return to the three groups established during Experience 3. Each group is given a grid marked "Foot Sizes" on the vertical axis and "Hand Sizes" on the horizontal axis. I direct the students to look over their group report sheet for the data, decide on the intervals to be listed in both margins, and plot the data. After this has been done, I lead a brief discussion of the results of each of the three groups.

For the worksheet activities, I regroup the students into teams of three. Both Spinner Relations and Word Lengths and Number of Vowels are essentially individual projects, but the teams should stay together and share the materials at each station. The setup for Button Toss is like that in Experience 2 except that the students make their tosses standing behind arcs of circles with center at the target and radii of two, four, six, eight, ten, and twelve feet. Each student makes five tosses from behind each of the six arcs; while one tosses the buttons a second measures the distances and a third records the data. If time permits each student gets a second turn and the tallies of distances can be recorded in column 2 of the Individual Data Sheet.

At the conclusion of these activities I have each student show one of the graphs and discuss the relationship he found. I have other students point out any differences in their results.

## EVALUATION

The discussion and individual worksheets should show that each student was able to graph data from various sources and discover relationships by graphing the data.

**MORE ON INTERPRETING DATA** **UNIT 11, EXPERIENCE 4**

**Activity 1** **Name** ______________________________

### *BUTTON TOSS*

**Complete the Individual Data charts, then have one member of your group complete the Group Data Sheet on the next page.**

**Individual Data**

**Line A (2 ft.)**

| Distance in Inches | Tally | |
|---|---|---|
| | 1 | 2 |
| 0–3 | | |
| 4–7 | | |
| 8–11 | | |
| 12–15 | | |
| 16–19 | | |
| 20–23 | | |
| 24–27 | | |
| 28–31 | | |
| 32–35 | | |
| 36–39 | | |
| 40 or More | | |

**Scores** ____ ____

**Line B (4 ft.)**

| Distance in Inches | Tally | |
|---|---|---|
| | 1 | 2 |
| 0–3 | | |
| 4–7 | | |
| 8–11 | | |
| 12–15 | | |
| 16–19 | | |
| 20–23 | | |
| 24–27 | | |
| 28–31 | | |
| 32–35 | | |
| 36–39 | | |
| 40 or More | | |

**Scores** ____ ____

**Line C (6 ft.)**

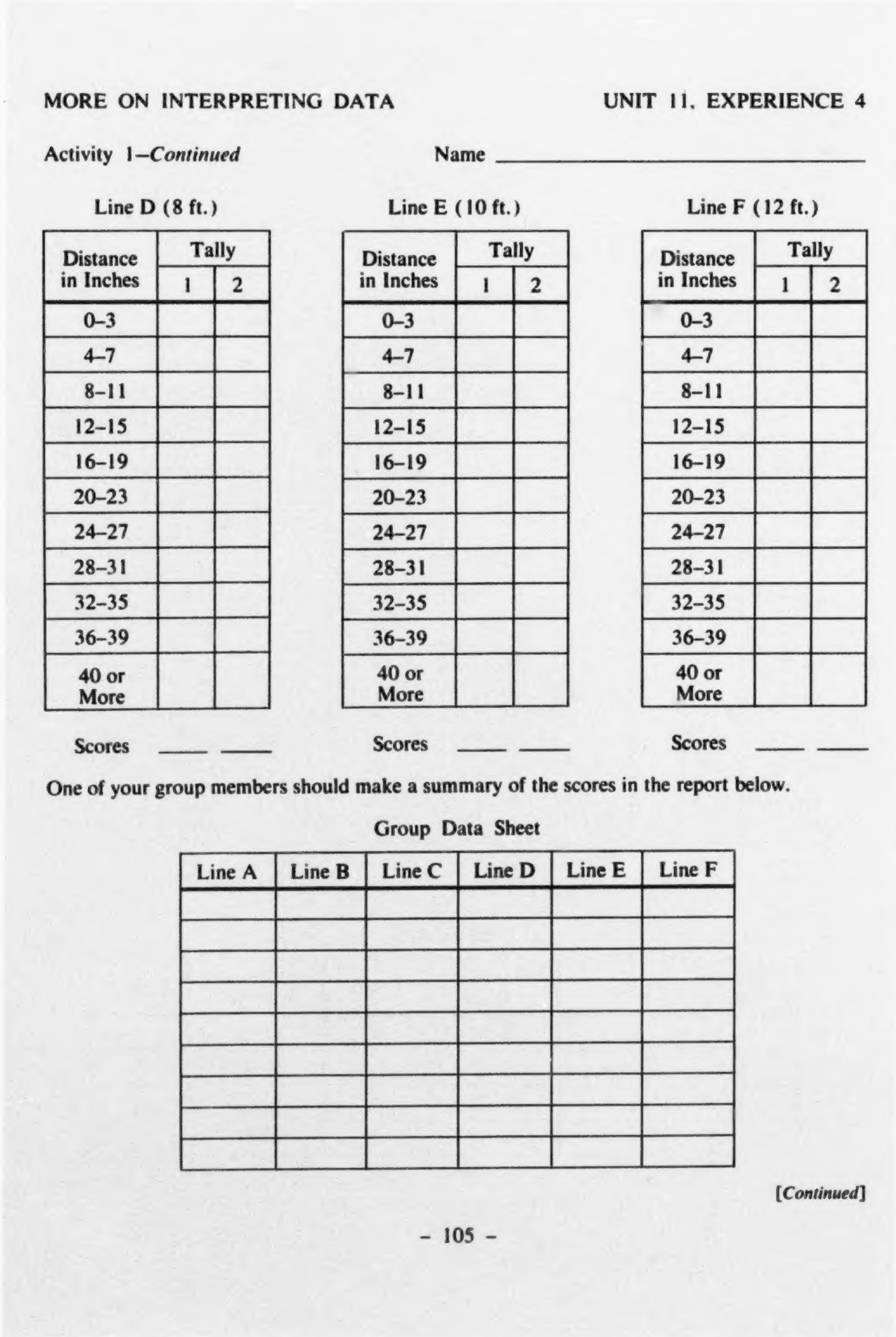
| Distance in Inches | Tally | |
|---|---|---|
| | 1 | 2 |
| 0–3 | | |
| 4–7 | | |
| 8–11 | | |
| 12–15 | | |
| 16–19 | | |
| 20–23 | | |
| 24–27 | | |
| 28–31 | | |
| 32–35 | | |
| 36–39 | | |
| 40 or More | | |

**Scores** ____ ____

**Score Key**

| Distance in Inches | Points | Distance in Inches | Points | Distance in Inches | Points |
|---|---|---|---|---|---|
| 0–3 | 10 | 16–19 | 6 | 32–35 | 2 |
| 4–7 | 9 | 20–23 | 5 | 36–39 | 1 |
| 8–11 | 8 | 24–27 | 4 | 40 or More | 0 |
| 12–15 | 7 | 28–31 | 3 | | |

*[Continued]*



MORE ON INTERPRETING DATA — UNIT 11, EXPERIENCE 4

Activity 1—*Continued* — Name ____________________

Line D (8 ft.)

| Distance in Inches | Tally | |
|---|---|---|
| | 1 | 2 |
| 0–3 | | |
| 4–7 | | |
| 8–11 | | |
| 12–15 | | |
| 16–19 | | |
| 20–23 | | |
| 24–27 | | |
| 28–31 | | |
| 32–35 | | |
| 36–39 | | |
| 40 or More | | |

Scores ___ ___

Line E (10 ft.)

| Distance in Inches | Tally | |
|---|---|---|
| | 1 | 2 |
| 0–3 | | |
| 4–7 | | |
| 8–11 | | |
| 12–15 | | |
| 16–19 | | |
| 20–23 | | |
| 24–27 | | |
| 28–31 | | |
| 32–35 | | |
| 36–39 | | |
| 40 or More | | |

Scores ___ ___

Line F (12 ft.)

| Distance in Inches | Tally | |
|---|---|---|
| | 1 | 2 |
| 0–3 | | |
| 4–7 | | |
| 8–11 | | |
| 12–15 | | |
| 16–19 | | |
| 20–23 | | |
| 24–27 | | |
| 28–31 | | |
| 32–35 | | |
| 36–39 | | |
| 40 or More | | |

Scores ___ ___

One of your group members should make a summary of the scores in the report below.

Group Data Sheet

| Line A | Line B | Line C | Line D | Line E | Line F |
|---|---|---|---|---|---|
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |

*[Continued]*

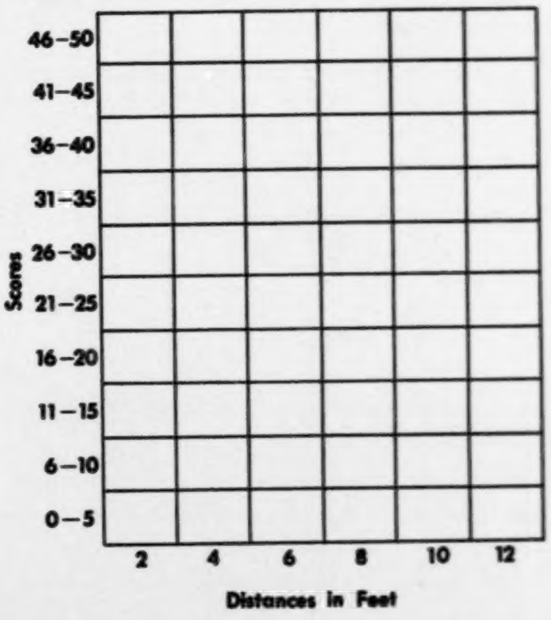
MORE ON INTERPRETING DATA UNIT 11, EXPERIENCE 4

Activity 1–*Continued* Name ______________________

Complete the chart and the graph below.

| Scores | |
|---|---|
| A Line (2 ft.) | |
| B Line (4 ft.) | |
| C Line (6 ft.) | |
| D Line (8 ft.) | |
| E Line (10 ft) | |
| F Line (12 ft.) | |

Group Results



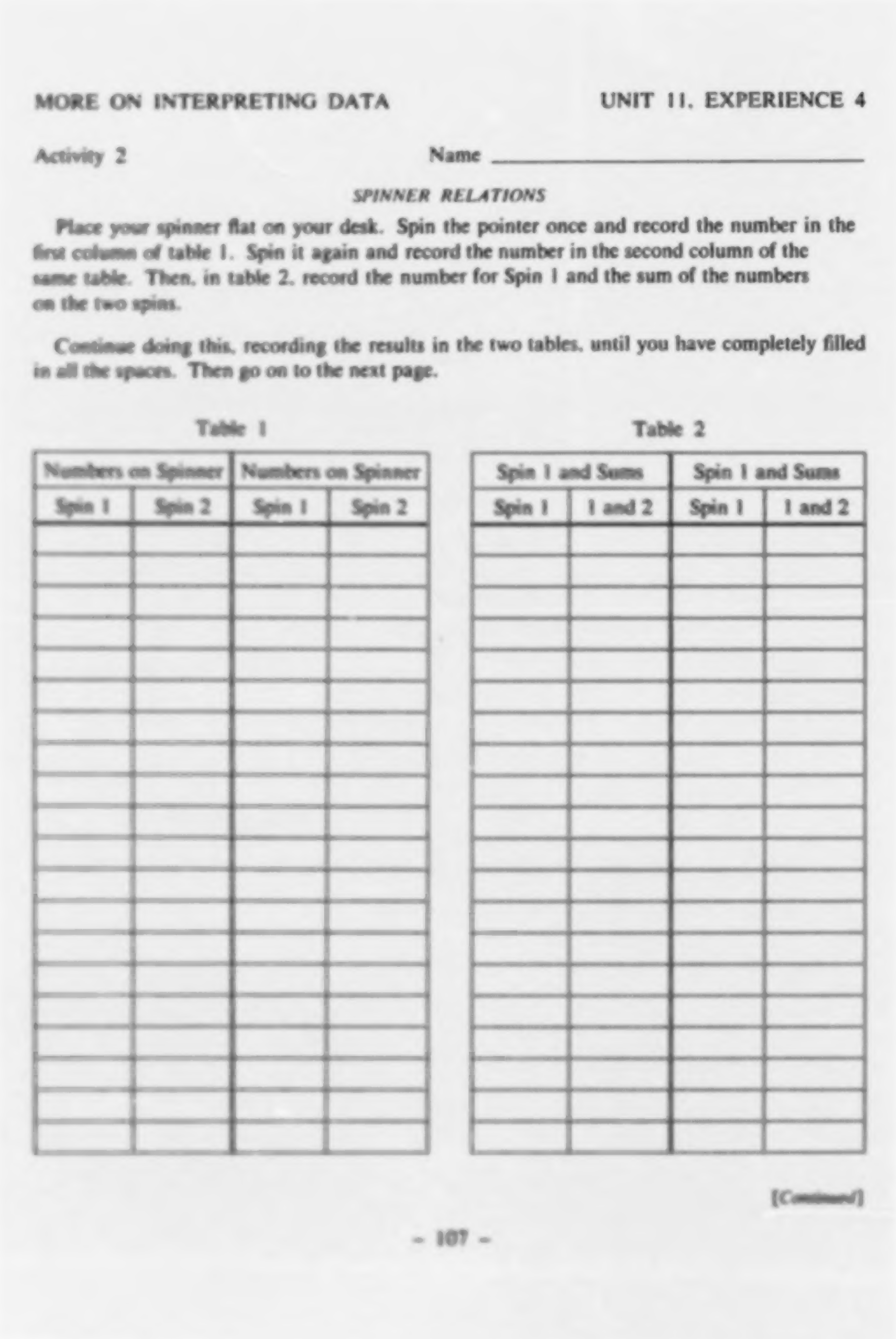
According to your group's graph, did there appear to be any relation between the distances you stood from the target line and the scores you made? ______

If so, what relation did you notice? ______________________

______________________________________________

From which line were the highest scores made? ______

From which line were the lowest scores made? ______



MORE ON INTERPRETING DATA UNIT 11, EXPERIENCE 4

Activity 2 Name ______________________________

*SPINNER RELATIONS*

Place your spinner flat on your desk. Spin the pointer once and record the number in the first column of table 1. Spin it again and record the number in the second column of the same table. Then, in table 2, record the number for Spin 1 and the sum of the numbers on the two spins.

Continue doing this, recording the results in the two tables, until you have completely filled in all the spaces. Then go on to the next page.

Table 1

| Numbers on Spinner | | Numbers on Spinner | |
|---|---|---|---|
| Spin 1 | Spin 2 | Spin 1 | Spin 2 |
| | | | |
| | | | |
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Table 2

| Spin 1 and Sums | | Spin 1 and Sums | |
|---|---|---|---|
| Spin 1 | 1 and 2 | Spin 1 | 1 and 2 |
| | | | |
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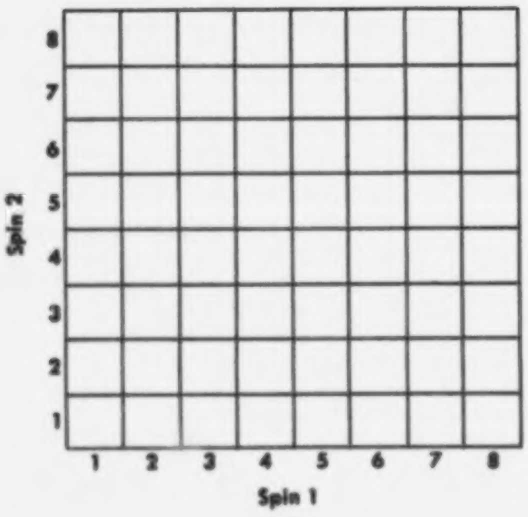
MORE ON INTERPRETING DATA UNIT 11, EXPERIENCE 4

Activity 2–*Continued* Name ______________________________

Use the pairs of numbers that you collected for table 1 of Spinner Relations to construct the graph below.

For each pair of numbers there should be a dot on the graph. For example: If the first pair of numbers was a 5 on Spin 1 and a 2 on Spin 2, you should place a dot, as shown, in the space where the fifth vertical column and the second horizontal column intersect.



Answer the questions below, then go on to the next page.

Does your graph seem to indicate that there is any relation between the numbers on Spin 1 and Spin 2? _____

If so, what relation do you notice? ______________________________

______________________________

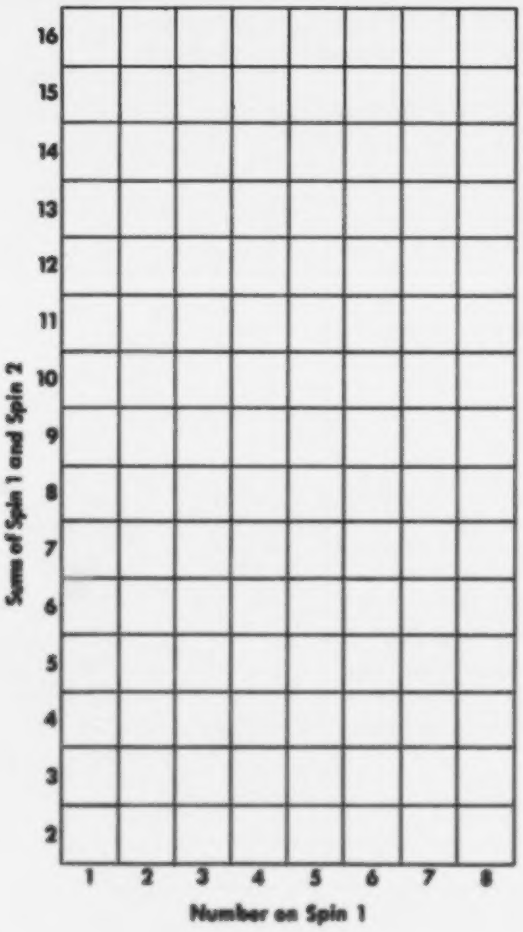
Did you expect to find a relation? _____

[*Continued*]

MORE ON INTERPRETING DATA UNIT 11, EXPERIENCE 4

Activity 2–*Continued* Name ______________________________

Use the same procedure as before, but this time use the data from table 2 and look for a relation between the numbers on Spin 1 and the sum of the numbers on Spins 1 and 2.



Does your graph seem to indicate that there is any relation between the numbers on Spin 1 and the sums of the numbers of Spins 1 and 2? _____

If it does, what relation do you notice? ____________________

____________________

____________________

Did you expect to find a relation? _____

Do you think that there would be a relation between the numbers on Spin 2 and the sums of the numbers of Spins 1 and 2? _____

MORE ON INTERPRETING DATA UNIT 11, EXPERIENCE 4

Activity 3 Name ______________________________

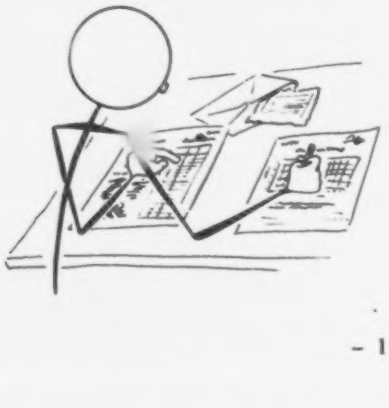
### *WORD LENGTHS AND NUMBER OF VOWELS*

How do you think the number of vowels a word has compares with the number of letters it has? The longer the word the more vowels? How about the words *a* and *check?* Both words have only 1 vowel, but one has 1 letter and the other has 5. Perhaps these are just exceptions?

Take a paragraph from the envelope. Look at the first word. Put the number of letters it has in the top space of the first column and the number of vowels in the top space of the second column. Make one entry in the chart for each word in your paragraph.

Make a graph of the data in the chart on the grid provided on the next page. There should be one dot in your graph for each pair of numbers that appears in your chart.

When the graph is completed, look to see if there seems to be any relation between the number of letters in the word and the number of vowels it has. Use the space provided under the graph to describe what you find.



Individual Data

| Letters in Word | Vowels in Word | Letters in Word | Vowels in Word |
|---|---|---|---|
| | | | |
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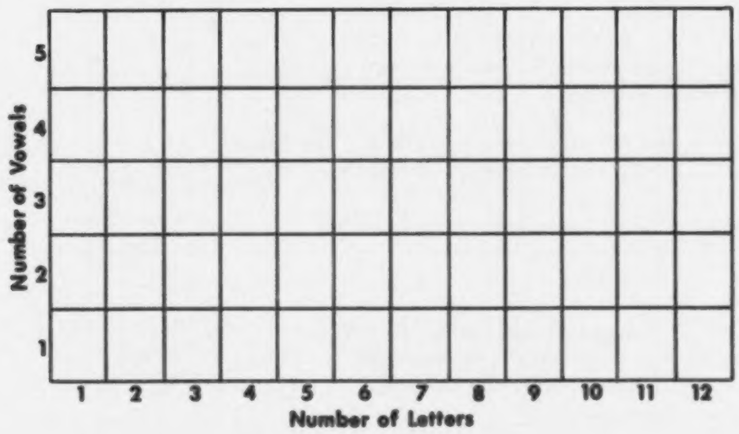
MORE ON INTERPRETING DATA UNIT 11, EXPERIENCE 4

Activity 3–*Continued* Name ______________________

*WORD LENGTHS AND NUMBER OF VOWELS*

Relation Graph



Describe the relation that seems to be shown by the points on your graph. If there seems to be no relation, say so and explain why. ______________________

______________________

______________________

______________________

# EXPERIENCE 5
## Culminating Experience

### OBJECTIVE

The student should be able to use the methods developed in the previous experiences to organize, tabulate, graph, and describe the data given him.

### MATERIALS

1 data sheet and 5 worksheets for each student

### TEACHER STRATEGY

This is a summary experience. There is nothing new in it, but it is aimed at finding whether students can put all of their techniques to use in meeting a new situation.

I like to give students the data sheet and all the worksheets and ask each one to organize his data in whatever manner he wishes so that he can answer the questions on the Summary Sheet (Activity 2). If a student needs help, I ask him to make a start on the first chart and then, if he still needs help, to ask me to check his start. Most students are then able to do the other graphs on their own.

After the students have completed their work, I have different ones place a table on the board and then discuss the description.

We use the summary sheets as a basis for discussion.

### EVALUATION

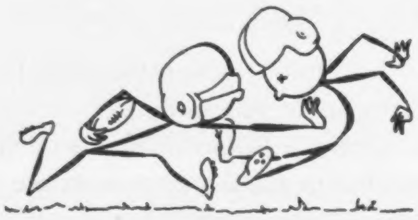
The graphs and tables should give quite a complete picture of the students' understanding.

Through class discussion of answers on the Summary Sheet, you should be able to see if the students are able to interpret what they have tabulated. This class-sharing time serves as an additional learning experience for students who have been still unsure of the ideas presented in this unit.

### *Football Roster*
### *STATE UNIVERSITY*

| Number | Height | Weight | Grade Average |
|---|---|---|---|
| 15 | 73 | 170 | A |
| 70 | 72 | 205 | B |
| 72 | 73 | 210 | B |
| 67 | 71 | 205 | C |
| 43 | 71 | 196 | C |
| 61 | 71 | 205 | B |
| 78 | 72 | 220 | A |
| 74 | 71 | 203 | B |
| 76 | 72 | 215 | C |
| 38 | 69 | 175 | B |
| 51 | 74 | 190 | C |
| 28 | 72 | 180 | A |
| 27 | 70 | 173 | D |
| 30 | 69 | 175 | B |
| 54 | 69 | 205 | B |
| 53 | 72 | 200 | C |
| 25 | 70 | 168 | B |
| 75 | 72 | 205 | A |
| 60 | 70 | 195 | D |
| 20 | 72 | 170 | A |

| Number | Height | Weight | Grade Average |
|---|---|---|---|
| 47 | 72 | 200 | B |
| 24 | 71 | 173 | C |
| 22 | 69 | 165 | B |
| 19 | 71 | 173 | B |
| 42 | 70 | 188 | A |
| 34 | 74 | 175 | D |
| 36 | 70 | 195 | D |
| 18 | 71 | 175 | B |
| 65 | 73 | 210 | C |
| 31 | 70 | 167 | B |
| 62 | 68 | 200 | C |
| 33 | 71 | 182 | A |
| 83 | 74 | 232 | A |
| 49 | 71 | 182 | B |
| 88 | 75 | 240 | B |
| 58 | 70 | 190 | C |



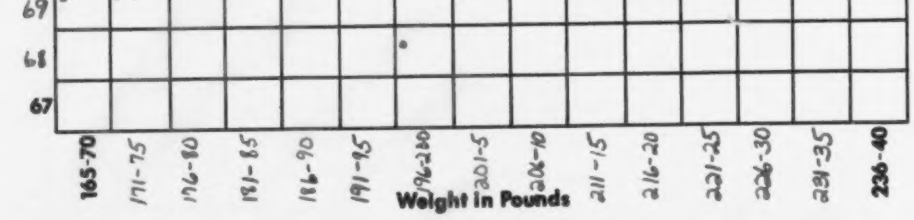
CULMINATING EXPERIENCE

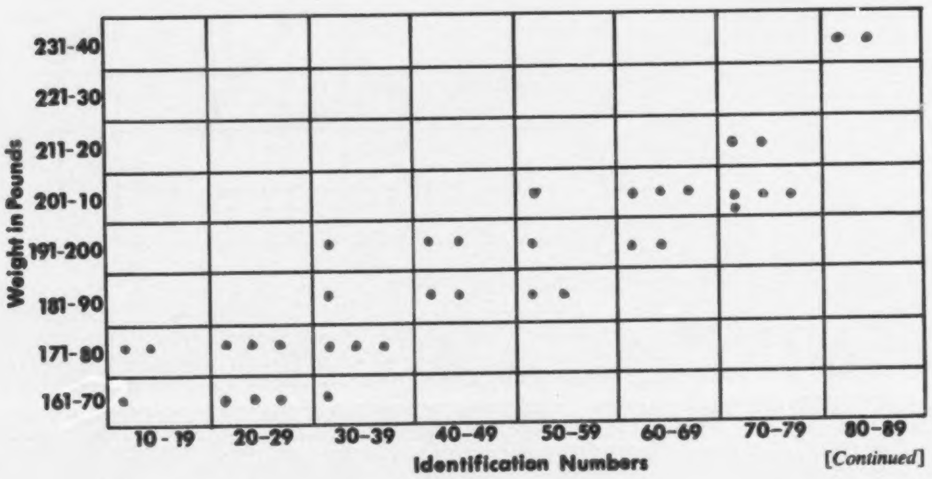
UNIT 11, EXPERIENCE 5

Activity

Name ______________________________

Using data from the Football Roster sheet, complete the two graphs below and a third one, at the top of the next page.





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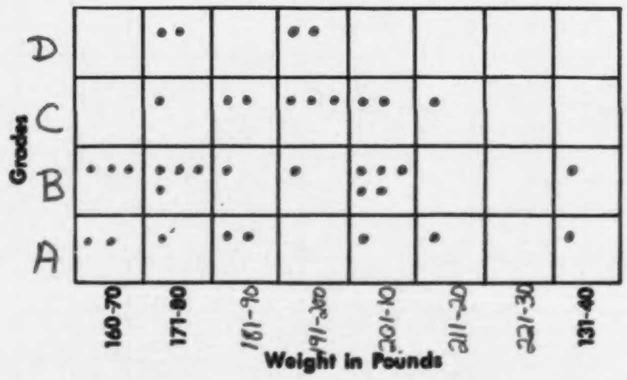
**CULMINATING EXPERIENCE** **UNIT 11, EXPERIENCE 5**

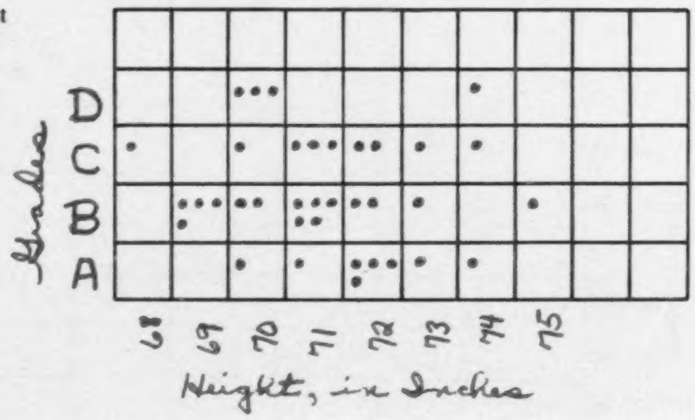
Activity 1—*Continued* Name ____________________

Grades and Weight



Use the grid at the right to make any other comparison that you would like to consider.

Grades and Height



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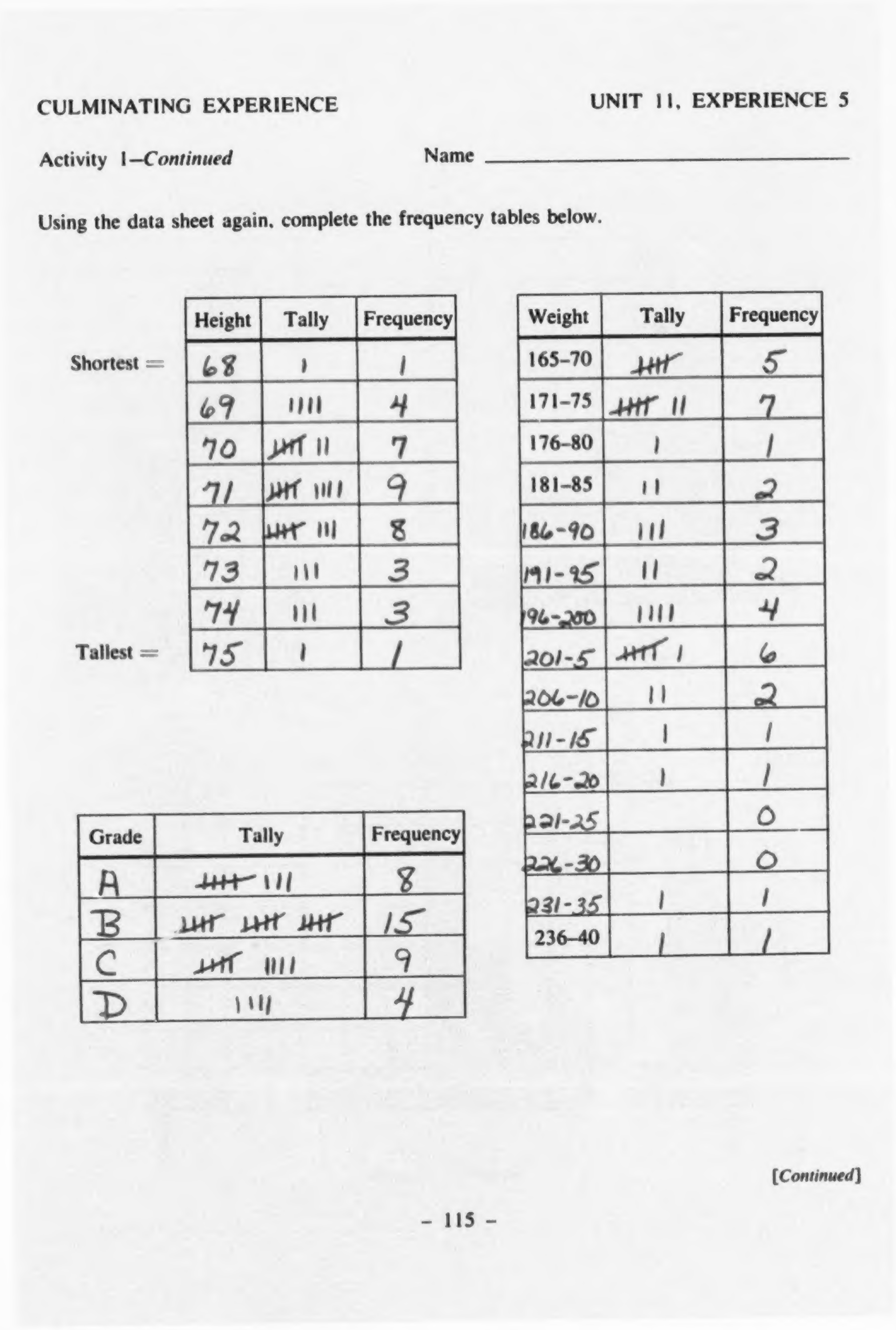


CULMINATING EXPERIENCE UNIT 11, EXPERIENCE 5

Activity 1—*Continued* Name ______________________

Using the data sheet again, complete the frequency tables below.

| | Height | Tally | Frequency |
|---|---|---|---|
| Shortest = | 68 | \| | 1 |
| | 69 | \|\|\|\| | 4 |
| | 70 | 卌 \|\| | 7 |
| | 71 | 卌 \|\|\|\| | 9 |
| | 72 | 卌 \|\|\| | 8 |
| | 73 | \|\|\| | 3 |
| | 74 | \|\|\| | 3 |
| Tallest = | 75 | \| | 1 |

| Weight | Tally | Frequency |
|---|---|---|
| 165–70 | 卌 | 5 |
| 171–75 | 卌 \|\| | 7 |
| 176–80 | \| | 1 |
| 181–85 | \|\| | 2 |
| 186-90 | \|\|\| | 3 |
| 191-95 | \|\| | 2 |
| 196-200 | \|\|\|\| | 4 |
| 201-5 | 卌 \| | 6 |
| 206-10 | \|\| | 2 |
| 211-15 | \| | 1 |
| 216-20 | \| | 1 |
| 221-25 | | 0 |
| 226-30 | | 0 |
| 231-35 | \| | 1 |
| 236–40 | \| | 1 |

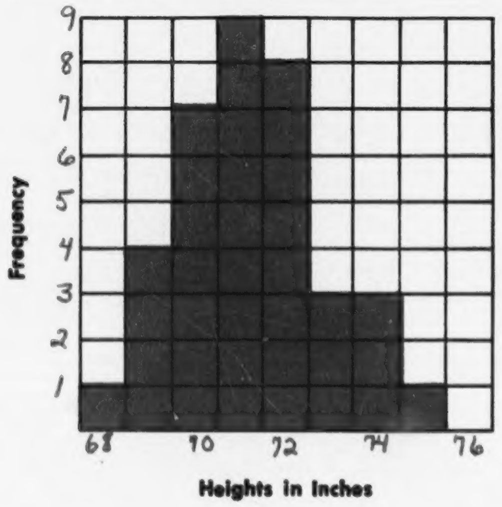
| Grade | Tally | Frequency |
|---|---|---|
| A | 卌 \|\|\| | 8 |
| B | 卌 卌 卌 | 15 |
| C | 卌 \|\|\|\| | 9 |
| D | \|\|\|\| | 4 |

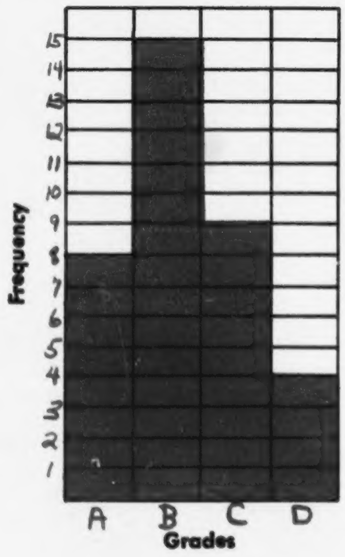
[*Continued*]

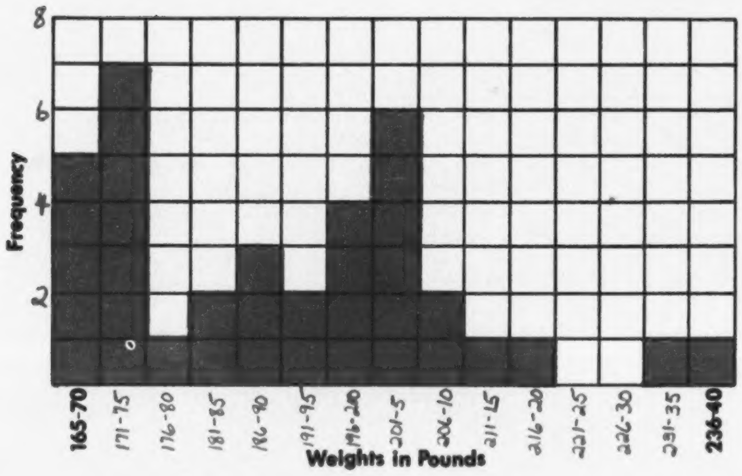
CULMINATING EXPERIENCE — UNIT 11, EXPERIENCE 5

Activity 1—*Continued* Name ________________________

Using the frequency tables you have just made, complete the graphs below.







CULMINATING EXPERIENCE

UNIT 11, EXPERIENCE 5

Activity 2

Name ______________________________

Summary Sheet

1. What is the height of the tallest player? 75 in.
2. What is the height of the shortest player? 68 in.
3. The most common height is 71 in.
4. Which player is the lightest? #22
5. Which player is the heaviest? #88
6. From your frequency graph, what would you consider to be an average weight for the team? 193 (or 191-95)
7. Are these players good students? Yes
8. How many have an A average? 8
9. More players have a(n) B average than any other.
10. The average for the team is in which range? Circle one.

    A–B [✓ B–C] C–D D–F
11. What relations did you find?
    - *a*) Does there appear to be any relation between height and weight? Yes

      If so, state some specific relation. Weight increases with height.
    - *b*) Does there appear to be any relation between the players' numbers and their weight? Yes If so, state some specific relation. The heavier players have higher numbers. (Customary for linemen.)
    - *c*) Is there any relation between grades and weight? No

      If so, state some specific relation. Mathematically speaking, there is a relation; any set of number pairs is a relation.
12. Give a description of the typical player on the team using the information you have obtained about his height, weight, and grades. He is 71 inches tall, weighs about 193 pounds, and earns B grades.

# 12

## EXPERIENCES IN

## Dealing with Uncertainty

How often are you really sure about anything? Don't you often find yourself thinking, "*Probably* it will rain tomorrow," or "There is a *chance* that our team will win the game," or "*More likely than not* Johnny will be absent again tomorrow"? Children, too, think this way, and many of them even use these same words in their own conversations. The activities in this unit are designed to help students understand these expressions and use them in a more meaningful way.

If we were to deal fully with the concepts associated with such expressions we would quickly find ourselves in a rather deep study of probability theory. We certainly do not propose such a study here; instead, we will deal with these ideas on an informal and intuitive level. Our goal is to give the student a taste of these ideas so that he will get something of the flavor of uncertainty and learn how to handle it in an elementary way.

Just as it is easier to think clearly if you first organize your thoughts, many problems in mathematics are easier if you order the ideas presented before you attempt to solve the problem. Therefore, in this unit students are given practice in organizing and in collecting and analyzing fairly simple data. They begin by working with number patterns found in everyday situations and by manipulating objects like coins and dice. Toward the end of the unit they have a chance to make predictions based on data they have collected, organized, and analyzed on their own. It is hoped that the students will find the experiences interesting and challenging.

Since the experiences in this unit involve an element of chance, a complete set of answers cannot be given on the reduced worksheets inserted in the text. However, note that at the end of the unit there is a section called Probability Theory: Supplementary Information. It contains a statement about the mathematical concepts involved in each experience.

## OVERVIEW

Each of the five experiences in this unit includes a detailed section called "Teacher Strategy," which is one teacher's first-person account of a procedure that proved effective with his class.

The following brief summary will help you decide whether these experiences meet the needs of your own class and will also give you some indication of the amount of preparation involved.

Sample student worksheets are provided in the Teaching Package. It is assumed that duplicating facilities are available to you so you can provide class lots of these worksheets.

### Experience 1: Recording Last Digits and Sums of Last Two Digits

Real-life situations are devised in the classroom, and attention is focused on both events that happen often and events that happen rarely. Students then work in pairs on the worksheets. They are encouraged to make predictions, based on their own recorded observations, about which events are common and which are rare.

Materials needed: telephone directory, worksheets.

### Experience 2: Tossing Coins

Students experiment with flipping one and two coins at a time, first working alone, then in pairs. They record the number of times "heads" turns up and the number of times "tails" turns up. They then make predictions based on these observations and gather additional data to test their predictions.

Materials needed: coins, worksheets.

**Experience 3: Rolling Dice**

Students roll one die many times to determine whether any one face turns up more often than others. They then roll a pair of dice to determine which sums are most likely to turn up in a cast of dice.

Materials needed: dice, worksheets.

**Experience 4: Drawing Beads**

Students try to predict the combinations of three colors of beads they are most likely to draw from a can. Then they try to determine the color of both the greatest number of beads and the least number of beads in the can.

Materials needed: large can, small cans, three different colors of beads, worksheets.

**Experience 5: Tossing Objects**

Students toss paper cups and thumbtacks to determine the position in which they are most likely to fall.

Materials needed: paper or plastic cups, thumbtacks, worksheets.

# EXPERIENCE 1
## Recording Last Digits and Sums of Last Two Digits

### OBJECTIVE

The student should be able to show that certain sums of two digits occur more frequently than others. He should also be able to give some explanation of why this happens.

### MATERIALS

Telephone directory
3 worksheets for each student

### TEACHER STRATEGY

I like to begin this experience by calling one student to the front of the room. I ask him to open the phone directory anywhere in the white pages and to point to any one of the phone numbers on the page, then copy this number on the chalkboard.

I ask another student to name the last two digits of this phone number. A discussion follows to make sure all the students know what I mean by phrases like "the first digit," "the last two digits," and "the sum of the last two digits."

Next I call two students to the front of the room. I give one student the phone directory and ask the second student to be the recorder. The first student opens the directory to a white page and picks a phone number on that page. He then reads the last digit of each of the next fourteen numbers. As he reads each digit the recorder writes it on the board.

Referring to the list on the board, I ask, "What is the least digit we found? What is the greatest digit we found? If we did this again, what is the least digit we could find?" Continuing in this way, I lead up to a convenient way of recording the results of such "experiments" (see fig. 1 on the following page). The recorder transfers the results of our experiments to this frequency table on the chalkboard.

I then begin asking questions about the digit found most often, least

| Digit | Tally | Total |
|---|---|---|
| 0 | | |
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |
| 9 | | |

Fig. 1

often, and so on. These lead up to a question like, "If I pick a phone number and ask you to guess the last digit, what would you guess?" Of course there is no "right" answer. But the discussion might reveal the interesting notions some students may have about their "lucky number" and other beliefs. I take these as they come and wait for students to acquire new ideas about chance events as they continue their experiments.

You may wish to ask students to begin the individual experiments immediately after this discussion, or you may want to wait until the next day to begin this phase of the experience. When you do introduce this phase, give each student a white page from the phone directory and a copy of the worksheet for Activity 1. Each student picks a partner and the partners take turns collecting and recording data.

Ask the data recorder to guess the sum of the last two digits of a phone number and write his guess in the space provided. His partner then closes his eyes and places his finger on the white page. When the data collector opens his eyes, he gives the actual sum for the last two digits of the number closest to his finger and of the nine numbers following it. The recorder writes in each of these sums and counts up and records the number of times his guess matched the actual sum.

Some students may make a guess like thirty-six for the sum of the last two digits of a phone number but then find out for themselves that the greatest possible sum is eighteen (9 + 9). They will probably also discover that sums like nine, ten, and eleven are more likely than numbers like two and seventeen, since many more pairs of digits add up to eleven, for example, than add up to two.

Each student takes three turns as recorder and three as collector. You will find that some students begin to be better guessers as they go from one trial to the next. Some may even be able to explain, in the discussion at the end of the experience, why a guess like nine is better than a guess like seventeen. (You can get a sum of nine in ten ways: 0 + 9, 1 + 8, 2 + 7, 3 + 6, 4 + 5, and the reverse of each. There are only two ways to get seventeen: 8 + 9 and 9 + 8. Refer to the table given in the supplementary information at the end of this unit for a complete list of the possible combinations of digits and their sums.)

When each student has completed his three trials, he tallies the result of each on the chalkboard. You could put a table like that shown in figure 2 on the board for this purpose.

| Number of Sums That Matched My Guess | Tally |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |

Fig. 2

I don't discuss the tally of actual sums that matched the guessed sum. I have found that students talk about it anyway, and after thinking things over they often say, "We could do better if you gave us another chance."

I either have the second worksheet ready for students who finish the first one quickly or I plan some kind of activity to take up slack time.

You may wish to have students do the worksheets for Activities 2 and 3 in class, or you may want to have them complete parts of them at home. I like to spread this experience over at least two days, and with slow learners I spend three days on it. Sometimes I use the first worksheet several times. I give the directions orally so I can ask students to guess the sum of the last three digits and, thus, extend the possible sum to numbers from zero to twenty-seven. Different classes react differently, but I always try to get across the idea that there can be some science in guessing. With some classes this requires more than one use of the first worksheet.

[*Continued on page 183*]

RECORDING DIGITS AND SUMS OF LAST TWO DIGITS

UNIT 12, EXPERIENCE 1

Activity 1

Name ______________________________

Follow the directions your teacher gives you.

TRIAL 1

My guess ________

Actual sums _____ _____ _____ _____ _____

_____ _____ _____ _____ _____

My guess matched the actual sum ________ times.

TRIAL 2

My guess ________

Actual sums _____ _____ _____ _____ _____

_____ _____ _____ _____ _____

My guess matched the actual sum ________ times.

TRIAL 3

My guess ________

Actual sums _____ _____ _____ _____ _____

_____ _____ _____ _____ _____

My guess matched the actual sum ________ times.

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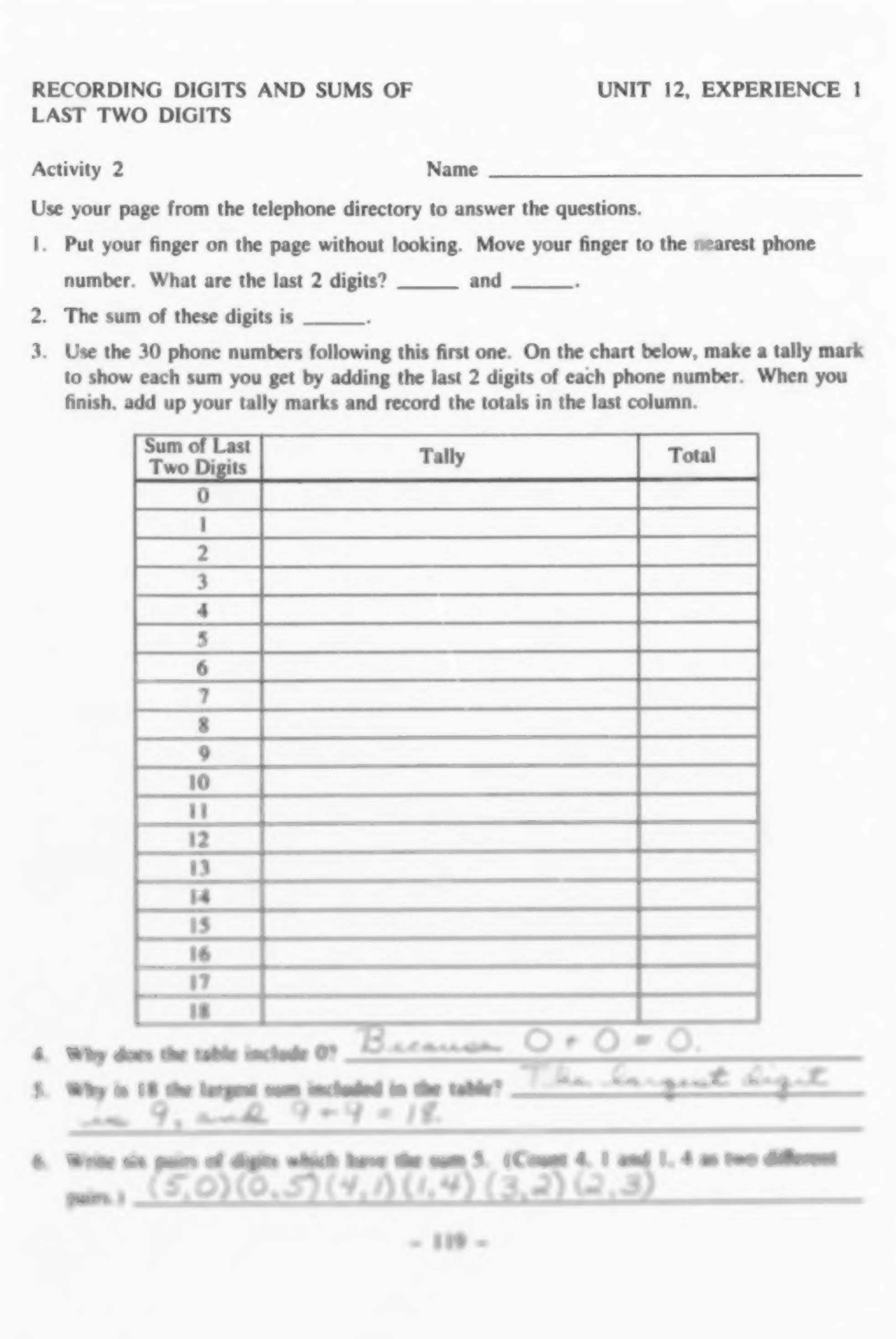


## RECORDING DIGITS AND SUMS OF LAST TWO DIGITS

UNIT 12, EXPERIENCE 1

Activity 2 Name ______________________

Use your page from the telephone directory to answer the questions.

1. Put your finger on the page without looking. Move your finger to the nearest phone number. What are the last 2 digits? _____ and _____.
2. The sum of these digits is _____.
3. Use the 30 phone numbers following this first one. On the chart below, make a tally mark to show each sum you get by adding the last 2 digits of each phone number. When you finish, add up your tally marks and record the totals in the last column.

| Sum of Last Two Digits | Tally | Total |
|---|---|---|
| 0 | | |
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |
| 9 | | |
| 10 | | |
| 11 | | |
| 12 | | |
| 13 | | |
| 14 | | |
| 15 | | |
| 16 | | |
| 17 | | |
| 18 | | |

4. Why does the table include 0? Because 0 + 0 = 0.
5. Why is 18 the largest sum included in the table? The largest digit is 9, and 9 + 9 = 18.
6. Write six pairs of digits which have the sum 5. (Count 4, 1 and 1, 4 as two different pairs.) (5,0) (0,5) (4,1) (1,4) (3,2) (2,3)

RECORDING DIGITS AND SUMS OF LAST TWO DIGITS

UNIT 12, EXPERIENCE 1

Activity 3

Name ______________________________

1. Record the license-plate numbers of 10 cars. Disregard letters—write down only the numerals.

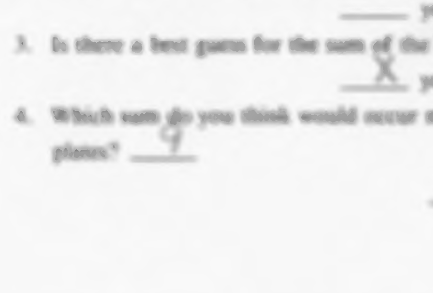
__________ __________ __________ __________ __________

__________ __________ __________ __________ __________

Now tally the last digits and the sums of the last 2 digits.

| Last Digit | Tally of Times You Got This Digit |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |

| Sum of Last Two Digits | Tally of Times You Got This Sum |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| 13 | |
| 14 | |
| 15 | |
| 16 | |
| 17 | |
| 18 | |



2. Is there a best guess for the last digit of a license-plate number?

______ yes ___X___ no

3. Is there a best guess for the sum of the last 2 digits of that number?

___X___ yes ______ no

4. Which sum do you think would occur most often if you looked at thousands of license plates? ___9___

When all students have completed the three worksheets, I lead a discussion centered around the question "What did you learn?"

## EVALUATION

I do not expect all students to be able to explain why some sums are more likely than others. But I do expect many students to notice the difference between guessing the last digit and guessing the sum of the last two (or three) digits. Some say, for example, "Even though more sums are possible, I'd rather guess sums. You just can't tell anything about the last digit. But I have a good system for guessing sums." If this sort of comment occurs, Experience 1 has gone well. Students have learned from their own experiments and have picked up ideas from one another. They are ready to go on to Experience 2.

# EXPERIENCE 2

## Tossing Coins

## OBJECTIVE

The student should be able to determine whether one side of a coin is more likely to turn up than another when the coin is flipped and allowed to fall to the table.

### MATERIALS

2 coins for each student (or any object that can be easily flipped and is weighted equally on both sides)
2 worksheets for each student

## TEACHER STRATEGY

You might introduce this experience by flipping a coin. Ask your students, "How many of you think it came up heads? How many think it came up tails?" Flip the coin a few more times. Then tell them that you are going to let each of them use a coin to perform an experiment in which they are to try to determine which side is more likely to turn up. Some students may already think they can predict pretty well.

Notice that the worksheet for Activity 1 asks the student to make a prediction after only twenty flips. Hopefully he may change his mind after the next thirty flips. There is a chance that students will feel tied quite closely to their exact data—that is, think that twenty-four tosses of heads and twenty-six tosses of tails definitely mean that tails is the more likely result, rather than that both results are equally likely. Don't worry about it. These concepts take a long time to develop properly. At this time our concern is that the students are able to collect their data and, on the basis of those data, come to some conclusions.

As the students complete the first worksheet, I like to distribute the worksheet for Activity 2 and have them work on it in pairs. Each member of a pair takes a turn at tossing his coins and tallying the results on his own worksheet.

After each student has had a chance to perform at least his first fifty trials (some probably will have done a hundred), I think it is a good idea to take a look at the data collected by the class as a whole. I first make on the board three columns headed H, T, and HT and ask how many students feel that one result is more likely than the others and how many feel that they are equally likely. I sometimes also collect the individual results and put the actual tabulation on the board. This usually makes it more obvious to students that the occurrences are equally likely.

I follow up with questions such as, "How many think that if I throw heads this time, I will throw tails the next time? What if I throw tails three times in a row—would the next throw *have* to be heads?"

## EVALUATION

Every student should meet with some success in this experience. If he tosses the coin, records his data, and comes to some conclusion based on his data, he will have done all that you can expect at this point. Some students will probably see that when they collect more data the number of heads and the number of tails tend to equalize, and hence they will see that in the long run one is no more likely to happen than the other (unless the coin is weighted). Don't be concerned if not all grasp this idea. In the experiences that are to come, students should see how more and more data usually help in making better predictions.

Students should not be expected to give good answers to Questions 2, 4, and 5 in Activity 2.

Activity 1 Name ______________________________

What happens when you flip a coin 20 times? Does the same side always land up? Try it and see. Flip your coin. Notice that one side is "heads" and the other is "tails." If heads turns up, put a tally mark in the H row below. If tails turns up, put a tally mark in the T row. Make 20 tosses, then fill in the totals.

| Side | Tally | Total |
|---|---|---|
| H | | |
| T | | |

Which side came up more often? _____ Which side do you think will turn up more often if you make 30 more flips? _____

Now flip the coin 30 more times and record your results below.

| Side | Tally | Total |
|---|---|---|
| H | | |
| T | | |

Which side came up more often this time? _____ Look at the totals from both sets of flips. If you add all the heads together and all the tails together, which turns up more often? _____

Now exchange coins with a neighbor or your teacher and flip the new coin 30 times. Record your results below.

| Side | Tally | Total |
|---|---|---|
| H | | |
| T | | |

Which side came up more often this time? _____

If you have time, make another chart and try someone else's coin. In looking over all your results, does it appear that one side comes up more often than the other? _____

**TOSSING COINS** **UNIT 12, EXPERIENCE 2**

Activity 2 Name ______________________________

What happens when you flip 2 coins at the same time? Try it and see. As you complete each flip of the 2 coins, tally your result below. Do it 40 times.

| The Coins Fall | Tally | Total |
|---|---|---|
| H,H | | |
| H,T or T,H | | |
| T,T | | |

1. Which way did the coins fall most often? __________
2. Why do you think they fell this way most often? ______________________________

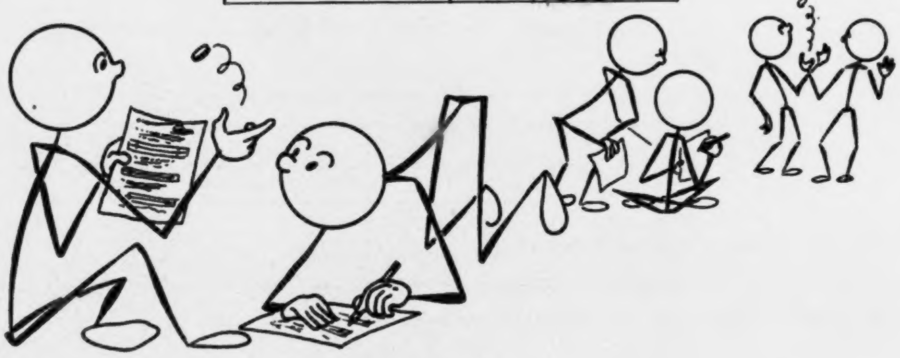
   ____________________________________________________________

3. Was there much difference in the number of times you got H,H and the number of times you got T,T? _____
4. How do you explain this? ______________________________

   ____________________________________________________________

5. Enter below the results you might expect to get if you tossed these 2 coins 4,000 times.

| The Coins Fall | Total |
|---|---|
| H,H | 1,000 |
| H,T or T,H | 2,000 |
| T,T | 4,000 |



# EXPERIENCE 3

## Rolling Dice

### OBJECTIVE

By throwing a die at least a hundred times, the student should be able to determine whether one number tends to turn up more often than any other number. By rolling a pair of dice one hundred times he should be able to decide which sums are more likely to be rolled and which sums are less likely to be rolled.

### MATERIALS

1 pair of dice for each student
5 worksheets for each student

### TEACHER STRATEGY

Have you ever felt that when you threw a die you were more likely to get one number than another? Most adults feel that way even though they know it isn't so. Before students begin the activities for this experience, I ask them if they feel they have a lucky number. Then I let them throw a die to see if their findings support their hunches.

The first activity requires students to make a prediction after only a few trials. They are then given an opportunity to change their prediction when they have more data. Some students should begin to see that a better decision about an uncertain situation can be made once more information has been collected and studied, although this is not a primary objective of the experience. However, to help the class reach this secondary objective, some time should be spent looking at the statistics the students have compiled.

I begin by taking an overall view, finding out how many students think certain numbers will come up more often. Then, on the board, I tabulate the data from the class as a whole (you may wish to use only a sample of six or seven students). I list the information in columns (see fig. 3, p. 188) and let the students find the totals—this gives them some good practice in column addition. Those students who think a particular number will come up more often are in charge of the column for that number,

| **Name** | ⚀ | ⚁ | ⚂ | ⚃ | ⚄ | ⚅ |
|---|---|---|---|---|---|---|
| **Pupil A** | | | | | | |
| . | | | | | | |
| . | | | | | | |
| . | | | | | | |
| **Pupil X** | | | | | | |
| **Totals** | | | | | | |

Fig. 3

and the others check their work. By using statistics gathered from the whole class, students can more easily see that all of the results are about the same and that a difference of ten in 1,000 tosses is not really very much.

Next I have my students work individually on Activity 1. When most of them are finished, we discuss the results. I then give each student two dice and ask some of them to report the sum of the numbers on the two faces they get in a cast of dice. After a short discussion I hand out the worksheets for Activities 2 and 3 and let students work individually.

The main objective of Activities 2 and 3 is for students to find through experimentation that certain sums do occur more often and others less often. Some students may decide that certain sums are equally likely (5 and 9, for example). They may even order the possible sums from the most likely to the least likely (7, 6 and 8, 5 and 9, 4 and 10, 3 and 11, 2 and 12). But do not expect most students to get this far on their own.

To enable students to get these ideas from this experience, I put a class chart listing all the possible sums (fig. 4) on the chalkboard. Each student contributes his results from Activities 2 and 3. With such a large amount of data the relative likelihoods of the different sums begin to show up more clearly (and students also get practice in adding large sums).

[*Continued on page 194*]

| Name | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Pupil A | | | | | | | | | | | |
| . | | | | | | | | | | | |
| . | | | | | | | | | | | |
| . | | | | | | | | | | | |
| Pupil X | | | | | | | | | | | |
| Totals | | | | | | | | | | | |

Fig. 4

ROLLING DICE UNIT 12, EXPERIENCE 3

Activity 1 Name ______________________________

Throw the die. What number faces up? _____ Throw it again. What number came up this time? _____ What other numbers could come up? ________________________

EXPERIMENT 1: Try to find out whether one number turns up on your die more often than any other number. Do you think one will? _____ If so, which one do you expect to turn up most often? _____

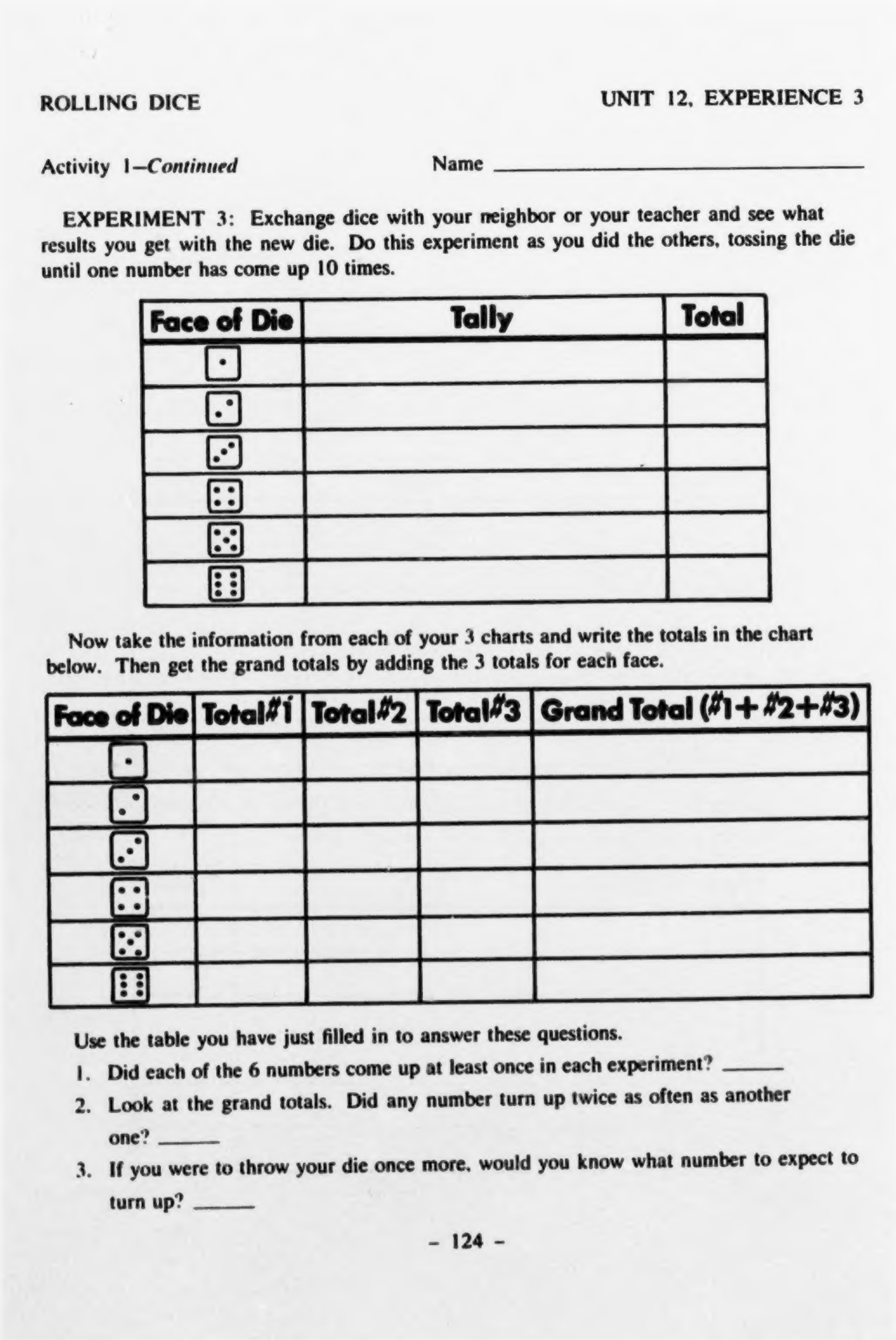
The table lists all the ways a die can turn up. Every time you throw the die, tally the number that turns up in the appropriate row. Keep throwing the die until one number has 10 tally marks after it. Then find your total for each number.

| **Face of Die** | **Tally** | **Total** |
|---|---|---|
| ⚀ | | |
| ⚁ | | |
| ⚂ | | |
| ⚃ | | |
| ⚄ | | |
| ⚅ | | |

EXPERIMENT 2: Let's do the same thing again. Toss the die until one number comes up 10 times. Do you think it will be the same number as before? _____

| **Face of Die** | **Tally** | **Total** |
|---|---|---|
| ⚀ | | |
| ⚁ | | |
| ⚂ | | |
| ⚃ | | |
| ⚄ | | |
| ⚅ | | |

[*Continued*]



ROLLING DICE — UNIT 12, EXPERIENCE 3

Activity 1—*Continued* Name ____________________

EXPERIMENT 3: Exchange dice with your neighbor or your teacher and see what results you get with the new die. Do this experiment as you did the others, tossing the die until one number has come up 10 times.

| **Face of Die** | **Tally** | **Total** |
|---|---|---|
| ⚀ | | |
| ⚁ | | |
| ⚂ | | |
| ⚃ | | |
| ⚄ | | |
| ⚅ | | |

Now take the information from each of your 3 charts and write the totals in the chart below. Then get the grand totals by adding the 3 totals for each face.

| **Face of Die** | **Total#1** | **Total#2** | **Total#3** | **Grand Total (#1+#2+#3)** |
|---|---|---|---|---|
| ⚀ | | | | |
| ⚁ | | | | |
| ⚂ | | | | |
| ⚃ | | | | |
| ⚄ | | | | |
| ⚅ | | | | |

Use the table you have just filled in to answer these questions.

1. Did each of the 6 numbers come up at least once in each experiment? _____
2. Look at the grand totals. Did any number turn up twice as often as another one? _____
3. If you were to throw your die once more, would you know what number to expect to turn up? _____

**ROLLING DICE** **UNIT 12, EXPERIENCE 3**

Activity 2 Name ______________________________

Shake the dice and roll them. What sum do you get on the two dice? _____ Shake them again and roll them. What sum did you get this time? _____ Roll the dice 100 times. Record your sums in the box below. When the same sum comes up more than once, use tally marks to show the number of times it comes up. Keep track of your throws by marking off, in the block on the right, each number as you throw. Put a line through one way for the first 50 throws and the other way for the second 50 throws. For example, on toss 1 do this: /; on toss 51 do this: X.

| Sums on Dice | | | Number of Throws | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | | | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| | | | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| | | | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| | | | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |

In the chart below, arrange your sums from smallest to largest. After each sum put the total number of times it came up in your experiment. Did you miss any possible sums in your first 100 tosses? If so, put them in their proper place in the chart and record the number of times as 0.

| Sums | Times Found in 100 Tosses | Tally | New Total |
|---|---|---|---|
| Smallest: | | | |
| Largest: | | | |

*[Continued]*

ROLLING DICE

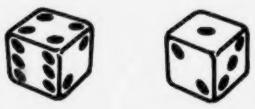
UNIT 12, EXPERIENCE 3

Activity 2–*Continued*

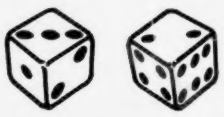
Name ______________________________

Look back at your chart for this activity. What sum(s) did you get most often? ______________ How many times did you get it (them)? ________________________ What sum(s) did you get least often or not at all? ____________

Now let's try something a little different. Try to throw sums of five:

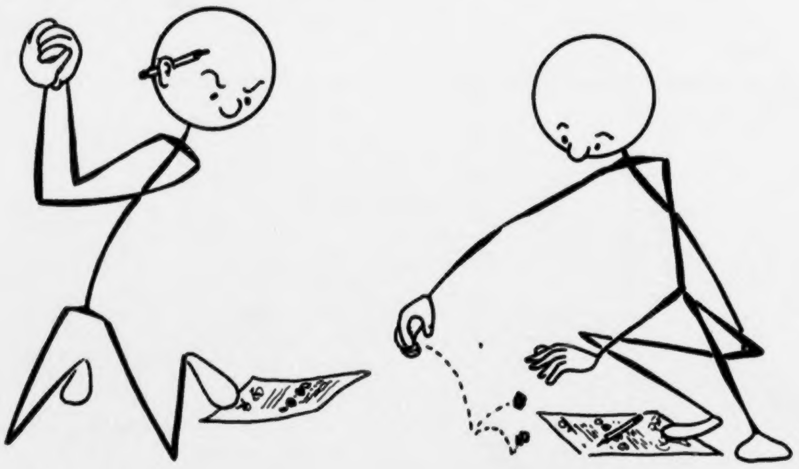


or



Keep throwing the dice until you have thrown a sum of five 10 times. In the tally column in the chart keep a record of all the sums that turn up while you are trying to throw 10 sums of five.

Now look at the tally column. Which sum has the most tally marks after it? _____ Which sum has the fewest tally marks after it? _____ What sum do you think is your "lucky sum"? _____ How many times do you think you'd have to throw to get it? _____ Try. How many throws did it take? _____ Do you think it would take that long to get it again? _____ Try it and see! How many throws did it take? _____



**ROLLING DICE** UNIT 12, EXPERIENCE 3

Activity 3 Name ______________________________

As you have seen, some sums seem to occur more often than others. Study your table from Activity 2. Use the first column of the table below to arrange the sums in order from those you found most often to those you found least often. Then throw the dice another 50 times and tally your results in the middle column. (Do more throws if you have time.) Then find your total for each sum.

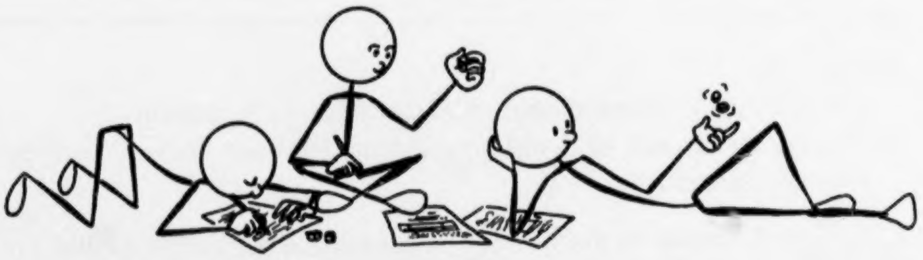
| Sum on Dice | Tally | Total |
|---|---|---|
| Most often: | | |
| | | |
| | | |
| | | |
| Least often: | | |

Do your sums seem to be in the right order? ______ What changes would you make in the order? ______________________________

Do you think that if you threw your dice a few more times, your results might change? ______

Look at the final totals in your tables for Activities 2 and 3 and then answer these questions.

1. Which sum occurred most often? ______
2. Which sum occurred least often? ______
3. What sums seemed to come up about the same number of times? ______________
4. The next time you roll a pair of dice, are you more likely to get a sum of 3 or a sum of 8? ______ Why? ______________________________



Although the numbers appearing in a cast of dice do come up randomly, it is hoped that through this experience the student will see that even chance favors certain results. A little experimenting and data collecting can help him to find out how the cards are stacked. Knowing this can be very helpful in making decisions.

## EVALUATION

The discussion following Activity 1 will tell you whether your students have learned that any one face is as likely to turn up as another. Their worksheets for Activities 2 and 3 will tell you whether they know which sums of two faces are more likely to occur. Most students should be able to answer correctly the final question for Activity 3.

# EXPERIENCE 4

## Drawing Beads

## OBJECTIVE

The student should be able to determine the possible color combinations he can get by drawing three beads at random from a can containing thirty beads (ten of each of three colors). Also, by drawing beads from a can containing an unknown number of beads of each color, the student should be able to make an intelligent guess about the color of the largest number of beads in the can.

## MATERIALS

1 large can
1 can (such as a frozen-orange-juice can) for each student
30 beads, 10 of each of 3 different colors, for each student (marbles may be substituted)
43 red } beads or marbles for each group of 8 students (other colors may be substituted)
58 blue
43 green
2-page worksheet for each student
2-page worksheet for each pair of students

## TEACHER STRATEGY

I begin this experience with a large can filled with beads on my desk. I tell the class the three colors of the beads in it, then I ask one student to come up and draw three beads without looking into the can. While he still has the beads in his hand, I ask the class to guess what colors have been drawn. Then we look at the beads and I write the color combination on the board. I then let two or three other students draw and let the rest of the class guess again, listing on the board all the combinations drawn or suggested.

A few examples of this nature should help students see what they are to do in this experience, and they can now begin working individually on Activity 1. I give each student the two-page worksheet and a small can filled with thirty beads, ten of each of three colors.

It may take some students a hundred trials to find all of the ten possible combinations. You should move around the room to see how the students are progressing. If a student does find all of the combinations fairly quickly, suggest that he try to find out which combinations are more likely and which are less likely. (See "Probability Theory" at the end of this unit for a list of combinations and some information on their expected frequencies.)

Have students check their own work by asking one student to list on the board all the combinations he found. Let the others add to his list. You need not explain why certain combinations occur more often than others. If the class sees that certain ones were drawn more often than others, leave it at that.

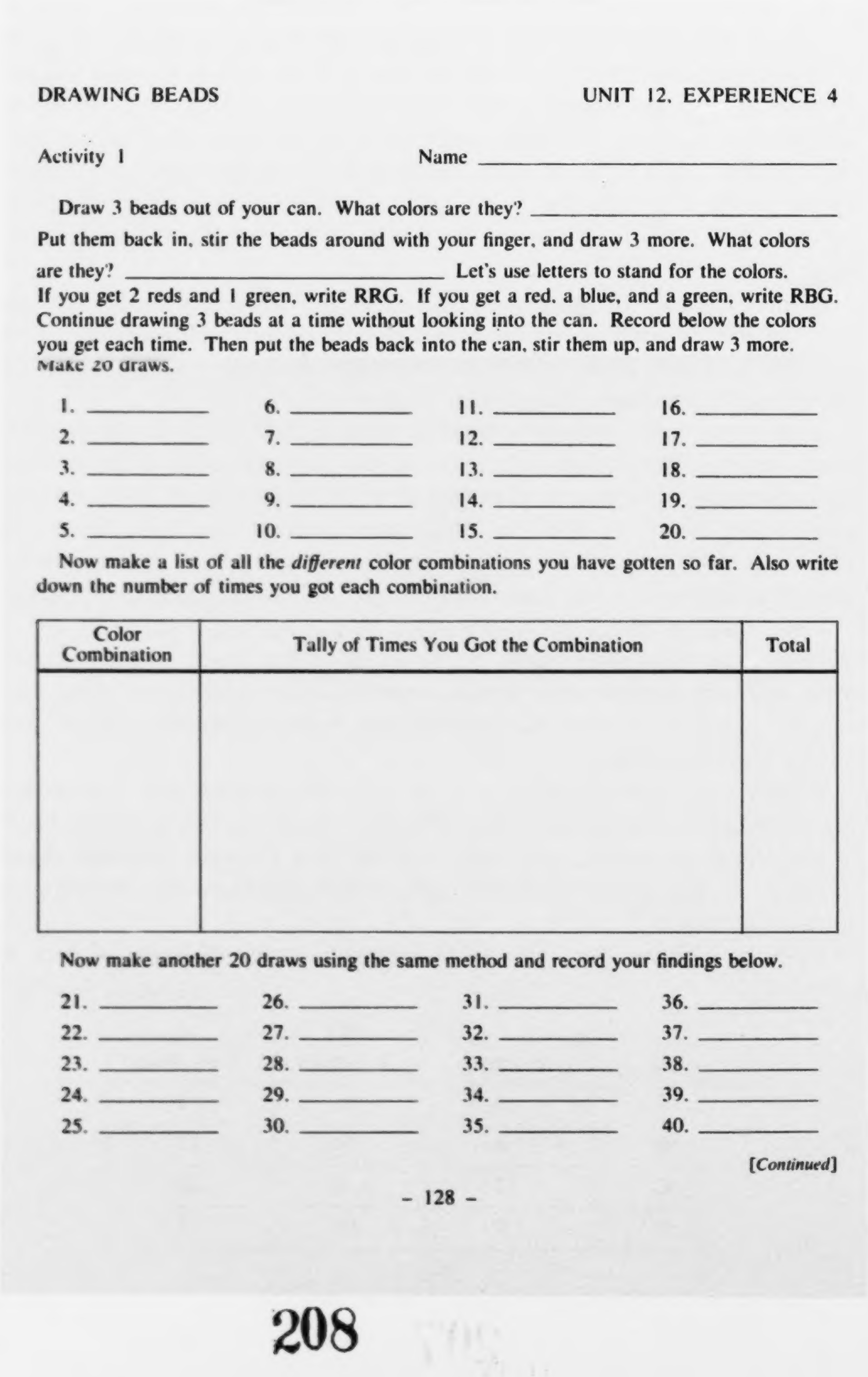
Students are to work in pairs in Activity 2. To each pair I distribute the two-page worksheet and a can of beads. Each can has thirty-six beads of three different colors, and I pass out the four different mixtures shown in figure 5. You should label the cans so that you know the contents but students do not.

I tell the students that there is an unequal number of beads of each of

[*Continued on page 200*]

| Can | Red Beads | Blue Beads | Green Beads |
|---|---|---|---|
| A | 16 | 12 | 8 |
| B | 6 | 18 | 12 |
| C | 12 | 4 | 20 |
| D | 9 | 24 | 3 |

Fig. 5



DRAWING BEADS — UNIT 12, EXPERIENCE 4

Activity 1 — Name ______________________

Draw 3 beads out of your can. What colors are they? ______________________
Put them back in, stir the beads around with your finger, and draw 3 more. What colors are they? ______________________ Let's use letters to stand for the colors. If you get 2 reds and 1 green, write RRG. If you get a red, a blue, and a green, write RBG. Continue drawing 3 beads at a time without looking into the can. Record below the colors you get each time. Then put the beads back into the can, stir them up, and draw 3 more. Make 20 draws.

1. __________
2. __________
3. __________
4. __________
5. __________
6. __________
7. __________
8. __________
9. __________
10. __________
11. __________
12. __________
13. __________
14. __________
15. __________
16. __________
17. __________
18. __________
19. __________
20. __________

Now make a list of all the *different* color combinations you have gotten so far. Also write down the number of times you got each combination.

| Color Combination | Tally of Times You Got the Combination | Total |
|---|---|---|
| | | |

Now make another 20 draws using the same method and record your findings below.

21. __________
22. __________
23. __________
24. __________
25. __________
26. __________
27. __________
28. __________
29. __________
30. __________
31. __________
32. __________
33. __________
34. __________
35. __________
36. __________
37. __________
38. __________
39. __________
40. __________

*[Continued]*

**DRAWING BEADS** UNIT 12. EXPERIENCE 4

Activity 1—*Continued* Name ________________________

Did any combinations come up that you didn't find in your first 20 drawings? _____
If so, what are they? ________________ Add these to the chart on the first page of this activity. Also put in tally marks, so that you can see how many times you got each combination.

Make 20 more draws and record them below.

41. __________
42. __________
43. __________
44. __________
45. __________
46. __________
47. __________
48. __________
49. __________
50. __________
51. __________
52. __________
53. __________
54. __________
55. __________
56. __________
57. __________
58. __________
59. __________
60. __________

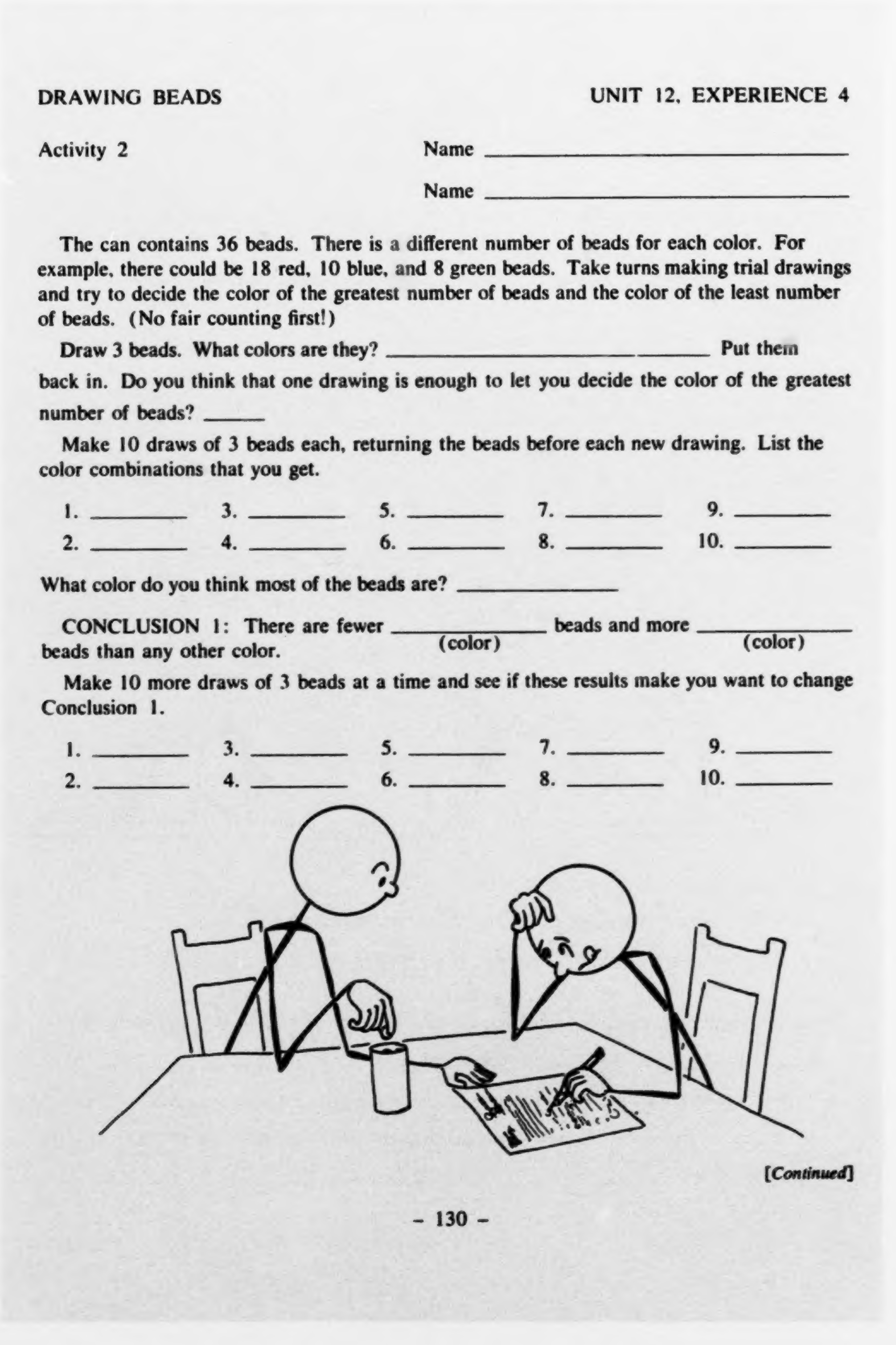
Did you get any new combinations this time? _____ If so, what are they? __________
________________ Add them to your chart and place tally marks after each one for as many times as you got it in draws 41–60.

Use your chart to answer these questions.

1. What color combination(s) did you draw most often? ____________________
2. What color combination(s) did you draw least often? ____________________

Answer these questions after you compare your findings with those of your classmates.

1. Did you draw all the possible combinations? ____________________
2. If not, which ones are you missing? ____________________
3. If you were to draw 3 beads at random from the can, what color(s) would you expect them to be? ____________________



**DRAWING BEADS** **UNIT 12, EXPERIENCE 4**

Activity 2

Name ______________________________

Name ______________________________

The can contains 36 beads. There is a different number of beads for each color. For example, there could be 18 red, 10 blue, and 8 green beads. Take turns making trial drawings and try to decide the color of the greatest number of beads and the color of the least number of beads. (No fair counting first!)

Draw 3 beads. What colors are they? ______________________ Put them back in. Do you think that one drawing is enough to let you decide the color of the greatest number of beads? _____

Make 10 draws of 3 beads each, returning the beads before each new drawing. List the color combinations that you get.

1. ________ 3. ________ 5. ________ 7. ________ 9. ________
2. ________ 4. ________ 6. ________ 8. ________ 10. ________

What color do you think most of the beads are? ____________

CONCLUSION 1: There are fewer ____________ (color) beads and more ____________ (color) beads than any other color.

Make 10 more draws of 3 beads at a time and see if these results make you want to change Conclusion 1.

1. ________ 3. ________ 5. ________ 7. ________ 9. ________
2. ________ 4. ________ 6. ________ 8. ________ 10. ________

*[Continued]*

**DRAWING BEADS** **UNIT 12, EXPERIENCE 4**

Activity 2–*Continued*

Name ______________________________

Name ______________________________

Look at your results so far. Then complete Conclusion 2.

CONCLUSION 2: We think that there are fewer ____________ beads and more ____________ beads than any other color.

Now make one more check.

1. ________
2. ________
3. ________
4. ________
5. ________
6. ________
7. ________
8. ________
9. ________
10. ________

CONCLUSION 3: Looking at all 30 drawings, we think there are fewer ____________ beads and more ____________ beads than any other color.

If Conclusions 1, 2, and 3 agree, or if 2 and 3 agree, record your Final Decision below and ask your teacher for another can to decode. If Conclusions 2 and 3 do not agree, make at least 20 more drawings before making your final decision.

1. ________
2. ________
3. ________
4. ________
5. ________
6. ________
7. ________
8. ________
9. ________
10. ________
11. ________
12. ________
13. ________
14. ________
15. ________
16. ________
17. ________
18. ________
19. ________
20. ________

FINAL DECISION: The color of the greatest number of beads is ______________.

The color of the fewest number of beads is ______________.

(We used the can labeled _____ to make our decision.)

three colors in their can and that by drawing enough samples of three beads they are to determine the color of the largest number of beads and the color of the least number of beads. Partners take turns drawing beads and recording results on the worksheet. When they complete the activity, I have them make a count of the various colors of beads in the can to check their final decision. You may wish to have the first pairs to finish exchange their cans of beads and try to determine the color of the largest number of beads in the new can by the same sampling method.

## EVALUATION

Most students should be able to determine the ten color combinations they can get in Activity 1. The class discussion and the students' work in Activity 2 should help you determine whether most of your students can make accurate predictions from the samples drawn.

# EXPERIENCE 5

## Tossing Objects

## OBJECTIVE

By collecting his own experimental evidence, the student should be able to determine which way physical objects such as cups and thumbtacks are most likely to land when tossed.

## MATERIALS

1 paper or styrofoam cup for each student
1 thumbtack for each student
3 worksheets for each student

## TEACHER STRATEGY

I like to begin this experience by asking my students which way they would expect a cup with a flat bottom to land if they were to toss it a foot into the air and let it land on their desks. I record on the board the number of students who predict each of the three possible outcomes. Then I pass out the two-page worksheet for Activity 1 and a paper cup to each student. I ask the students to read the directions carefully and to work individually while completing the worksheet.

Which way do you think it will land most often? If you're not sure, toss a cup a few times until you decide on your answer—there is no mathematical answer to this question. I don't know what answer your students will come up with. It will depend upon the kind of cup used.

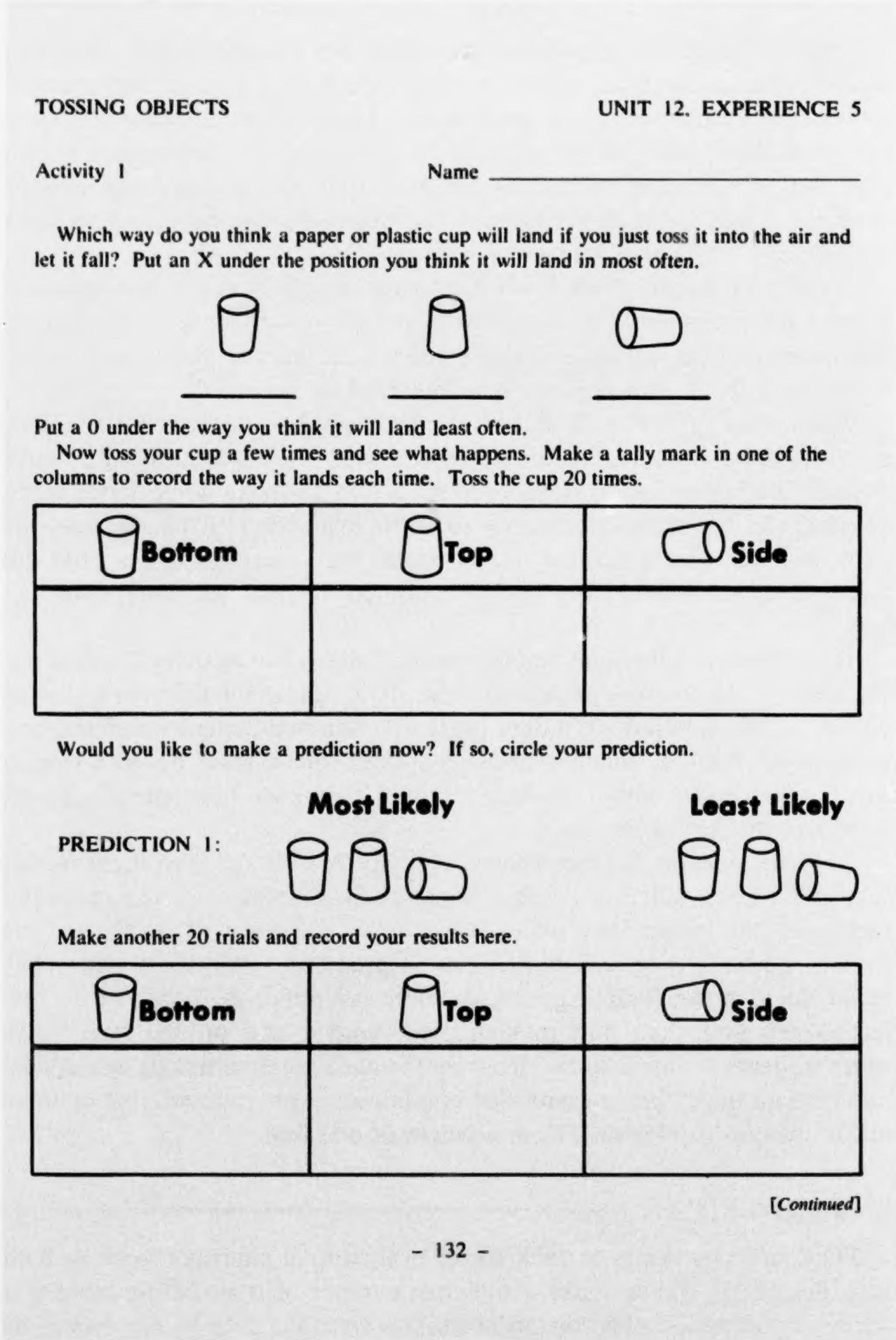
When most of the students have made a final prediction, put the class data on the board and come to a class decision. If a student disagrees quite strongly with the class results, encourage him to make some more trials. (Perhaps he based his decision on too little evidence.) If he still does not agree tell him that it may be his particular cup—perhaps it is a little different from the others. Let him try another cup and see what results he gets.

Then I pass out the thumbtacks and worksheets for Activity 2 and direct the class to go to work independently. It is important that each student collect a sufficient amount of data (at least a hundred tosses) before making a decision. Also, a student's decision should follow from his own data—this is much more important than reaching a decision that agrees with the results of the rest of the class.

When all students have completed Activity 2, I discuss with them factors that could bring differing results. Some of these factors are the particular tack used, the height from which it is thrown, the force of the throw, and so on. Perhaps they will have other suggestions. Ask them what they could do to make their experiments more scientific, as if they were collecting scientific data and making predictions based on the data. Ask some students to come to the front of the class to demonstrate the collection of data under better-controlled conditions. For example, one of them might always drop his tack from a height of one foot.

## EVALUATION

There are two things to look for in evaluating a student's work in both activities. First, did he make a sufficient number of trials before making a decision? Second, do his conclusions follow from the data he has collected?



TOSSING OBJECTS UNIT 12. EXPERIENCE 5

Activity 1 Name ______________________________

Which way do you think a paper or plastic cup will land if you just toss it into the air and let it fall? Put an X under the position you think it will land in most often.

Put a 0 under the way you think it will land least often.

Now toss your cup a few times and see what happens. Make a tally mark in one of the columns to record the way it lands each time. Toss the cup 20 times.

| **Bottom** | **Top** | **Side** |
|---|---|---|
| | | |

Would you like to make a prediction now? If so, circle your prediction.

PREDICTION 1: **Most Likely** **Least Likely**

Make another 20 trials and record your results here.

| **Bottom** | **Top** | **Side** |
|---|---|---|
| | | |

*[Continued]*

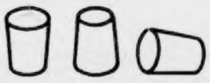
**TOSSING OBJECTS** **UNIT 12, EXPERIENCE 5**

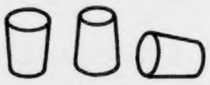
Activity 1–*Continued* Name ____________________

**Study the information from both experiments and then make your second prediction.**

**Most Likely** **Least Likely**

**PREDICTION 2:**





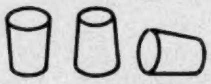
**Make some more tosses. Make at least 20, but more if you have time.**

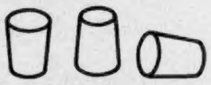
| **Bottom** | **Top** | **Side** |
|---|---|---|
| | | |

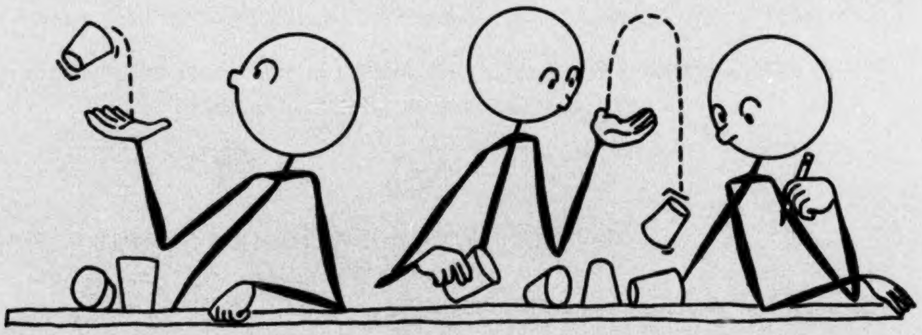
**FINAL PREDICTION: I have made __________ tosses of the cup.**

**On the basis of these tosses, I have decided the following.**

**Most Likely** **Least Likely**

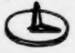
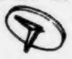






TOSSING OBJECTS UNIT 12, EXPERIENCE 5

Activity 2 Name ______________________________

Have you ever tossed a thumbtack in the air and had it land point down, sticking into the floor? __________ It doesn't happen very often, does it? How do you think a thumbtack will usually land, like this  or like this  ? Circle one.

EXPERIMENT 1: Toss the tack 25 times. Show the way that it lands each time by placing a tally mark in the correct column of row 1.

| Experiment | Lands Up | Lands on Side | Lands Down |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |

Which way did it land most often? ____________________

EXPERIMENT 2: Make another 25 tosses and record them in row 2. Then look at the results of your first 50 tosses. Which way has the tack landed most often now?

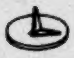
____________________ Did it come up one way twice as often as any other way?_____

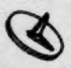
If so, which way? ____________________ Do you think you can predict the way that this tack will usually land? _____ If so, which way? ____________________

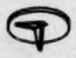
EXPERIMENT 3: If you didn't make a prediction after Experiment 2, do another 50 tosses and record the results in row 3.

If you decided to make a prediction after Experiment 2, check it by doing the 50 tosses.

FINAL CONCLUSION: I have come to the following conclusion based on my tosses.
The most likely way for a thumbtack to land is:







My data indicate that this thumbtack is no more likely to land one way than another. _____

Would a different thumbtack make any difference? _____

# PROBABILITY THEORY
## Supplementary Information

The data in this supplementary section will help you evaluate the results your students obtain in the activities for this unit. We discuss theoretical distributions of certain digits and sums of digits in telephone numbers; of heads and tails when coins are tossed; of the faces on a pair of dice; and of color combinations of beads drawn from a can containing three colors of beads. To convince yourself that the results in practice may vary considerably from the theoretical results, you are urged to perform these experiments in advance and record the outcomes.

**Experience 1.** Note first that we have not tried to make a careful distinction between a number and a numeral, which is a name or a symbol for a number. Since digits are numerals, not numbers, it is incorrect to speak of "sums of digits" as we did in this experience when referring to the last two digits of the numerals in a telephone book. However, incorrect usages have become so firmly established that we hesitate to insist on the strictly correct language. Wouldn't it surprise you to hear an operator say, "Numeral please"?

If the telephone numbers (represented by numerals) are fairly randomly distributed, we can expect each digit to occur one-tenth of the time as the last digit in the numerals on any page of the directory. But the following distribution chart (fig. 6, p. 206) shows rather vividly why the eighteen possible sums of numbers represented by the last two digits in telephone numbers have different frequencies. The numerals connected by the lines in the table show that we would expect a sum of twelve 7 times in 100 sums, while we would expect a sum of sixteen 3 times in 100 sums of two single-digit numerals. Not only does this chart show the relative frequency of any sum, but also it shows the addition combinations that give that sum.

**Experience 2.** If the coin tossed is fair—that is, is symmetrical—we can expect as many heads as tails in a long series of tosses. Avoid the temptation to expect a higher probability of a tail on the toss following an unlikely run of, say, ten heads in a row. The coin has no memory, and if it is a fair coin the probability of a tail on the next toss is always $\frac{1}{2}$.

In tossing two coins the outcomes H,H; H,T; T,H; and T,T are equally likely, so we can expect each outcome about one-fourth of the time. Since we have grouped two outcomes as "H,T or T,H" on the tables in Activity 2, we can expect about half of the tally marks to be placed in this row.

| + | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 4 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 5 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 6 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 7 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 8 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 9 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |

Fig. 6

**Experience 3.** Assuming that a die is not loaded, we can expect any of its six faces to come up equally often. The probability of getting any particular sum from two to twelve can be displayed by making a chart similar to that shown in figure 6. Just omit 0, 7, 8, and 9 in the two borders and the corresponding portions of each row and column.

**Experience 4.** This experience involves some rather difficult but important concepts of probability and statistical inference. Although they of course will not be discussed with the students, a few of these ideas are mentioned here so you can better understand the results students are likely to get.

The probability of drawing three red beads from a can containing ten red, ten blue, and ten green beads is

$$\frac{10 \cdot 9 \cdot 8}{1 \cdot 2 \cdot 3} \div \frac{30 \cdot 29 \cdot 28}{1 \cdot 2 \cdot 3}, \quad \text{or} \quad \frac{120}{4{,}060}.$$

There are 4,060 distinct combinations of three articles chosen from thirty, and there are 120 ways to pick three red beads from the ten red beads.

Similarly, the ways in which we get the color combination RBB are

$$10 \cdot \frac{10 \cdot 9}{1 \cdot 2}, \quad \text{or} \quad 450.$$

Figure 7 shows the probability of each color combination both as a fraction and as a decimal approximation. The sum of these probabilities, with an allowance for rounding off, is one, as it should be. The chart indicates that in 1,000 drawings we could expect about 30 RRRs, 111 RRBs, 246 RBGs, and so on. Of course it could happen that one or more of these color combinations would not occur in the sixty drawings required for Activity 1.

| RRR | RRB | RRG | RBB | RBG | RGG | BBB | BBG | BGG | GGG |
|---|---|---|---|---|---|---|---|---|---|
| $\frac{120}{4{,}060}$ | $\frac{450}{4{,}060}$ | $\frac{450}{4{,}060}$ | $\frac{450}{4{,}060}$ | $\frac{1{,}000}{4{,}060}$ | $\frac{450}{4{,}060}$ | $\frac{120}{4{,}060}$ | $\frac{450}{4{,}060}$ | $\frac{450}{4{,}060}$ | $\frac{120}{4{,}060}$ |
| .030 | .111 | .111 | .111 | .246 | .111 | .030 | .111 | .111 | .030 |

Fig. 7

The probability of drawing the various color combinations in each of the four mixtures suggested for Activity 2 are given in figure 8.

After students list the color combination for each drawing, they will probably count the number of red, blue, and green beads in ten drawings and compare these totals in deciding the color of the largest number of beads. As the number of drawings increases, the ratio of a particular bead drawn tends to get closer to the exact ratio of that bead to the total number of beads in the can.

**Experience 5.** We have no theoretical results for either object. The combined results of students' experiments will give the best approximation to the various probabilities.

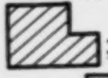
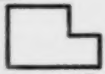
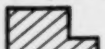
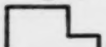
| Mixture | | | Probability | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| R | B | G | RRR | RRB | RRG | RBB | RBG | RGG | BBB | BBG | BGG | GGG |
| 16 | 12 | 8 | .078 | .202 | .135 | .148 | .215 | .063 | .031 | .074 | .048 | .008 |
| 6 | 18 | 12 | .003 | .038 | .025 | .130 | .181 | .055 | .114 | .257 | .166 | .031 |
| 12 | 4 | 20 | .031 | .037 | .185 | .010 | .135 | .319 | .001 | .017 | .107 | .160 |
| 9 | 24 | 3 | .012 | .121 | .015 | .348 | .091 | .003 | .284 | .116 | .010 | .000 |

Fig. 8

# 13

## EXPERIENCES WITH Geometry

Many students enjoy studying geometry until asked to calculate perimeters, areas, and volumes. Shapes are interesting; but formulas are confusing. The experiences of this unit are directed at shape—shape studied in such a way that an appropriate foundation is laid for later study of perimeter and area.

The underlying idea is as follows: In measuring area, one measures a set of points which looks like ; in measuring perimeter, one measures a set of points which looks like . The shapes studied in this unit are therefore not studied according to their names (circle, triangle, and so forth), but according to whether they are like  or . The difference is presented to the student as the difference between the shape of a shadow of a piece of cardboard and the shape of a rubber band on the geoboard. To maintain the students' interest while they are becoming familiar with the basic idea, nonstandard units are studied in problem situations represented by questions such as "Can this be made on the geoboard?" and "Which of these can be represented on the geoboard?" The culmination of the unit is the construction of replicas of figures from appropriate pieces: bits of cardboard for figures like shadows and lengths of thin sticks for figures like the shape of a rubber band.

Pedagogically, the student is free of reliance on the tools so useful to the high achiever—memory, vocabulary, the ability to compute; he is never asked to recall anything, never asked to learn the name for anything, and never asked to do any arithmetic beyond counting. Instead, he is asked to manipulate objects and, as he works, to observe, record, and predict. The manipulation of objects becomes a reliable means for checking guesses. So that he may be able to answer the question "Can this be made on the geoboard?" he has a geoboard to experiment with. The teacher may need to encourage such experimentation, however, for the student may be so accustomed to "guessing and failing" that he does not realize there are reliable aids to learning mathematics which he *can* use.

It is intended that the student work through the experiences individually. The teacher's role is to see that the necessary materials are available and to ask helpful questions as the student reports completion of an activity. The work the student is directed to show the teacher, when used in conjunction with appropriate questions, provides a check on his progress.

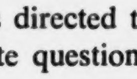
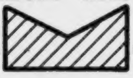
## OVERVIEW

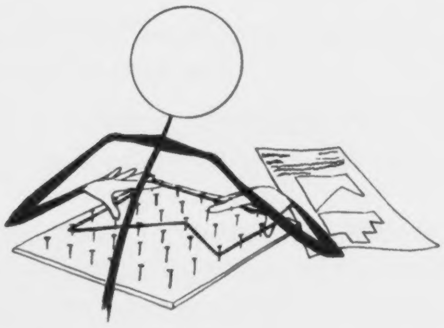
Each of the five experiences in this unit includes a detailed section called "Teacher Strategy," based on procedures that have proved effective in actual classroom experience.

The following brief summary will help you decide whether these experiences meet the needs of your own class and will also give you some indication of the amount of preparation involved.

Student worksheets and other "handout" materials are provided in the Teaching Package. It is assumed that duplicating facilities are available to you.

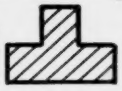
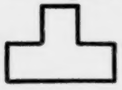
### Experience 1: Shapes Like a Rubber Band

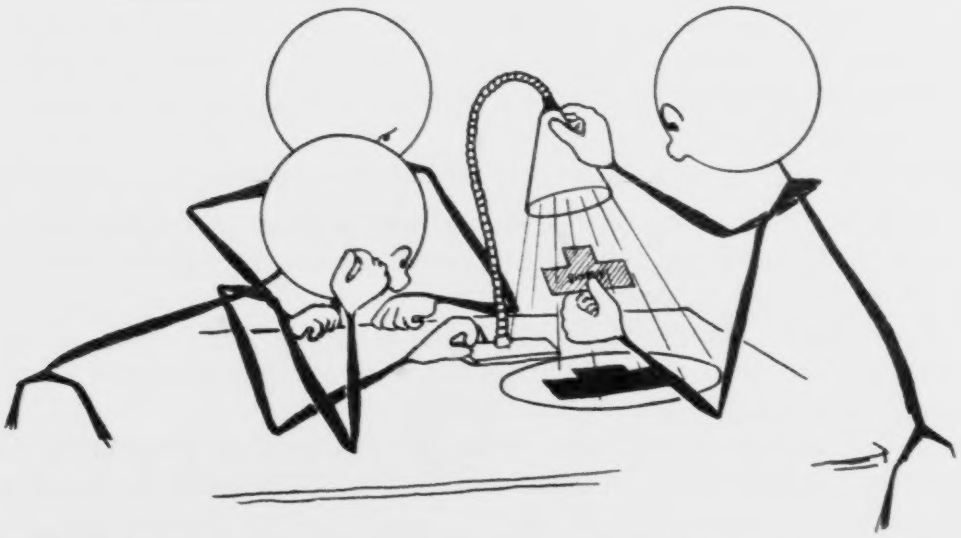
In this experience attention is given to geometric figures that can be represented by a rubber band on a geoboard. After some practice in arranging rubber bands to look like given pictures, the student is asked to sort out from a set of pictures those that are like a rubber band (, for example) and those that are not like a rubber band (, for example).



Materials: geoboards, rubber bands, worksheets.

### Experience 2: Shapes Like Shadows of Pieces of Cardboard

The student learns that the shadow of a piece of cardboard changes shape according to the angle at which the light strikes the cardboard. However, the shadow cannot have just any shape; there will be no "hole" in the shadow, for example, if there is no hole in the cardboard. By experimentation, the student learns some of the shapes that shadows can have. He is then asked to select from a set of pictures those that can be the shadow of a piece of cardboard (, for example) and those that cannot (, for example).



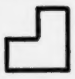
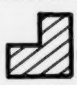
Materials: lamps, cardboard, scissors, paper, colored crayons, Shapes X and Y, worksheets.

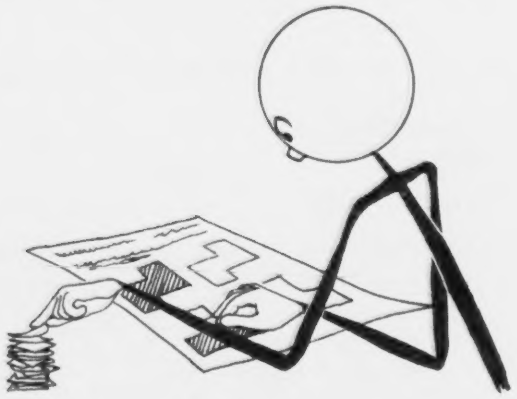
### Experience 3: Shapes Not Like a Rubber Band and Not Like a Piece of Cardboard

This experience provides a review of the first two experiences and sharpens the distinctions made in them. This is accomplished by introducing pictures that represent neither the shape of a rubber band nor the shadow of a piece of cardboard—that is, pictures that are mixtures of both. The student sorts the pictures into three categories, again using the geoboard and lamp as he needs to in making his decisions.

Materials: geoboards, lamps, worksheets.

**Experience 4: Building Replicas of Geometric Figures**

With Experience 3 having reinforced the student in his understanding of the distinction between a pair of figures such as  and , he is asked to construct replicas of each type, using appropriate "building blocks." Thin sticks are used for figures like rubber bands, and square-shaped pieces of tagboard for figures like shadows. To provide a basis for later work with the measurement of geometric figures, the student is asked to record the number of pieces he uses in replicating the figures.



Materials: tagboard squares, balsa strips, worksheets.

**Experience 5: More Replicas of Geometric Figures**

The advantages of a shortcut are more apparent if one is familiar with the long way of doing something. In the previous experience, the student became familiar with a particular kind of problem. In the final experience he is asked to do the same kind of problem; however, the arrangement is such that he can find shortcuts if he tires of laying out pieces. The shortcuts are not required, for the emphasis is on successful performance by methods comprehensible to the student. Nevertheless, the student who discovers for himself that computation can be a labor-saving device has taken a big step toward understanding those shortcuts known as "formulas."

Materials: tagboard squares, balsa strips, worksheets.

# EXPERIENCE 1
## Shapes Like a Rubber Band

## OBJECTIVE

The student should be able to distinguish between pictures that can be represented by a rubber band on a geoboard and those that cannot.

## MATERIALS

1 geoboard for each student
Large rubber bands
7 worksheets for each student

Geoboards made from a 6-by-6-in. piece of plywood with 6 rows of nails at 1-in. intervals (see fig. 1) are adequate for this unit.

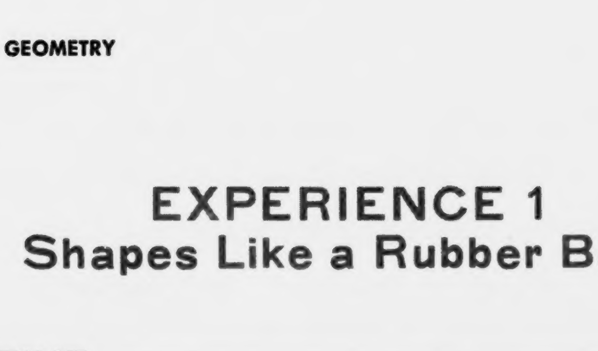


Fig. 1

## TEACHER STRATEGY

The first experience in this series consists of three activities to be done individually by the student. Faster students may be able to complete all three in one class period; younger or slower students may be able to complete only one activity in that time.

Direct the students to check with you as they complete each activity. You can determine whether a student should continue or whether that should be "all for today." It is important that the student does not rush through merely to finish. He must give thoughtful attention to the task at hand. In addition, since slow learners are unaccustomed to academic success, they may not realize that they can actually learn something from the activities. Thus it is doubly important that you question the student at the end of each activity to find out if he really knows or is only guessing, to assure him that his own activities provide a reliable source of correct answers.

To introduce this experience I stretch a rubber band around nails of a geoboard to make a figure and call attention to the shape assumed by the rubber band. The shape need not be identified by name—tracing it with a finger is sufficient. Then I pass out geoboards and rubber bands and have the students copy a simple figure drawn on the chalkboard. After a brief discussion of various kinds of figures that can be formed I pass out the three-page worksheets for Activity 1 (see the reproductions). When a student returns his worksheets I check the accuracy of his drawings and ask him to make two or three of the shapes. If he cannot reproduce them, I have him begin again with Shape 1 under close supervision, giving help and/or encouragement as needed. When I am satisfied that he can make the figures, I give him the materials for Activity 2.

Another way of presenting the material for Activity 1 would be to put each of the shapes on a separate card, and this might be better for some classes. In Activities 2 and 3 the student must discriminate between figures that can, and those that cannot, represent the shape of a rubber band on the geoboard. These activities also, presented here in worksheet form, can be organized by using separate cards. These would be divided into two piles.

The student should be encouraged to use his geoboard as needed to make the decisions for Activity 2. When he has finished, I check his work, perhaps asking him to demonstrate that he can in fact make some of the ones he claims can be made with the rubber band. In case he thinks a filled-in area can be made on the geoboard, I ask him to show me how; when it is finished, I point to the interior of the picture and ask, "Where is that part?" He may need to be reminded that the picture is supposed to be of the rubber band itself. If the student seems ready, I give him the worksheet for Activity 3. If he has had trouble, I might ask him to go back over the Activity 1 material in order to help him make the proper distinctions.

If the student has learned the ideas inherent in Activities 1 and 2, it should be easy for him to do Activity 3 correctly, distinguishing between figures that are like, and figures that are unlike, the rubber band. However, to make sure that he has not just marked the ones that aren't black, I

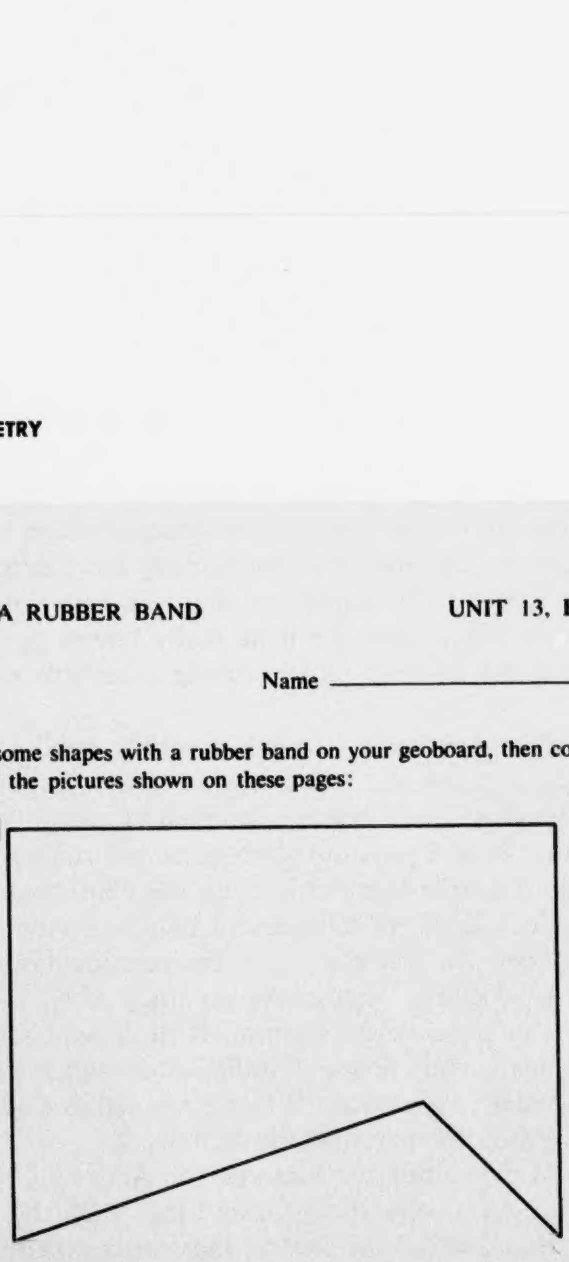
[*Continued on page 221*]

**SHAPES LIKE A RUBBER BAND** **UNIT 13, EXPERIENCE 1**

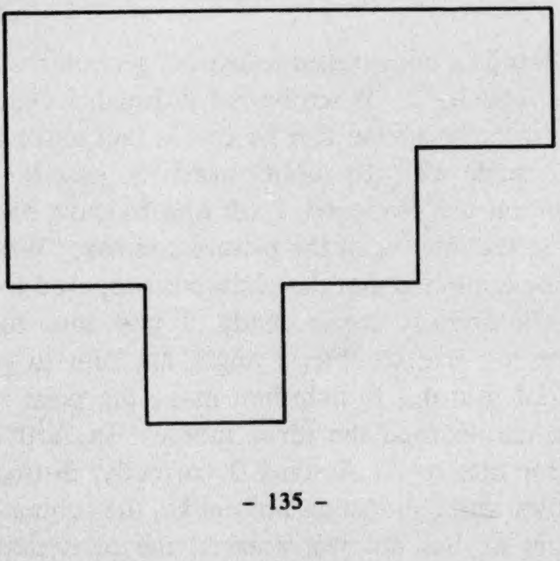
Activity 1 Name ______________________________

Practice making some shapes with a rubber band on your geoboard, then copy on the geoboard each of the pictures shown on these pages:

**1**



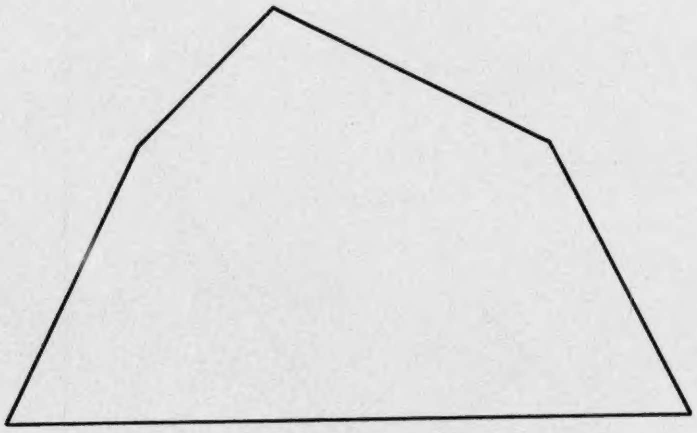
**2**



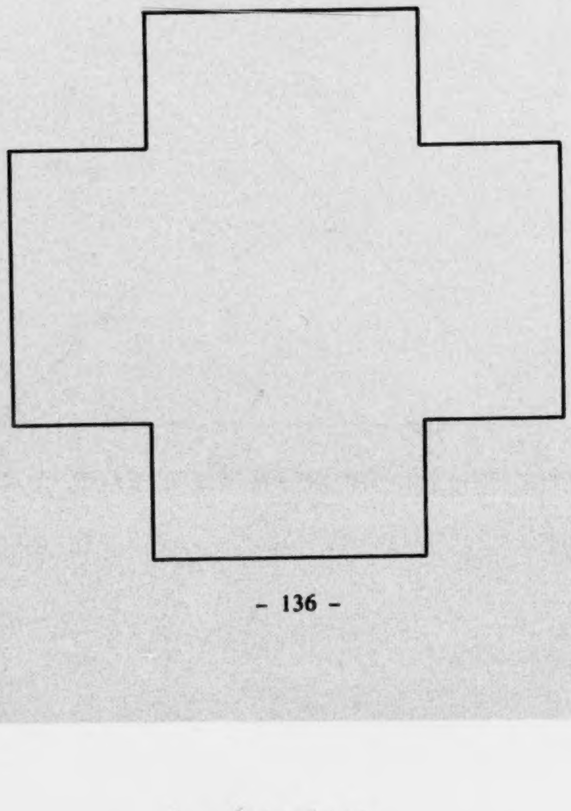
*[Continued]*

Activity 1–*Continued* Name ____________________

**3**



**4**

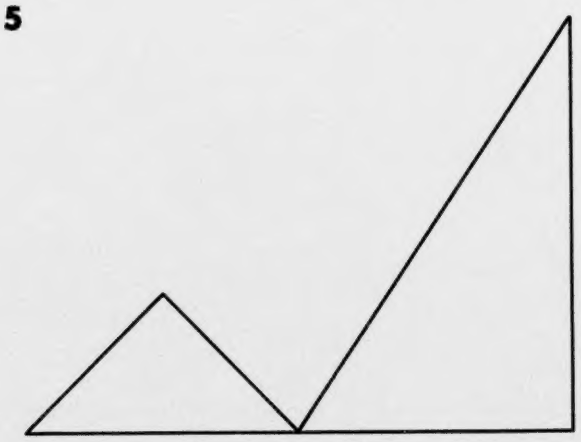


*[Continued]*

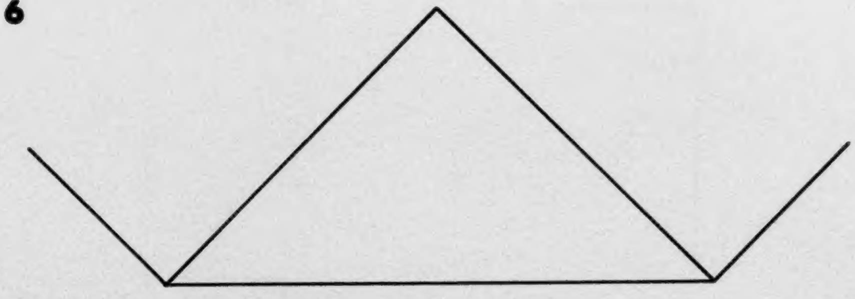
SHAPES LIKE A RUBBER BAND UNIT 13, EXPERIENCE 1

Activity 1—*Continued* Name ______________________________

**5**



**6**



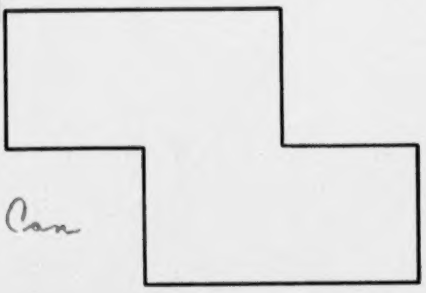
**Now draw pictures of three other shapes you can make on the geoboard. Show them to your teacher.**

**SHAPES LIKE A RUBBER BAND** **UNIT 13, EXPERIENCE 1**

Activity 2 Name ______________________________

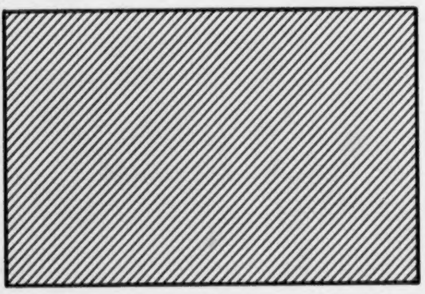
Two kinds of pictures are shown here. Not all of them can be copied on the geoboard. Write "Can" beside each picture you can copy on the geoboard.

**1**

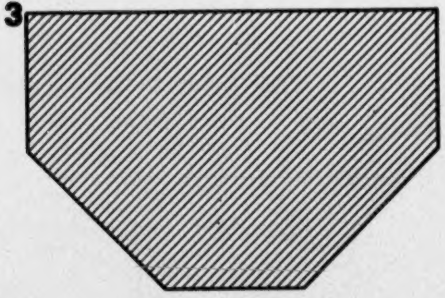


Can

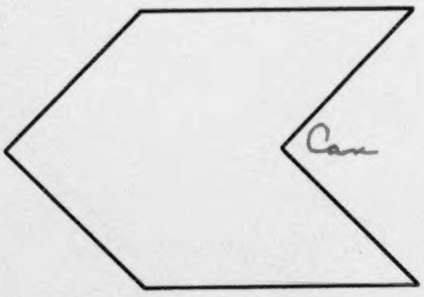
**2**



**3**

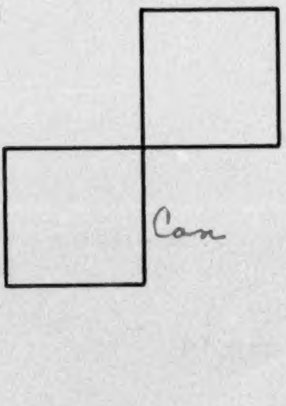


**4**



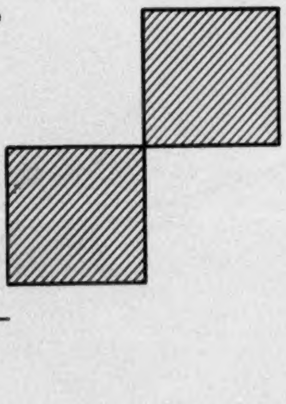
Can

**5**



Can

**6**

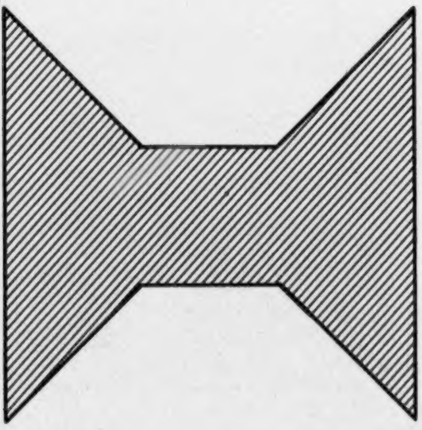


[*Continued*]

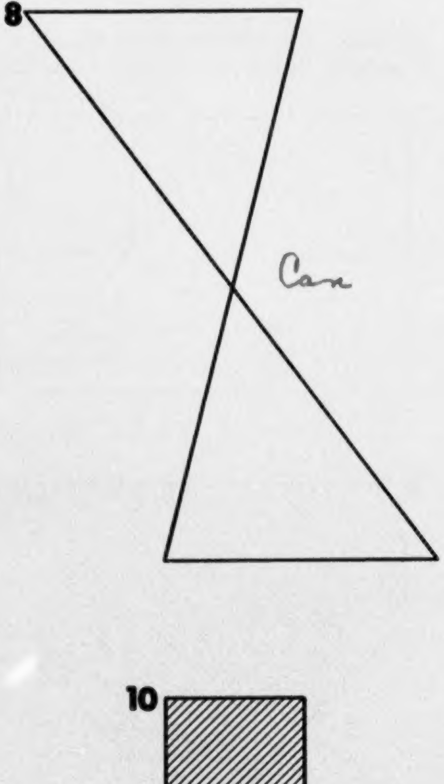
**SHAPES LIKE A RUBBER BAND** **UNIT 13, EXPERIENCE 1**

Activity 2–*Continued* Name ______________________

7

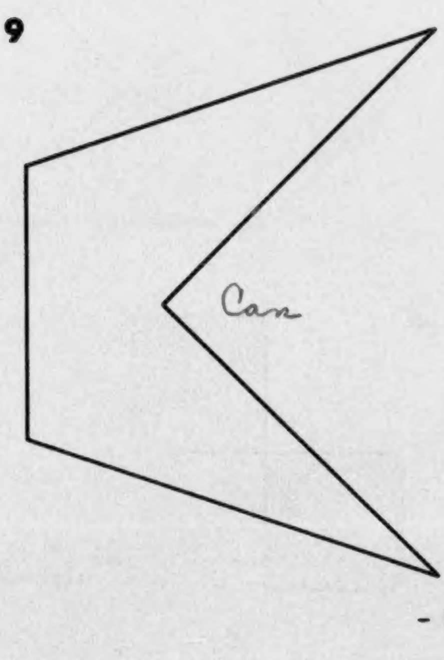


8



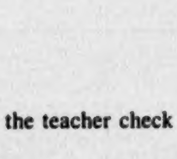
Can

9



Can

10



**Now have the teacher check your work.**

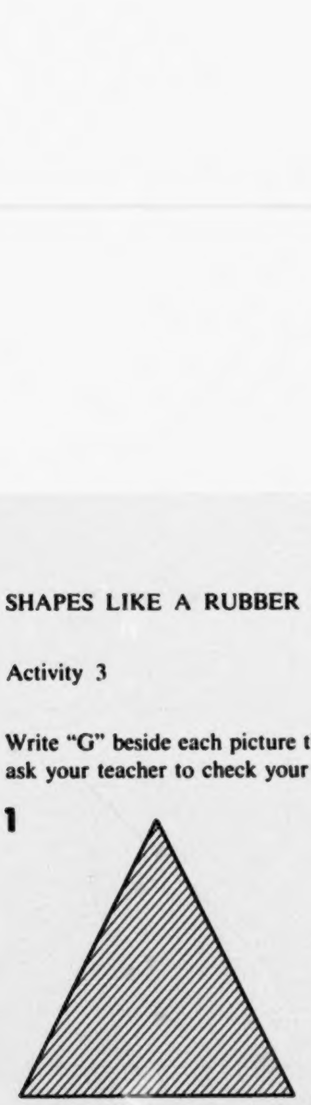
## SHAPES LIKE A RUBBER BAND

UNIT 13, EXPERIENCE 1

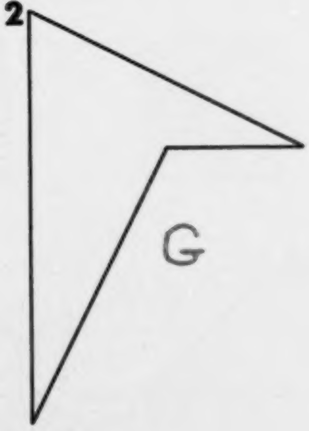
Activity 3 Name ______________________________

Write "G" beside each picture that you can make on the geoboard. When you have finished, ask your teacher to check your work.

1

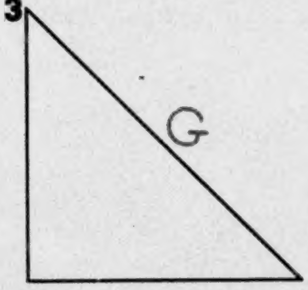


2



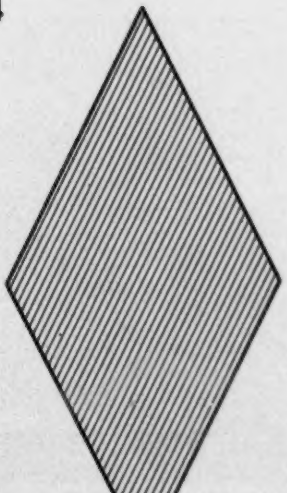
G

3

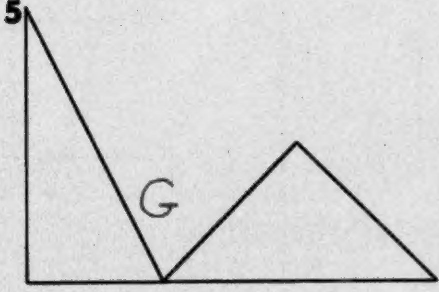


G

4



5



G

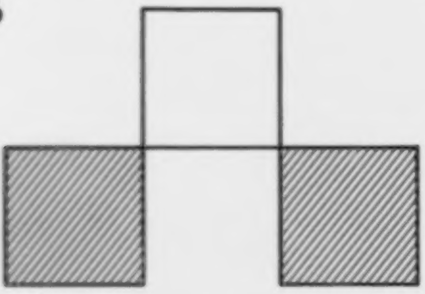
[*Continued*]

**SHAPES LIKE A RUBBER BAND** **UNIT 13, EXPERIENCE 1**

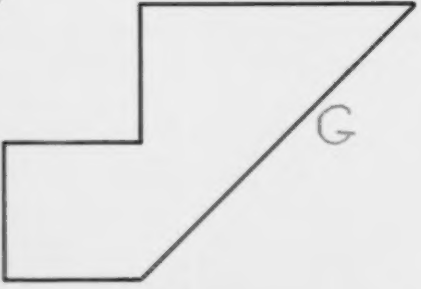
Activity 3–*Continued*

Name ______________________________

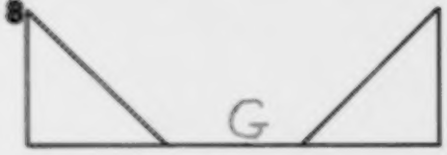
**6**



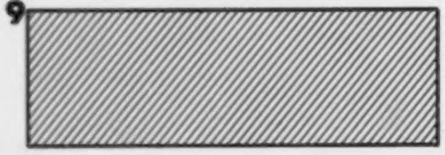
**7**



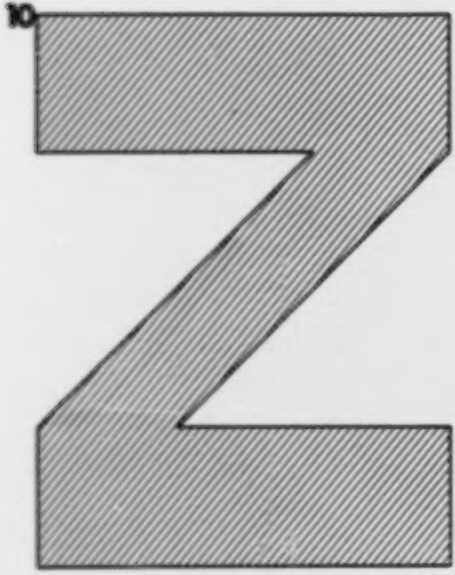
**8**



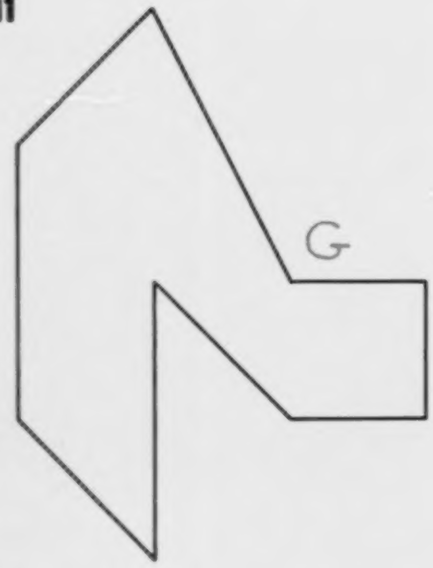
**9**



**10**



**11**



might pick out two or three that he has marked and ask him to make them on the geoboard. I also ask him why he cannot make the ones he hasn't marked, realizing of course that he may not be able to verbalize the explanation very well.

## EVALUATION

The student's performance on the worksheets will indicate whether he distinguishes between plane regions and those that can be represented on a geoboard with a rubber band.

# EXPERIENCE 2

## Shapes Like Shadows of Pieces of Cardboard

## OBJECTIVE

The student should be able to demonstrate that a piece of cardboard can cast shadows of varying shapes according to the angle at which it is held and to recognize shapes that such a shadow cannot possibly have.

## MATERIALS

1 lamp for each student
Cardboard
Scissors
Sheets of paper such as newsprint
Colored crayons
Shapes X and Y for each student
2 worksheets for each student

Gooseneck desk lamps are ideal for this experience.

Patterns for Shapes X and Y are found in the Teaching Package. These shapes should be mounted on cardboard.

## TEACHER STRATEGY

This experience consists of two separate activities, which may or may not be done in one class period according to individual progress. It is important to realize that a certain amount of experimentation in projecting shadows is needed before the student is ready to predict what can and what cannot happen.

Activity 1 is exploratory. The student projects different shadows of the same pieces of cardboard and then colors the shadows. Shapes X and Y for making shadows are provided, and the student himself cuts out a third shape. As the student works, you may check to make sure that he is coloring all of the shadow and not just the outline. Since this is to be a learning experience and not just busywork, it is important that the student examine his work when he is finished. You may encourage this critical study by spreading out his drawings and looking at them, asking him to identify the piece of cardboard used to make each of the various shadows and also to tell you how he held the piece of cardboard in order to make it. Then you may ask the student to tell what would happen to the shadow if the cardboard were turned "more this way," demonstrating. Acceptable answers would be of the form "getting thinner and thinner," "getting shorter and shorter," and so forth. If a student is not very good at these predictions, he should be encouraged to experiment more before going to the next activity.

In Activity 2 the student is asked to discriminate between pictures that could be shadows of Shapes X or Y and pictures that cannot, and he is told to experiment as necessary in making his decisions. In checking the student's work you should ask for a demonstration of some of the pictures that have been marked and also of some that have not. You will note that some students may not recognize that the line segment shown in 4 may be the shadow of either X or Y. If he has left this picture blank, asking for an explanation may help him recognize his mistake. If it does not, suggest, "Let's try Shape X and be sure." Also, for reinforcement of ideas, point to pairs of pictures such as 6 and 7, 9 and 10, or 11 and 12 and ask the student to explain why one is marked and the other is left blank. If a student does poorly on Activity 2, it may be necessary to go through the pictures one at a time, asking questions and encouraging experimentation to help the student see his errors.

## EVALUATION

The student's worksheet and his explanations indicate whether he has attained the objective of this experience.

## SHAPES LIKE SHADOWS OF PIECES OF CARDBOARD

UNIT 13, EXPERIENCE 2

Activity 1

Name ______________________________

Part 1

1. Take out the cardboard called "Shape X." Use the lamp to make a shadow of Shape X on a piece of paper. Color the shadow with the crayon. Be sure to color all the shadow.
2. Turn the cardboard a little to make a different shadow, on a new piece of paper. Color it.
3. Make some more shadows of Shape X on different pieces of paper and color them. Try to make them all different.

Part 2

1. Now take Shape Y and make some shadows of it. Make the shadows different and use different pieces of paper.
2. Go on to Part 3.

Part 3

1. Now take another piece of cardboard and cut a shape from it.
2. Make some different shadows from this cardboard shape on different pieces of paper. Color all these shadows.

Part 4

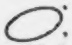
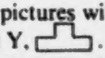
1. Look at all the pictures you have colored. Can you tell which piece of cardboard made each shadow?
2. Have your teacher check your work.

SHAPES LIKE SHADOWS OF PIECES OF CARDBOARD

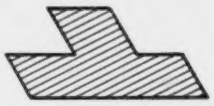
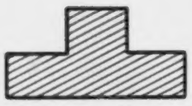
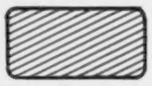
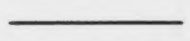
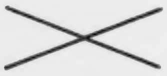
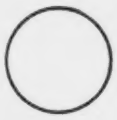
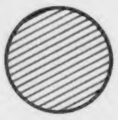
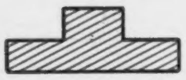
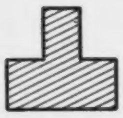
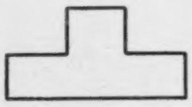
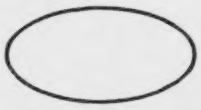
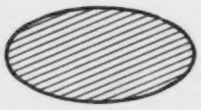
UNIT 13, EXPERIENCE 2

Activity 2

Name ______________________

Mark each of the pictures with an X, if it is a shadow of Shape X, , or with a Y, if it is a shadow of Shape Y, . If it can be made with either, mark it with both an X and a Y. Do not mark any picture that is not a shadow of X or Y.

If you are not sure of the answers, use the lamp and cardboard patterns to find out. When you have finished, have your teacher check your work.

| | | |
|---|---|---|
| 1  Y | 2  Y | 3  |
| 4  X,Y | 5  | 6  |
| 7  X | 8  Y | 9  Y |
| 10  | 11  | 12  X |

# EXPERIENCE 3

## Shapes Not Like a Rubber Band and Not Like a Piece of Cardboard

### OBJECTIVE

The student should be able to recognize figures that are different from those studied in Experiences 1 and 2.

### MATERIALS

Materials from Experiences 1 and 2
1 worksheet for each student

### TEACHER STRATEGY

For this experience the student divides pictures into three classifications: (1) those that can be made with a rubber band on the geoboard and are marked by a "G"; (2) those that can be shadows of pieces of cardboard and are marked by a "C"; and (3) the rest, which are left blank. If you prefer, you could mount the pictures on separate cards for division into three piles. As in the previous activities, the geoboard, rubber band, lamp, and pieces of cardboard should be used as necessary in the decision-making process.

If the ideas incorporated in the earlier experiences have been well learned, Experience 3 should be easy; but it is important that the student have this opportunity to review and integrate the two earlier experiences before proceeding. Leaving some pictures blank should, on the one hand, sharpen the student's understanding of the differences between the G and C categories and, on the other, suggest that there are figures that are like neither a rubber band nor a piece of cardboard—that is, are a mixture of the two types.

### EVALUATION

The way the student marks the pictures and explains his reasons will determine how well he has attained the objective of this experience.

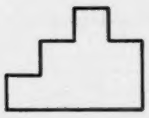
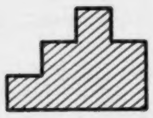
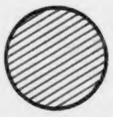
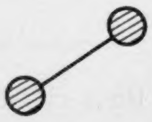
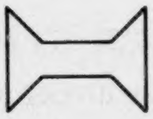
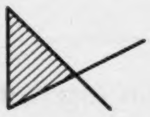
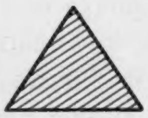
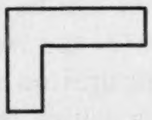
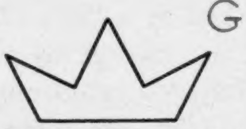
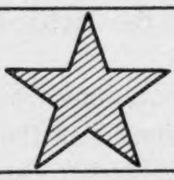
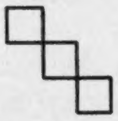
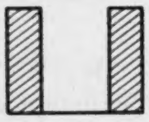
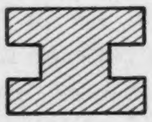
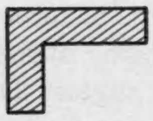
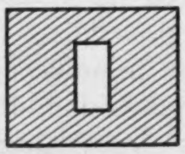
**SHAPES NOT LIKE A RUBBER BAND AND NOT LIKE A PIECE OF CARDBOARD**

**UNIT 13, EXPERIENCE 3**

Name ______________________________

Three kinds of pictures are shown below. Write "G" in each box that contains a picture you can make, in a larger size, on the geoboard. Write "C" in each box that contains a picture that could be a shadow of a piece of cardboard. Do not write anything in the other boxes.

When you have finished, have your teacher check your work.

| | | |
|---|---|---|
| **1**  G | **2**  C | **3**  C |
| **4**  | **5**  G | **6**  |
| **7**  C | **8**  | **9**  G |
| **10**  C | **11**  G | **12**  |
| **13**  C | **14**  C | **15**  C |

# EXPERIENCE 4
## Building Replicas of Geometric Figures

### OBJECTIVE

The student should be able to construct replicas of geometric figures from smaller pieces (all of which are the same size and shape) and record the number of pieces required.

### MATERIALS

24 1-in. sticks for each student
12 1-in. tagboard squares for each student
4-page worksheet for each student

The sticks can be cut from kitchen matches or from ⅛-in. balsa sticks available at hobby shops, art supply stores, and some lumber yards. They can be easily colored with a felt pen, if desired.

Counting is easier if there is a variety of colors of squares.

### TEACHER STRATEGY

Experience 4 represents a transition to methods of measuring point sets, although the students should not be told this. For the student—except that he will record the number of pieces he uses—the activity should be like putting together a puzzle.

If you can watch the students unobtrusively, look for signs that a student has found it unnecessary to actually lay out all the pieces in order to answer the questions. If he has, a positive comment is in order; however, no student should be urged to find shortcuts if he does not do so voluntarily. The emphasis, as always, is on providing the student with reliable methods which he can use to be successful; laying out pieces and counting them is such a method. In checking the work, students with all correct answers should be commended. Any student who has errors should be asked to repeat his work while you watch to determine the source of error.

Note that for the fourth picture the cardboard figure would have to have a hole in it and two rubber bands would be needed for the companion figure.

[*Continued on page 232*]

**BUILDING REPLICAS OF GEOMETRIC FIGURES** **UNIT 13, EXPERIENCE 4**

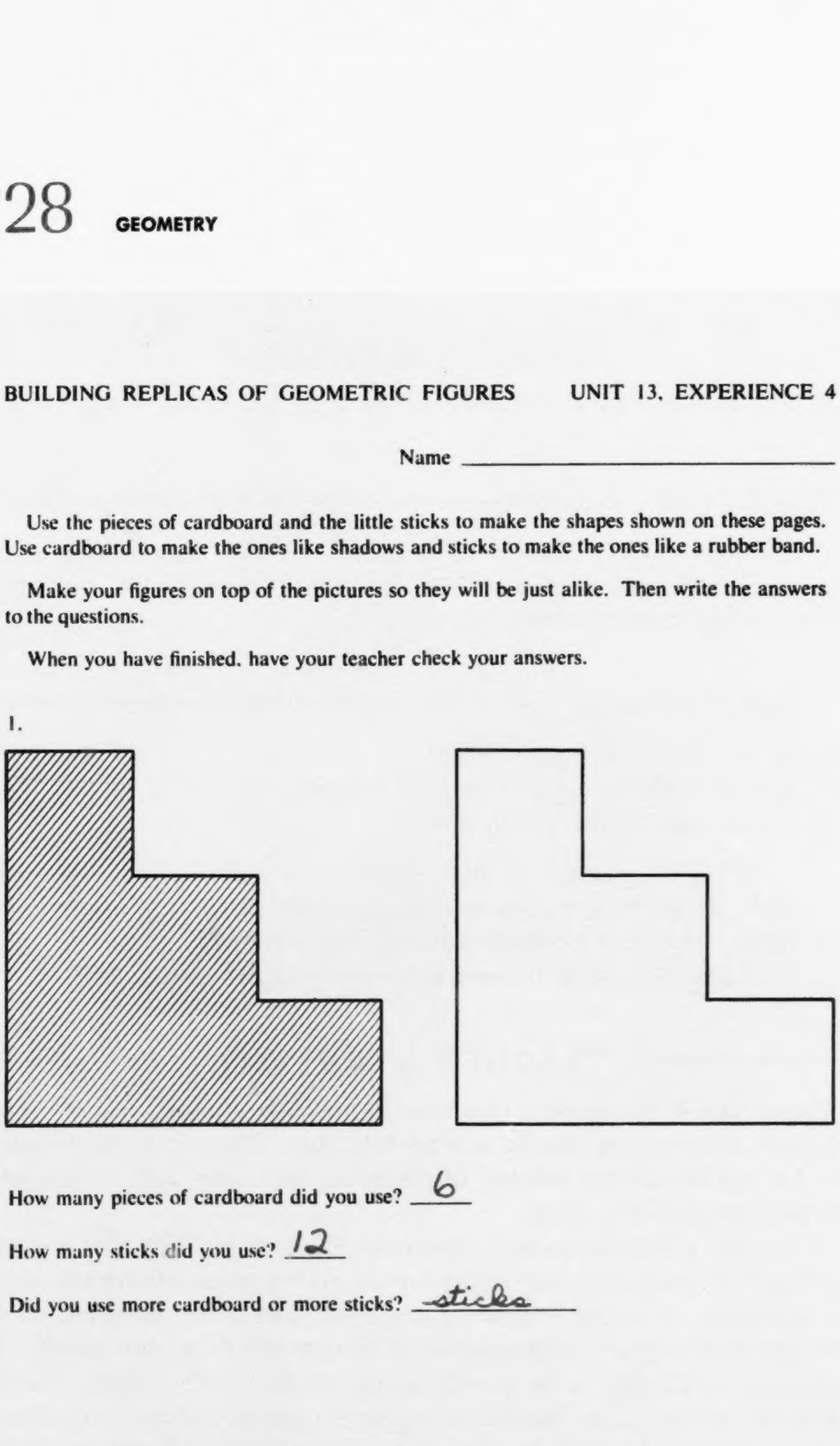
Name ______________________________

Use the pieces of cardboard and the little sticks to make the shapes shown on these pages. Use cardboard to make the ones like shadows and sticks to make the ones like a rubber band.

Make your figures on top of the pictures so they will be just alike. Then write the answers to the questions.

When you have finished, have your teacher check your answers.

1.



How many pieces of cardboard did you use? 6

How many sticks did you use? 12

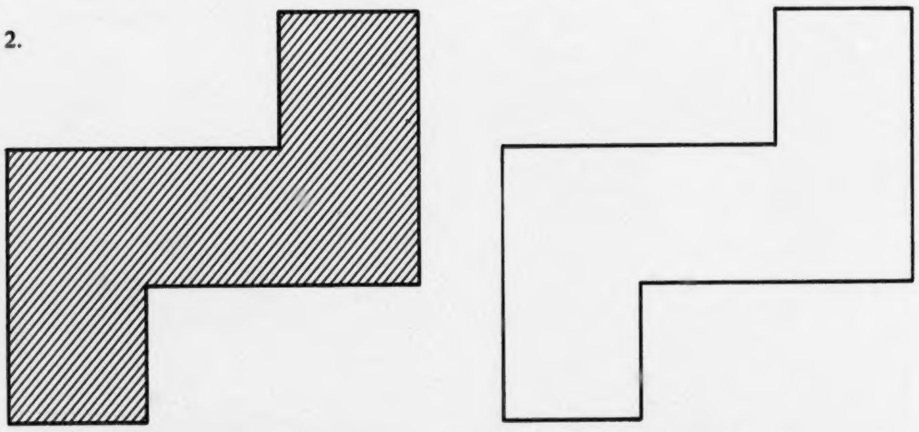
Did you use more cardboard or more sticks? sticks

*[Continued]*

## BUILDING REPLICAS OF GEOMETRIC FIGURES UNIT 13, EXPERIENCE 4

*Continued* Name ______________________

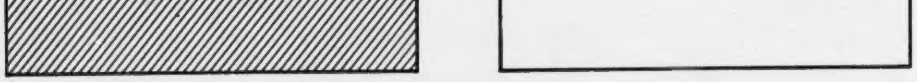
2.



How many pieces of cardboard did you use? 5 How many sticks? 12

Did you use more cardboard or more sticks? sticks

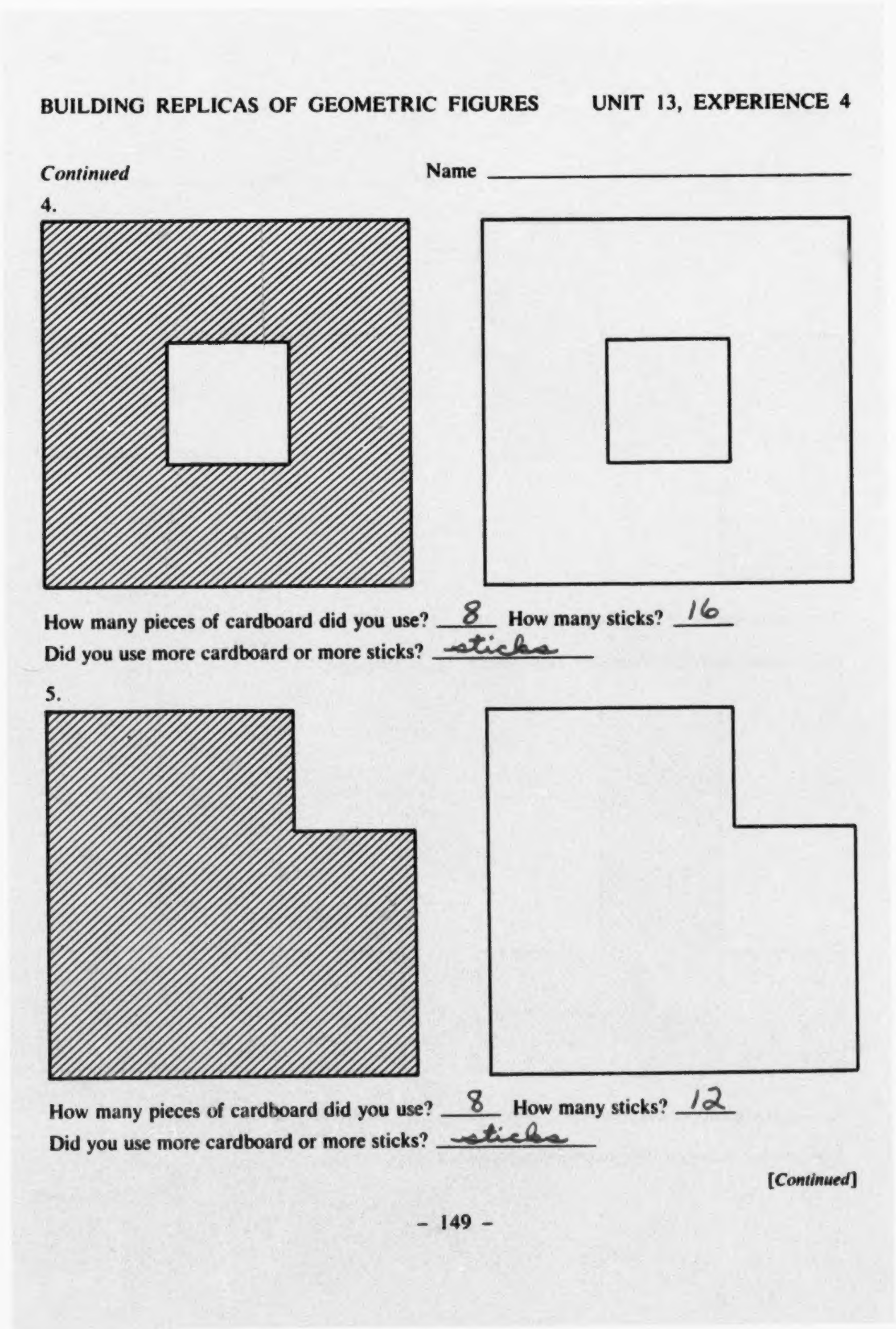
3.



How many pieces of cardboard did you use? 5 How many sticks? 12

Did you use more cardboard or more sticks? sticks

*[Continued]*



BUILDING REPLICAS OF GEOMETRIC FIGURES UNIT 13, EXPERIENCE 4

*Continued* Name ______________________

4.

How many pieces of cardboard did you use? 8 How many sticks? 16

Did you use more cardboard or more sticks? sticks

5.

How many pieces of cardboard did you use? 8 How many sticks? 12

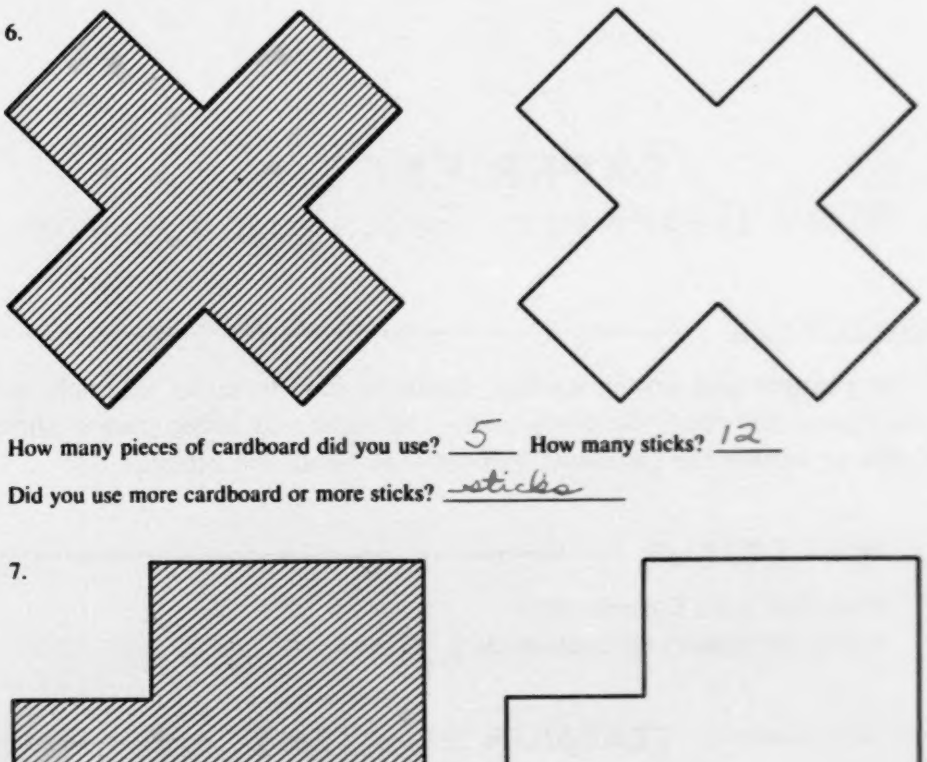
Did you use more cardboard or more sticks? sticks

*[Continued]*

## BUILDING REPLICAS OF GEOMETRIC FIGURES UNIT 13, EXPERIENCE 4

*Continued* Name ______________________

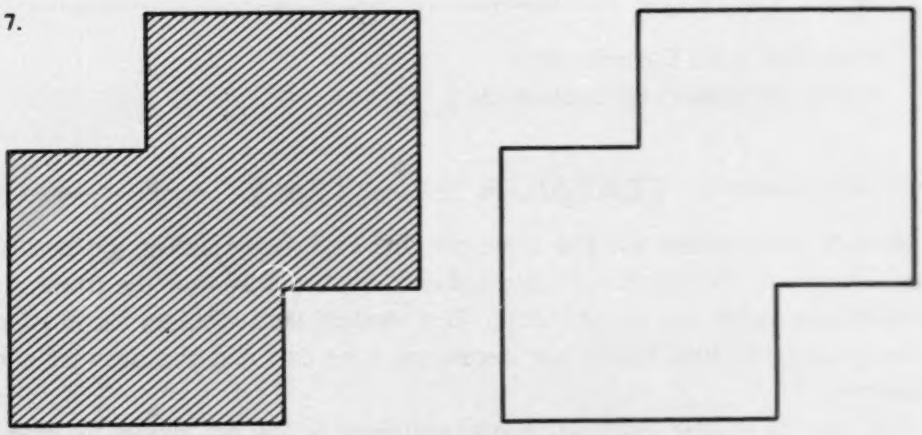
6.



How many pieces of cardboard did you use? 5 How many sticks? 12

Did you use more cardboard or more sticks? sticks

7.



How many pieces of cardboard did you use? 7 How many sticks? 12

Did you use more cardboard or more sticks? sticks

## EVALUATION

If the student gets the correct number of pieces of cardboard and of sticks for at least five of the seven sets of figures he has met the objective adequately.

# EXPERIENCE 5
## More Replicas of Geometric Figures

## OBJECTIVE

The younger and slower student should be able to make the replicas of these figures and count the parts used. The older and faster student should be able to answer the questions without making all the replicas.

### MATERIALS

Materials from Experience 4
4-page worksheet for each student

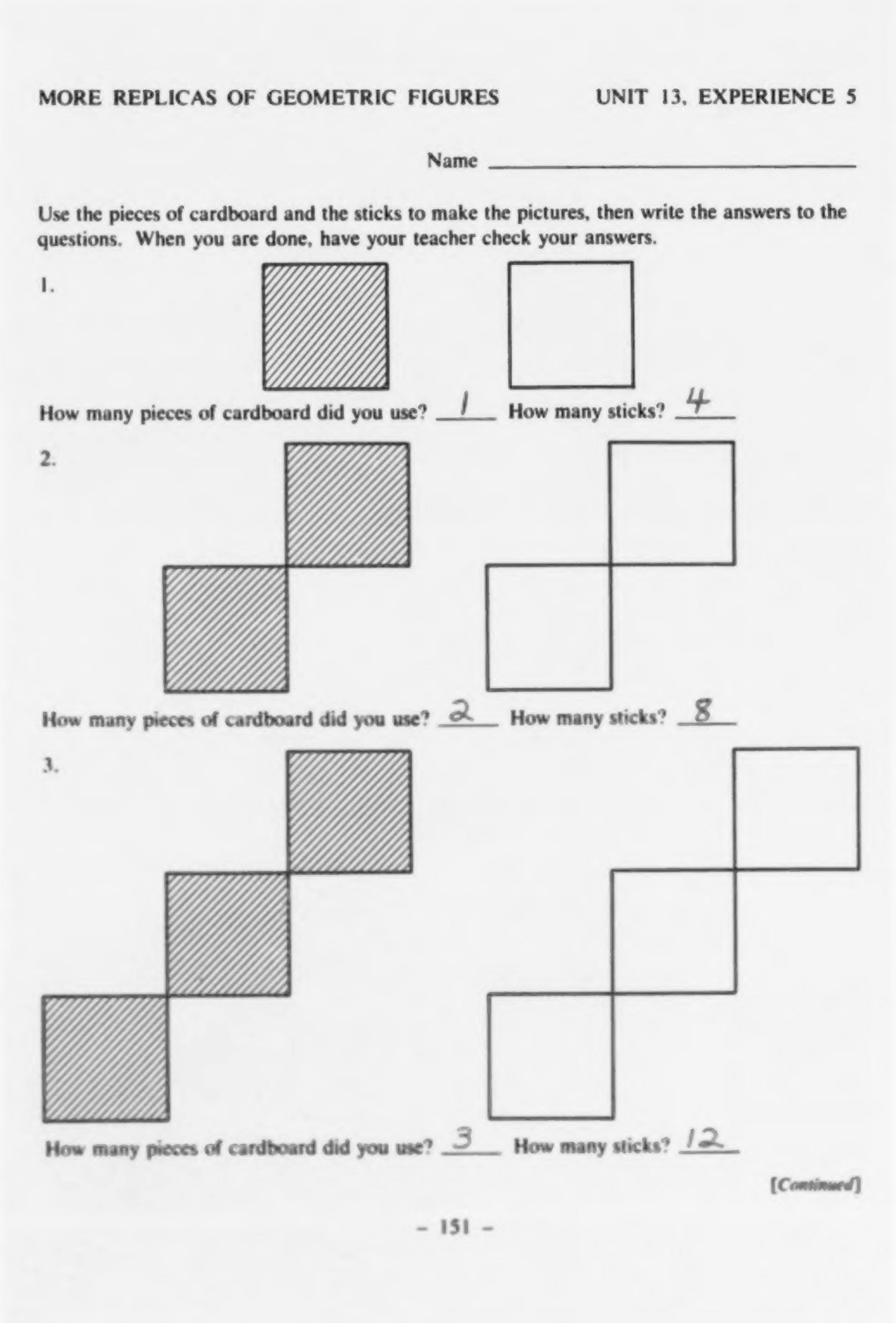
## TEACHER STRATEGY

Student instructions are the same for this experience as for the previous one. However, the figures are presented in such a way that the more alert students can make use of shortcuts. If a student asks whether he must put down pieces, tell him this is not necessary if he can find the answers some other way.

Younger or slower students should continue to lay out pieces as long as necessary, since this is a reliable method even though it is not the fastest.

## EVALUATION

Correctness of answers to the exercises, with or without the necessity of making replicas, determines the extent to which the objective has been reached.



MORE REPLICAS OF GEOMETRIC FIGURES UNIT 13, EXPERIENCE 5

Name ________________________

Use the pieces of cardboard and the sticks to make the pictures, then write the answers to the questions. When you are done, have your teacher check your answers.

1.

How many pieces of cardboard did you use? 1 How many sticks? 4

2.

How many pieces of cardboard did you use? 2 How many sticks? 8

3.

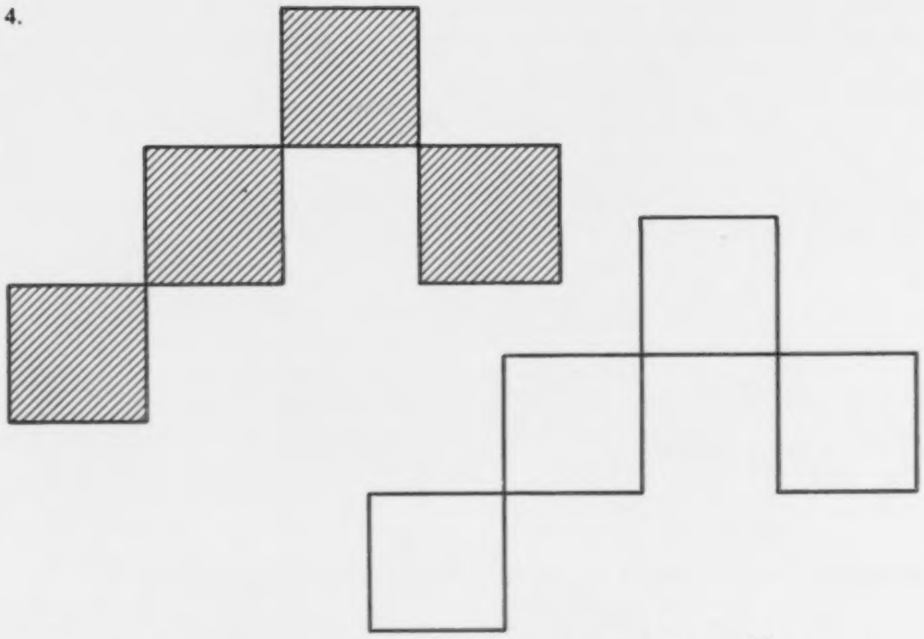
How many pieces of cardboard did you use? 3 How many sticks? 12

*[Continued]*

**MORE REPLICAS OF GEOMETRIC FIGURES** **UNIT 13, EXPERIENCE 5**

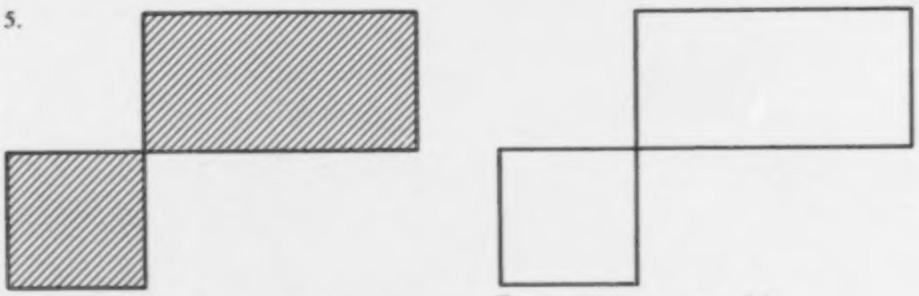
*Continued* Name ______________________

4.



How many pieces of cardboard did you use? __4__ How many sticks? __16__

5.



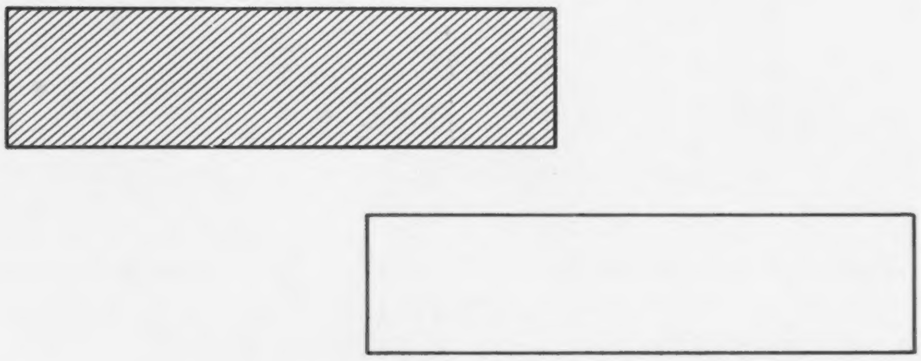
How many pieces of cardboard did you use? __3__ How many sticks? __10__

*[Continued]*

## MORE REPLICAS OF GEOMETRIC FIGURES UNIT 13, EXPERIENCE 5

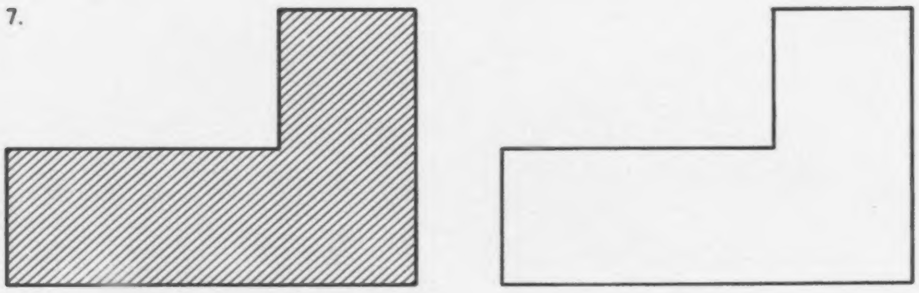
*Continued* Name ______________________________

6.



How many pieces of cardboard did you use? 4 How many sticks? 10

7.



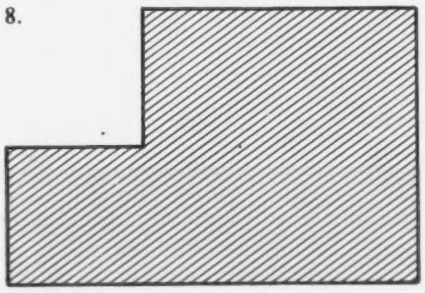
How many pieces of cardboard did you use? 4 How many sticks? 10

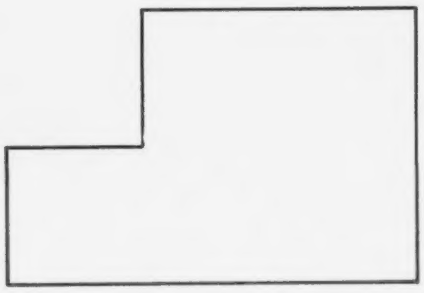
[*Continued*]

**MORE REPLICAS OF GEOMETRIC FIGURES** **UNIT 13, EXPERIENCE 5**

*Continued* Name ______________________________

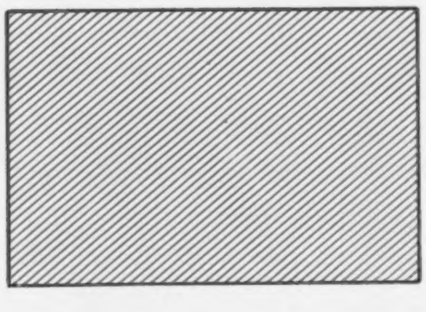
8.

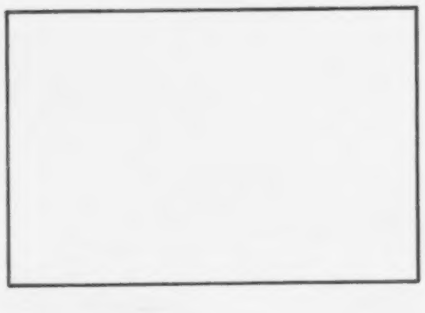




How many pieces of cardboard did you use? __5__ How many sticks? __10__

9.





How many pieces of cardboard did you use? __6__ How many sticks? __10__

## MAKING AND RECORDING OBSERVATIONS

## UNIT 7, EXPERIENCE 1

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Name ______________________________

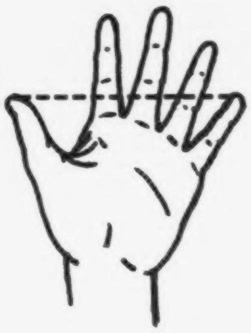
Name ______________________________

Working as a team, record in the spaces at the right the information required for five different surveys.

*Blocks from School.* The number of blocks you and your partner live from school.

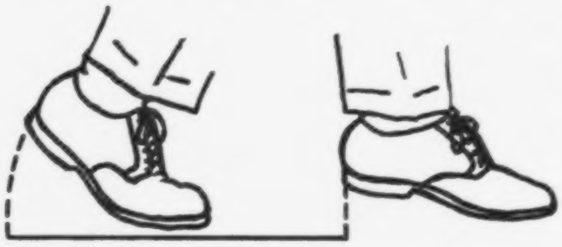
| *Blocks from School* |
|---|
| |
| |

*Length of Span.* Use the 12-inch ruler and measure the length of your span and the length of your partner's span. Here is a picture of a span:



| *Length of Span* |
|---|
| |
| |

*Length of Pace.* Use the steel tape measure or a yardstick to measure the length of your pace and your partner's pace. A pace is the length of your stride, heel to heel, as you walk. Here is a picture of a pace:



| *Length of Pace* |
|---|
| |
| |

*Length of Shoe.* Use the 12-inch ruler and measure the length of one of your shoes and one of your partner's shoes.

| *Length of Shoe* |
|---|
| |
| |

*Birth Month.* Record the month of your birthday and the month of your partner's birthday.

| *Birth Month* |
|---|
| |
| |

Now copy the answers on a card and put it in the Project Box.

SE 010 741

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Name ______________________

Use tallies to mark the tables below, then complete them by writing in the proper numerals.

*Length of Pace*

| Length in Inches | Number of People |
|---|---|
| Less than 20 | |
| 20 up to 23 | |
| 23 up to 26 | |
| 26 up to 29 | |
| 29 up to 32 | |
| 32 up to 35 | |
| More than 35 | |

*Length of Shoe*

| Length in Inches | Number of People |
|---|---|
| Less than 6 | |
| 6 up to 7 | |
| 7 up to 8 | |
| 8 up to 9 | |
| 9 up to 10 | |
| 10 up to 11 | |
| More than 11 | |

1. How many people have a pace less than 26 inches long? __________
2. How many people have a pace more than 26 inches long? __________
3. How many tally marks are on the Length of Pace table? __________
4. How many paces were measured to get the information on the Length of Pace table? __________
5. Look at your Length of Shoe table. Which is the most popular shoe size? __________
6. How many people have a shoe that is from 8 to 9 inches long? __________
7. How many tally marks are on your Length of Shoe table? __________
8. Does that table show that anyone has a shoe longer than 11 inches? __________

Tally Tables

Name ______________________________

Tally the length of the words in the writing samples below. Do not write out any answers to the questions at the end of each sample, but think about the questions so that you will be ready for class discussion.

### SAMPLE FROM A GEOGRAPHY BOOK

Let us now take the train for Paris. What funny trains we do find in Europe. They are not at all like ours. Their carriages, as the cars are called, are of three kinds, first, second, and third class, and each car is divided into little rooms which hold six, eight or ten persons. A door and two small windows are in each end of a compartment. The first and second classes have cushioned seats, but there are only wooden benches in the third.

*Do you notice anything about the table at the right? What?*

| Number of Letters | Number of Words |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| More | |

### SAMPLE FROM A NEWSPAPER EDITORIAL

Of all the reforms most desired in an essentially agricultural nation, such as South Vietnam, the most valuable symbolically and actually is land reform. Give the farmer land and he is on the road to independence and democracy. The cry for the redistribution of large land holdings is as old as the revolt led by Spartacus against Rome and as new as any reform in Latin America.

*What do you notice about the table you have just made?*

| Number of Letters | Number of Words |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| More | |

[*Continued*]

Tally Tables—*Continued* Name ______________________

Tally the length of the words in the writing samples below. Do not write out any answers to the questions at the end of each sample, but think about the questions so that you will be ready for class discussion.

### SAMPLE FROM A SPORTS STORY

The Colt touchdowns came hard. The Rams either scored easily or they did not move the ball at all. As the third quarter opened, Gabriel hoisted another long, towering pass to Jack Snow. Well covered by Boyd, Snow tipped the ball into the air, juggled it momentarily, then secured it and outran everybody for an 80-yard touchdown.

*What do you notice about the length of words in sports writing?*

| Number of Letters | Number of Words |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| More | |

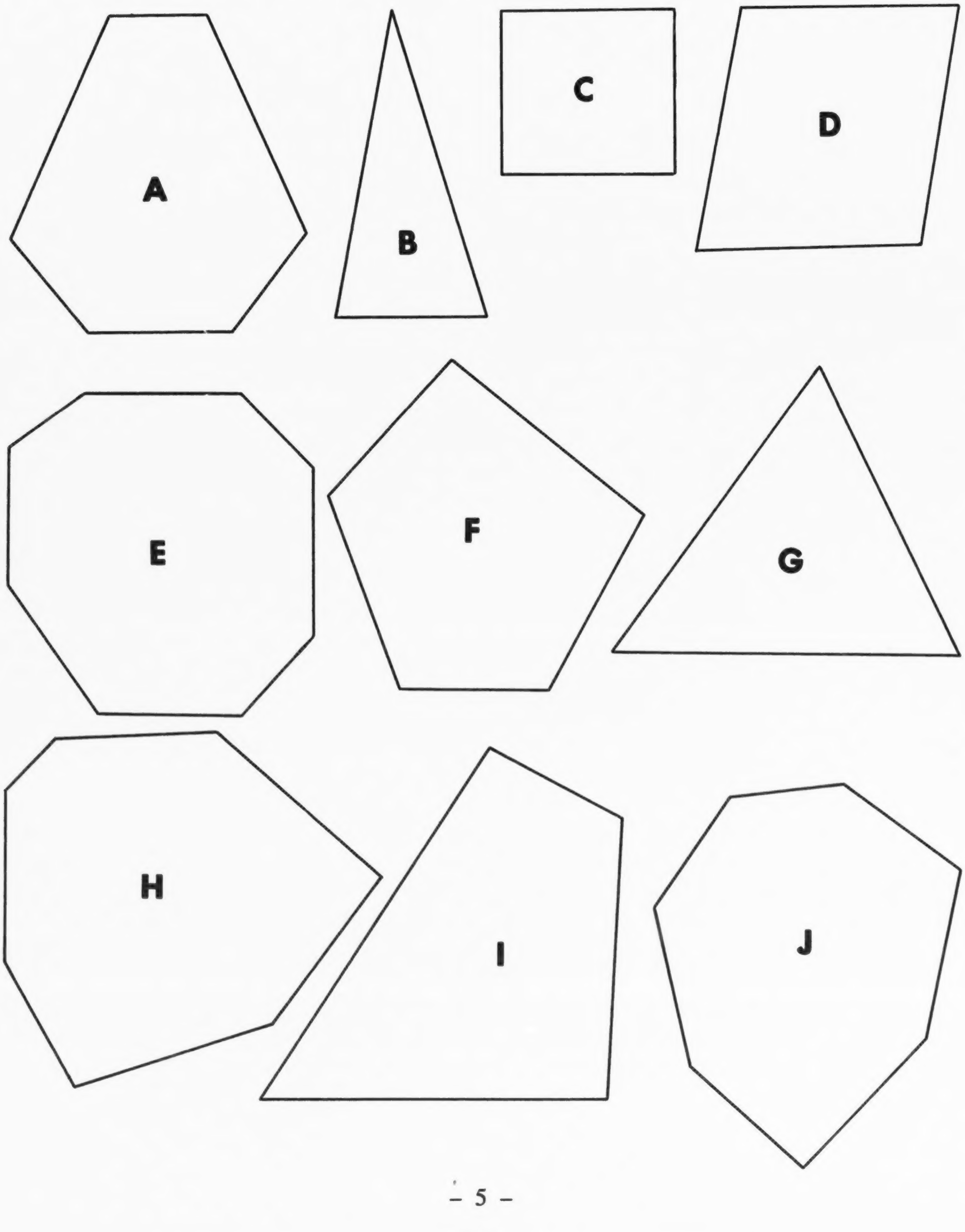
### SAMPLE FROM A NOVEL

The sun was up so high when I waked that I judged it was after eight o'clock. I laid there in the grass and the cool shade thinking about things, and feeling rested and ruther comfortable and satisfied. I could see the sun out at one or two holes, but mostly it was big trees all about, and gloomy in there amongst them. There was freckled places on the ground where the light sifted down through the leaves, and the freckled places swapped about a little, showing there was a little breeze up there. A couple of squirrels set on a limb and jabbered at me very friendly.

*Look at your table. Do you notice anything? What?*

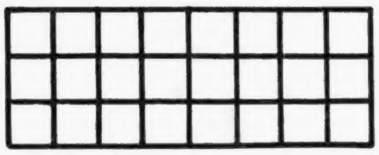
| Number of Letters | Number of Words |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| More | |

Polygons for Use in Making a Table


A
B
C
D
E
F
G
H
I
J

Name ______________________________

Name ______________________________



Here is a picture of a rectangle that is 8 blocks long and 3 blocks high.

Go to each of the four stations in any order.

At each station use *all* the tiles in front of you to make as many *different* rectangles as you can. Fill in the table for that station, showing how the rectangles are formed.

*Station A—12 Tiles*

| Number in Length | Number in Height |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |
| | |

*Station B—24 Tiles*

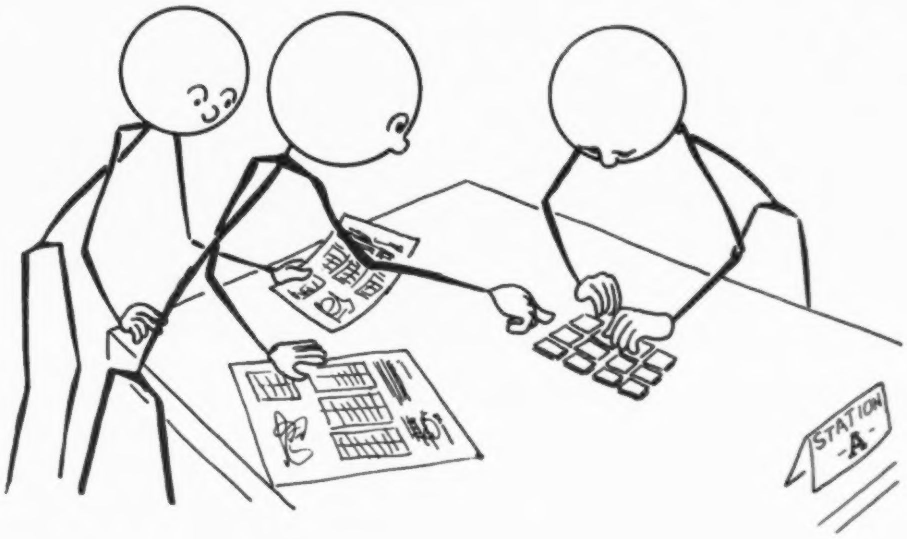
| Number in Length | Number in Height |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |
| | |

*Station C—15 Tiles*

| Number in Length | Number in Height |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |
| | |

*Station D—13 Tiles*

| Number in Length | Number in Height |
|---|---|
| | |
| | |
| | |
| | |
| | |



## TABLES OF SUMS AND DATA ON MAKING CHANGE

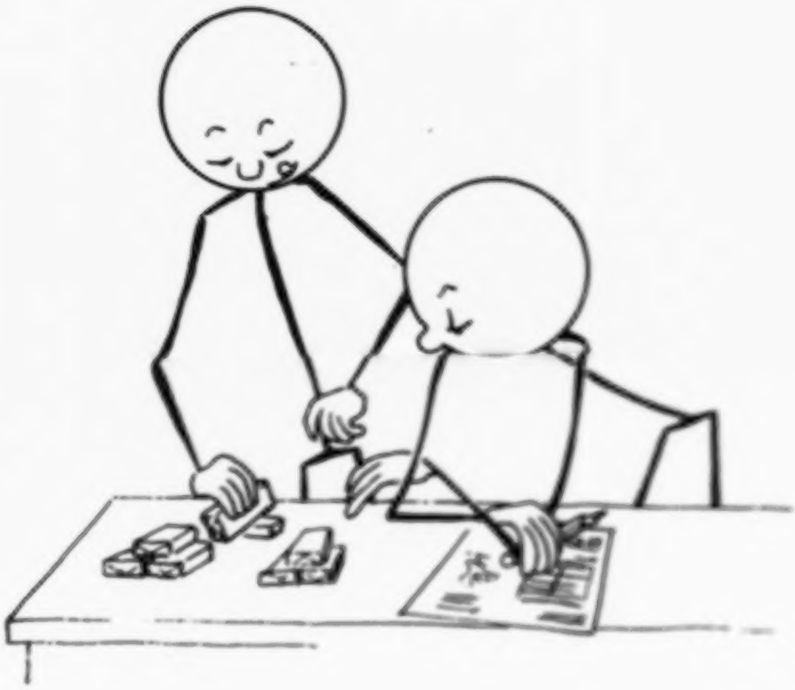
Name ____________________________

Name ____________________________

Activity 1

*Shares with __ Pieces of Candy*

| Member 1 | Member 2 |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |



Activity 2

**Amounts and Numbers of Coin Combinations**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1¢ | | 8¢ | | 15¢ | | 22¢ | |
| 2¢ | | 9¢ | | 16¢ | | 23¢ | |
| 3¢ | | 10¢ | | 17¢ | | 24¢ | |
| 4¢ | | 11¢ | | 18¢ | | 25¢ | |
| 5¢ | | 12¢ | | 19¢ | | | |
| 6¢ | | 13¢ | | 20¢ | | | |
| 7¢ | | 14¢ | | 21¢ | | | |

## *Teacher Materials*

Station A, Clue 1

Can You Multiply?

Station A, Clue 2

Add This Pair
to Your Table:

| | |
|---|---|
| 10 | 20 |

Station A, Clue 3

Add These Pairs
to Your Table:

| | |
|---|---|
| 15 | 30 |
| 20 | 40 |
| 25 | 50 |

Station A, Clue 4

Multiply by 2

Station B, Clue 1

Can You Add?

Station B, Clue 2

Add This Pair
to Your Table:

| | |
|---|---|
| 10 | 1 |

Station B, Clue 3

Add These Pairs
to Your Table:

| | |
|---|---|
| 20 | 2 |
| 21 | 3 |
| 22 | 4 |

Station B, Clue 4

Add Digits

***Teacher Materials***

Station C, Clue 1

Can You Multiply?

Station C, Clue 2

Add This Pair
to Your Table:

| | |
|---|---|
| 11 | 1 |

Station C, Clue 3

Add These Pairs
to Your Table:

| | |
|---|---|
| 21 | 2 |
| 22 | 4 |
| 23 | 6 |

Station C, Clue 4

Multiply All the Digits

Station D, Clue 1

Can You Multiply?

Station D, Clue 2

Add This Pair
to Your Table:

| | |
|---|---|
| 5 | 20 |

Station D, Clue 3

Add These Pairs
to Your Table:

| | |
|---|---|
| 3 | 12 |
| 30 | 120 |
| 35 | 140 |

Station D, Clue 4

Multiply by 4

Name ______________________________

Try to finish each of the tables on this sheet.

If you need some help, go to the station named at the top of the table and use the clues you will find there. Use the clues in order, with Clue 1 first, and don't use any more than you need to.

Check, at the bottom of the table, the numbers of the clues you have used.

| Station A | |
|---|---|
| 22 | 44 |
| 8 | 16 |
| 9 | |
| | 32 |
| 31 | |
| | 182 |
| | |
| | |
| | |
| | |
| | |
| | |

Clues Used:

1__ 2__ 3__ 4__

| Station B | |
|---|---|
| 265 | 13 |
| 14 | 5 |
| 82 | |
| 146 | |
| 75 | |
| | 17 |
| | 24 |
| | |
| | |
| | |
| | |
| | |

Clues Used:

1__ 2__ 3__ 4__

| Station C | |
|---|---|
| 13 | 3 |
| 214 | 8 |
| 223 | 12 |
| 75 | |
| 64 | |
| 127 | |
| | 32 |
| | 11 |
| | |
| | |
| | |
| | |

Clues Used:

1__ 2__ 3__ 4__

| Station D | |
|---|---|
| 2 | 8 |
| 50 | 200 |
| 9 | |
| | 28 |
| 40 | |
| | 96 |
| 120 | |
| | |
| | |
| | |
| | |
| | |

Clues Used:

1__ 2__ 3__ 4__

MATCHING TABLES AND EVERYDAY SITUATIONS UNIT 8, EXPERIENCE 1
Sheet 1 Activity 1

***Teacher Materials***

**Peas**
**2 Cans for 34¢**

**Candy Bars**
**6 for 25¢**

**Milk**
**32¢ a Quart**

**Eggs**
**58¢ a Dozen**

**Shoe Polish**
**12¢ a Can**

**GROCERY STORE**

MATCHING TABLES AND EVERYDAY SITUATIONS UNIT 8, EXPERIENCE 1
Sheet 2 Activity 1

*Teacher Materials*

| | |
|---|---|
| **3 Teaspoons** | **1 Tablespoon** |
| **2 Tablespoons** | **1 Ounce** |
| **16 Tablespoons** | **1 Cup** |
| **24 Cookies** | **1 Tray** |
| **6 Cupcakes** | **1 Pan** |

**BAKERY**

MATCHING TABLES AND EVERYDAY SITUATIONS UNIT 8, EXPERIENCE 1
Sheet 3 Activity 1

***Teacher Materials***

**Stamps**
**Regular Mail 6¢**

**Stamps**
**Airmail 10¢**

**Stamps**
**Postcards 5¢**

**Stamps**
**Special Delivery 30¢**

**Parcel Post**
**3 Pounds or Less 40¢**

**POST OFFICE**

MATCHING TABLES AND EVERYDAY SITUATIONS UNIT 8, EXPERIENCE 1
Sheet 4 Activity 1

*Teacher Materials*

**Nails**
**1 Pound 20¢**

**White Paint**
**1 Quart $2.60**

**Rope**
**48 Feet 69¢**

**Sandpaper**
**6¢ a Sheet**

**Topsoil**
**50 Pounds $1.50**

**HARDWARE STORE**

MATCHING TABLES AND EVERYDAY SITUATIONS UNIT 8, EXPERIENCE 1

Activity 1, *GROCERY STORE* Name ______________________________

For each table, find the sign that matches and write headings in the spaces over the columns.

TABLE 1

| | |
|---|---|
| 1 pt. | 16 |
| 2 pt. or 1 qt. | 32 |
| 2 qt. | 64 |
| 3 qt. | 96 |
| 4 qt. or 1 gal. | 128 |

TABLE 2

| | |
|---|---|
| 1 | 58 |
| 2 | 116 |
| 3 | 174 |
| 4 | 232 |
| 5 | 290 |

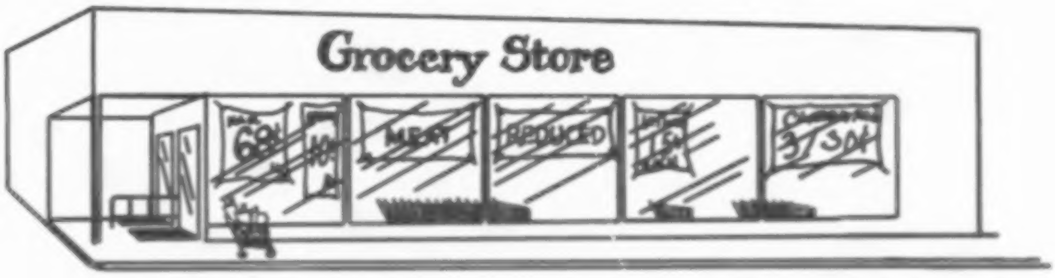


TABLE 3

| | |
|---|---|
| 3 | 15 |
| 6 | 25 |
| 9 | 40 |
| 12 | 50 |
| 15 | 65 |
| 16 | 70 |

TABLE 4

| | |
|---|---|
| 2 | 34 |
| 4 | 68 |
| 6 | 102 |
| 8 | 136 |
| 10 | 170 |
| 1 case | 408 |

## MATCHING TABLES AND EVERYDAY SITUATIONS UNIT 8, EXPERIENCE 1

Activity 1, *BAKERY* Name ______________________________

For each table, find the sign that matches and write headings in the spaces over the columns.

TABLE 1

| | |
|---|---|
| 3 | 1 |
| 6 | 2 |
| 9 | 3 |
| 12 | 4 |
| 15 | 5 |

TABLE 2

| | |
|---|---|
| 2 | 1 |
| 4 | 2 |
| 6 | 3 |
| 8 | 4 |
| 10 | 5 |

TABLE 3

| | |
|---|---|
| 16 | 1 |
| 32 | 2 |
| 48 | 3 |
| 64 | 4 |
| 80 | 5 |

TABLE 4

| | |
|---|---|
| 6 | 1 |
| 12 | 2 |
| 24 | 4 |
| 36 | 6 |
| 48 | 8 |
| 60 | 10 |
| 144 | 24 |

MATCHING TABLES AND EVERYDAY SITUATIONS UNIT 8, EXPERIENCE 1

Activity 1, *POST OFFICE* Name ____________________

For each table, find the sign that matches and write headings in the spaces over the columns.

TABLE 1

| | |
|---|---|
| 1 | 6 |
| 10 | 60 |
| 20 | 120 |
| 100—1 sheet | 600 |
| 500—1 roll | 3,000 |

TABLE 2

| | |
|---|---|
| 5 | 25 |
| 10 | 50 |
| 25 | 125 |
| 100 | 500 |

SPECIAL DELIVERY

TABLE 3

| | |
|---|---|
| 1 | 10 |
| 10 | 100 |
| 100—1 sheet | 1,000 |
| 500—1 roll | 5,000 |

TABLE 4

| | |
|---|---|
| 3 lb. or less | 40 |
| 4–6 lb. | 45 |
| 7–8 lb. | 50 |
| 9–11 lb. | 55 |
| 12–13 lb. | 60 |

# MATCHING TABLES AND EVERYDAY SITUATIONS UNIT 8, EXPERIENCE 1

Activity 1, *HARDWARE STORE* Name ______________________________

For each table, find the sign that matches and write headings in the spaces over the columns.

TABLE 1

| | |
|---|---|
| 1 pt. | 130 |
| 2 pt. or 1 qt. | 260 |
| 3 pt. | 390 |
| 4 pt. or 2 qt. | 520 |
| 1 gal. | 1,000 |

TABLE 4

| | |
|---|---|
| 1 | 20 |
| 2 | 40 |
| 5 | 100 |
| 10 | 200 |
| 20 | 400 |

TABLE 3

| | |
|---|---|
| 50 | 150 |
| 100 | 300 |
| 200 | 600 |
| 300 | 900 |

TABLE 4

| | |
|---|---|
| 12 | 20 |
| 24 | 35 |
| 48 | 69 |
| 96 | 130 |
| 144 | 185 |

## MATCHING TABLES AND EVERYDAY SITUATIONS UNIT 8, EXPERIENCE 1

Activity 2 Name ______________________________

Look at the four signs we have just written on the board. Match the tables on this sheet with the signs. In each table, write headings in the spaces over the columns.

TABLE 1

| | |
|---|---|
| 1 | 30 |
| 2 | 60 |
| 3 | 90 |
| 4 | 120 |
| 5 | 150 |

TABLE 2

| | |
|---|---|
| 6 | 1 |
| 12 | 2 |
| 18 | 3 |
| 24 | 4 |
| 30 | 5 |

TABLE 3

| | |
|---|---|
| 1 | 12 |
| 2 | 24 |
| 3 | 36 |
| 4 | 48 |
| 5 | 60 |

TABLE 4

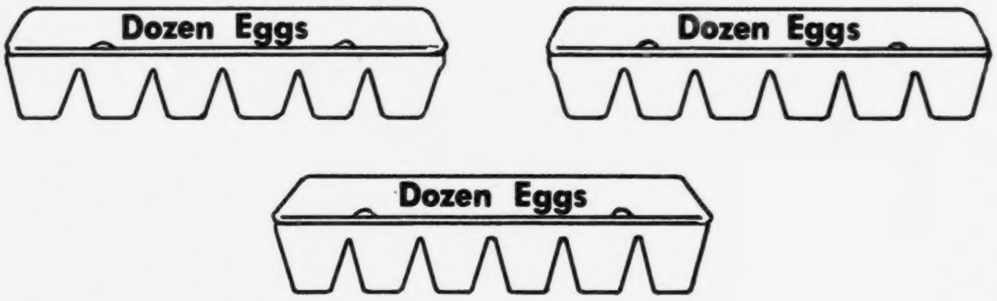
| | |
|---|---|
| 24 | 1 |
| 36 | $1\frac{1}{2}$ |
| 48 | 2 |
| 60 | $2\frac{1}{2}$ |
| 72 | 3 |

MATCHING TABLES AND EVERYDAY SITUATIONS UNIT 8, EXPERIENCE 1

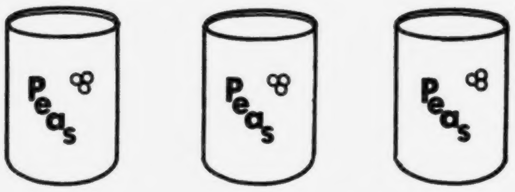
Activity 2, *GROCERY STORE* Name ________________________________

Use your tables about the Grocery Store to help you solve these problems.

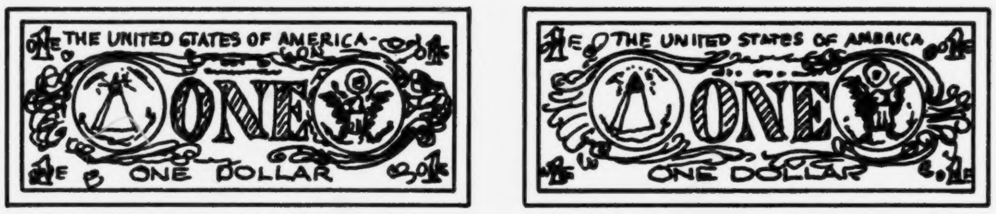
1. How much do these eggs cost? ________

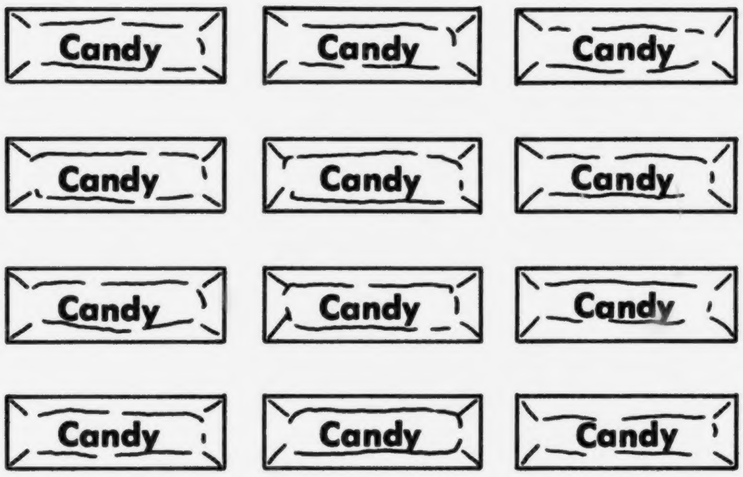

Dozen Eggs
Dozen Eggs
Dozen Eggs

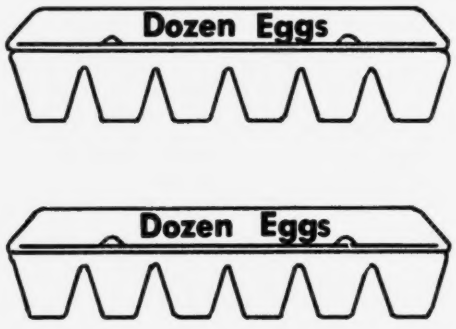
2. Find the total cost of these peas. ________


Peas
Peas
Peas

3. You have the money shown in the picture. You buy candy and eggs:


THE UNITED STATES OF AMERICA
ONE
ONE DOLLAR
THE UNITED STATES OF AMERICA
ONE
ONE DOLLAR


Candy
Candy
Candy
Candy
Candy
Candy
Candy
Candy
Candy
Candy
Candy
Candy

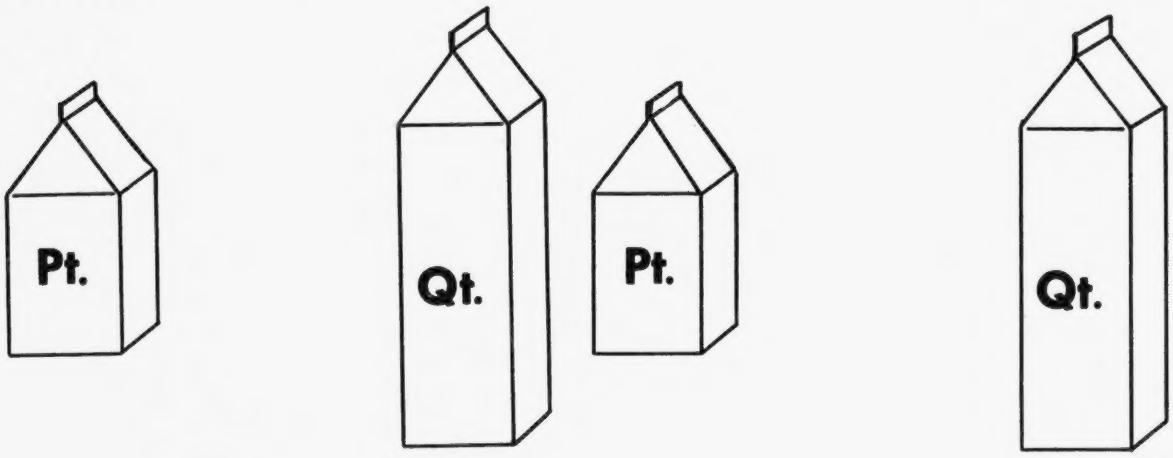

Dozen Eggs
Dozen Eggs

How much money do you have left? ________

[*Continued*]

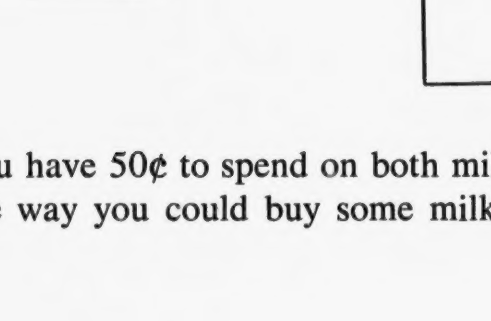
Activity 2, *GROCERY STORE–Continued* Name ______________________________

Continuing with the same problem, circle how much milk you can buy with the money you have left.



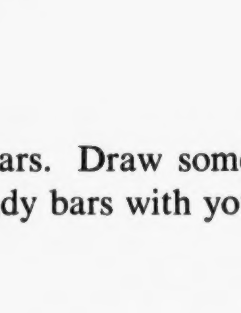
4. You have 50¢ to spend on both milk and candy bars. Draw some pictures to show one way you could buy some milk and some candy bars with your 50¢.

5. Circle which is the better buy:


Candy
Candy
Candy
Candy
Candy
Candy

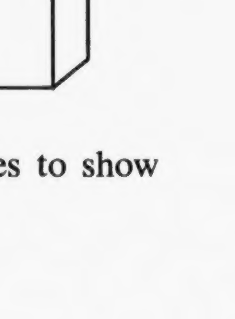
6 bars at one time

or


Candy
Candy
Candy

3 bars at one time

and


Candy
Candy
Candy

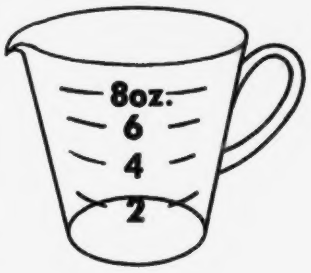
3 bars at another time

## MATCHING TABLES AND EVERYDAY SITUATIONS UNIT 8, EXPERIENCE 1

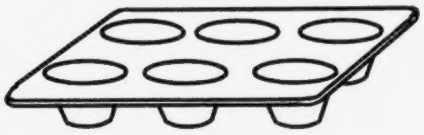
Activity 2, *BAKERY* Name ______________________________

Use your tables about the Bakery to help you solve these problems.

1. A recipe calls for 8 tablespoons of butter. Draw a line on this picture of a measuring cup to show how many ounces of butter you would use.
   What fraction of a cup have you marked? _____



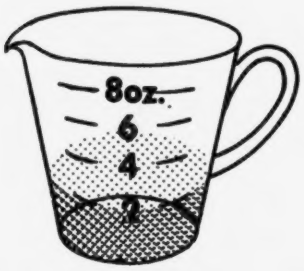
2. Here is a picture of a cupcake pan:



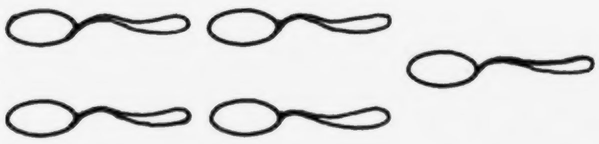
   How many of these pans would the baker need to make 60 cupcakes? _____

   The baker's oven holds 5 pans at one time. How many cupcakes can he make at one time if he fills his oven with cupcake pans? _____

3. Here is a picture of a measuring cup with some vanilla flavoring in it:
   How many tablespoons of vanilla are in the cup? _____

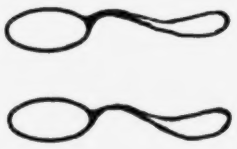


4. Circle which holds more:



Teaspoons

or



Tablespoons

5. Draw a picture of the number of tablespoons in 6 ounces of a liquid.

## MATCHING TABLES AND EVERYDAY SITUATIONS UNIT 8, EXPERIENCE 1

Activity 2, *POST OFFICE* Name ______________________________

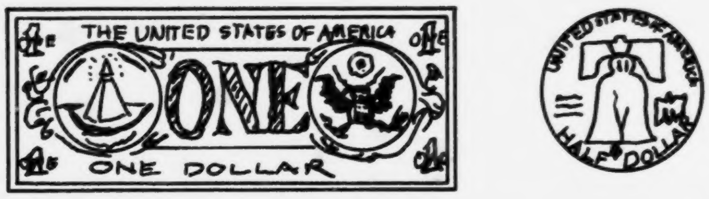
Use your tables about the Post Office to help you solve these problems.

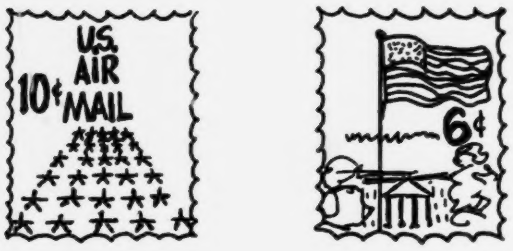
1. You have the money in the picture.

   You want to buy the stamps shown.

   Do you have enough money?

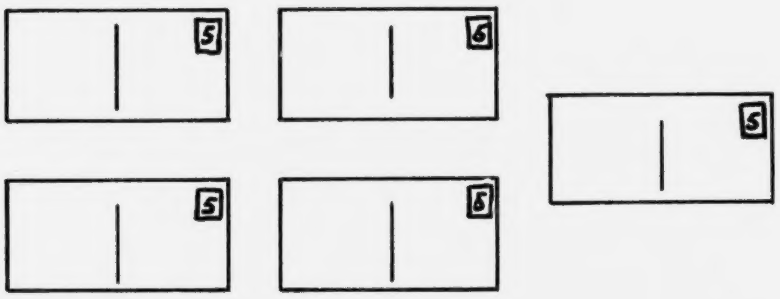
   ____


THE UNITED STATES OF AMERICA
ONE
ONE DOLLAR
UNITED STATES OF AMERICA
HALF DOLLAR

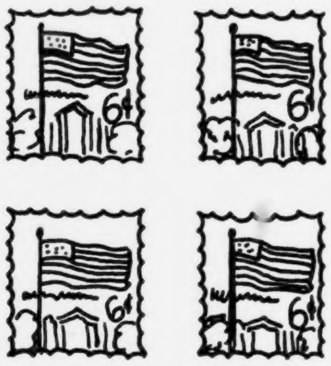

U.S. AIR MAIL
10¢
6¢

10 Airmail and 10 Regular

2. Circle which would cost more:

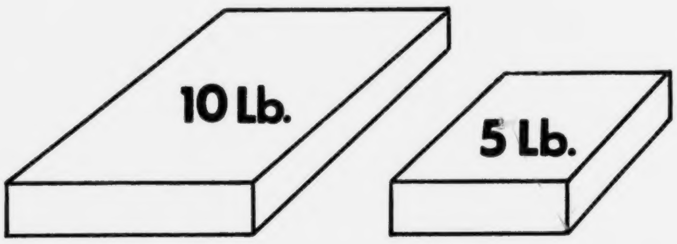


or



3. How much would it cost to mail these two packages at the same time?

   ____



4. Circle the total cost of 2 rolls of regular stamps and 1 sheet of airmail stamps. There are 500 stamps in a roll and 100 stamps in a sheet.

   $16 $70 $46 $10

[*Continued*]

Activity 2, *POST OFFICE–Continued* Name ____________________________

5. You went to the post office with $1.00. You bought 6 airmail stamps, and the clerk gave you your change in postcards. Draw a picture of the correct number of postcards the clerk gave you.

## MATCHING TABLES AND EVERYDAY SITUATIONS UNIT 8, EXPERIENCE 1

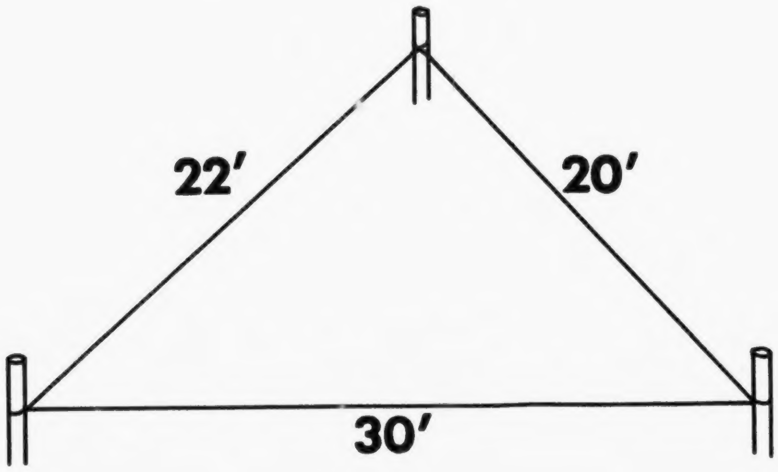
Activity 2, *HARDWARE STORE* Name ______________________________

Use your tables about the Hardware Store to help you solve these problems.

1. Here is a picture of some clothesline in a backyard.

   If you use rope for the clothesline, how much will it cost?

   ____



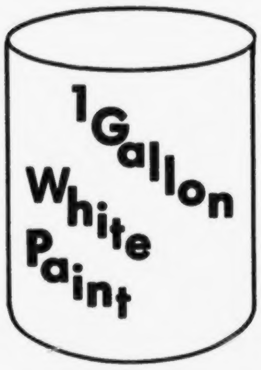
   Explain your answer. ______________________________

   ______________________________

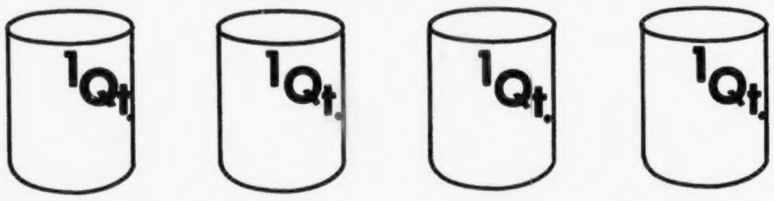
   ______________________________

2. Topsoil is sold only in 50-pound bags. How many bags of topsoil could you buy with $10? ____

3. Which is cheaper? Circle your answer.



or

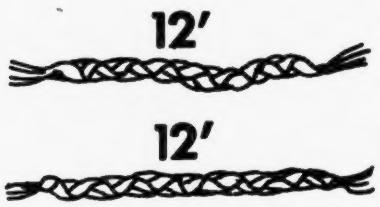

1 Qt.
1 Qt.
1 Qt.
1 Qt.

[*Continued*]

Name ______________________________

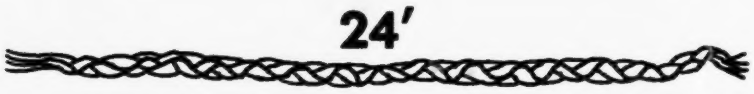
Activity 2, *HARDWARE STORE–Continued*

4. Which is more expensive? Circle your answer.



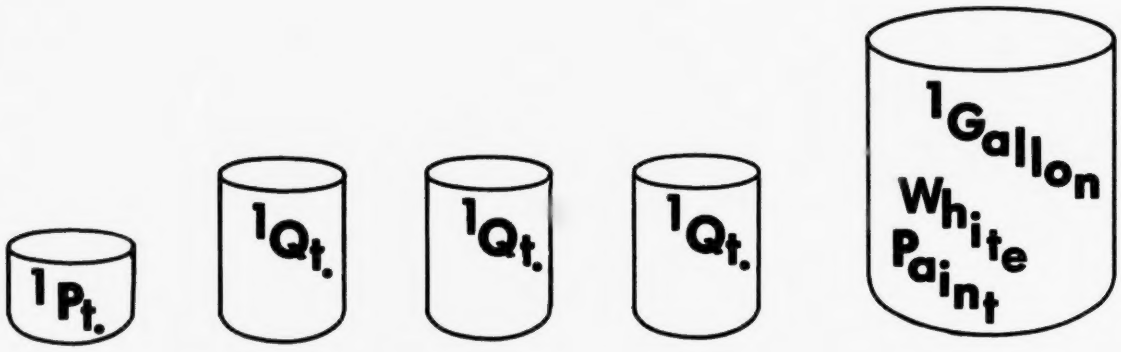
Rope

or



Rope

5. Find the total cost of the paint shown here. _____


1 Pt.
1 Qt.
1 Qt.
1 Qt.
1 Gallon White Paint

FINDING PATTERNS IN TABLES AND USING THEM TO SOLVE PROBLEMS

UNIT 8, EXPERIENCE 2

Activity 1

Name ________________________________

Use the objects at the stations to help you decide what the headings for each table should be, then write them in. Complete the tables.

TABLE A

| | |
|---|---|
| | 1 |
| | 2 |
| | 3 |
| 48 | 4 |
| 60 | 5 |
| 72 | 6 |
| 84 | 7 |
| | 8 |
| | 9 |
| | 10 |

TABLE B

| | |
|---|---|
| 2 | 10 |
| 20 | 28 |
| 8 | 16 |
| 7 | 15 |
| 10 | |
| 3 | |
| 4 | |
| 6 | |
| 1 | |
| 15 | |

[*Continued*]

Activity 1—*Continued*

Name ______________________________

Use the objects at the stations to help you decide what the headings for each table should be, then write them in. Complete the tables.

TABLE C

| | |
|---|---|
| 2 | |
| 12 | |
| 6 | |
| 4 | 9 |
| 9 | |
| 1 | |
| 3 | 12 |
| 18 | |
| 36 | 1 |

TABLE D

| | |
|---|---|
| | 1 |
| 14 | 2 |
| 21 | 3 |
| 28 | 4 |
| | 5 |
| | 6 |
| | 7 |
| | 8 |
| | 9 |
| | 10 |

[*Continued*]

## FINDING PATTERNS IN TABLES AND USING THEM TO SOLVE PROBLEMS

UNIT 8, EXPERIENCE 2

Activity 1–*Continued*

Name ______________________________

Use the objects at the stations to help you decide what the headings for each table should be, then write them in. Complete the tables.

TABLE E

| | |
|---|---|
| 1 | |
| 2 | 16 |
| 3 | |
| 4 | |
| 5 | 13 |
| 6 | |
| 7 | |
| 8 | 10 |
| 9 | |
| 10 | |

TABLE F

| | |
|---|---|
| | 1 |
| 4 | 2 |
| 6 | 3 |
| 8 | 4 |
| 10 | 5 |
| | 6 |
| | 7 |
| | 8 |
| | 9 |
| | 10 |

[*Continued*]

Activity 1–*Continued*

Name ______________________________

Use the objects at the stations to help you decide what the headings for each table should be, then write them in. Complete the tables.

TABLE G

| | |
|---|---|
| | 5 |
| | 2 |
| | 7 |
| 22 | 3 |
| | 8 |
| 16 | 9 |
| | 4 |
| 24 | 1 |
| | 6 |
| 15 | |

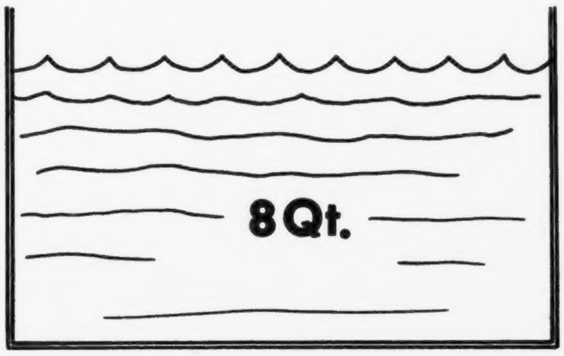
TABLE H

| | |
|---|---|
| | 1 |
| 16 | 2 |
| 24 | 3 |
| 32 | 4 |
| 40 | 5 |
| | 6 |
| | 7 |
| | 8 |
| | 9 |
| | 10 |

Activity 2 Name ______________________________

Use tables A, D, F, and H to help you solve these problems.

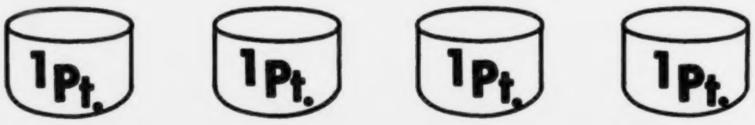
1. Joe's fish tank holds 8 quarts of water.
   Joe takes out half the water.
   How many quarts of water are now left in Joe's tank? _____
   How many *pints* of water are now left in Joe's tank? _____



2. Sara has a 12-ounce bottle of coke. Circle how many cups of coke she has in her bottle.

   1 cup
   2 cups
   Between 1 and 2 cups
   More than 2 cups

3. John has strawberries as pictured below.


1 Pt.
1 Pt.
1 Pt.
1 Pt.

   Draw cups to show the number of cups of strawberries John has.

4. Which contains less milk, 11 cups or 5 pints? _____
   11 cups or 3 quarts? _____

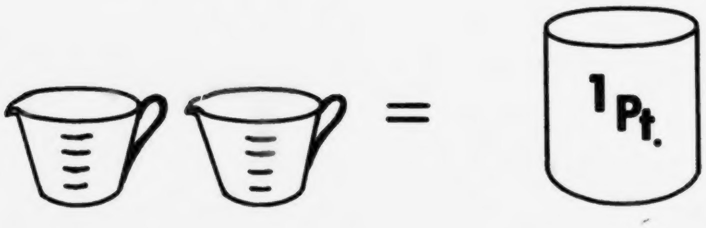
[*Continued*]

Activity 2–*Continued*

Name ________________________________

5. Following the example at the top, connect the equations below to make true statements.

2 pints
2 cups = 1 pint
2 quarts
1 quart



8 pints
4 quarts = 4 pints
6 cups
10 cups

2 cups
24 ounces = 4 cups
3 pints
3 cups

2 quarts
10 pints = 20 cups
4 quarts
40 cups

1 quart
6 cups = 1 pint
3 pints
24 ounces

[*Continued*]

Activity 2–*Continued* Name ____________________

Use tables A, D, F, and H to help you solve these problems:

6. A customer on your paper route will be gone on his vacation 6 weeks and 3 days.

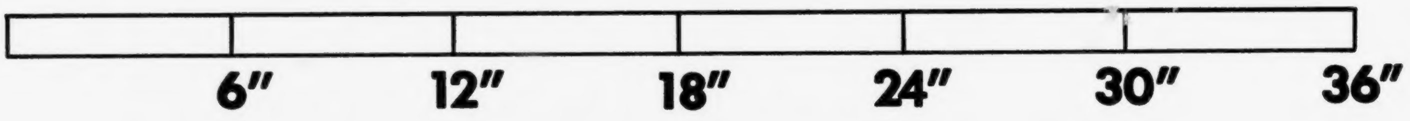
   How many days will he be gone? _____

7. Becky's dad wants to put screen around their back porch. He measured and found that the porch is 40 feet around. If the store sells screen by the running inch, how many inches of screen should Becky's dad buy? _____

8. John is 8 years and 4 months old; Mike is 7 years and 18 months old.

   Who is the older of the two? _____

9. Here is a diagram of a yardstick:



   Draw a line on it to show a measure of 2 feet, 3 inches.

10. Draw another diagram of a yardstick and show a measure of $1\frac{1}{2}$ feet on it.

MATCHING NUMBER PAIRS WITH TABLES UNIT 8, EXPERIENCE 3
Sheet 1

***Teacher Materials***

| | |
|---|---|
| 84 | 12 |
| $1\frac{1}{2}$ | 24 |
| 40 | 20 |
| 18 | 0 |
| 8 | $4\frac{1}{2}$ |

Sheet 2

***Teacher Materials***

| | |
|---|---|
| 9 | 17 |
| 132 | 11 |
| 11 | 19 |
| 54 | $1\frac{1}{2}$ |
| 15 | 3 |

Sheet 3

**Teacher Materials**

| | |
|---|---|
| 0 | 8 |
| 140 | 20 |
| 0 | 18 |
| 100 | 50 |
| $\frac{1}{2}$ | 72 |

**Teacher Materials**

| Committee 1<br>Data-Recording Form | |
|---|---|
| Names of Members | Number of Heartbeats in 30 Seconds |
| | |
| | |
| | |
| | |
| | |
| | |
| | |

| Committee 2<br>Data-Recording Form | |
|---|---|
| Names of Members | Time to Walk 52 Feet, 10 Inches |
| | |
| | |
| | |
| | |
| | |
| | |
| | |

**Teacher Materials**

| Committee 3<br>Data-Recording Form | |
|---|---|
| Block | Length in Feet |
| 1st | |
| 2nd | |

| Committee 4<br>Data-Recording Form | | | |
|---|---|---|---|
| Standard Cars | Length in Feet | Small Cars | Length in Feet |
| 1st | | 1st | |
| 2nd | | 2nd | |
| 3rd | | 3rd | |
| 4th | | | |
| 5th | | | |

Name ______________________________

Use the tables you have just made (tables 1–4) to help you answer these questions.

1. What is the average number of heartbeats in 1 minute for members of your committee? _____ How many beats in 3 minutes? _____ How many beats in 30 minutes? _____

2. How can you figure out how many times your heart beats in a week?

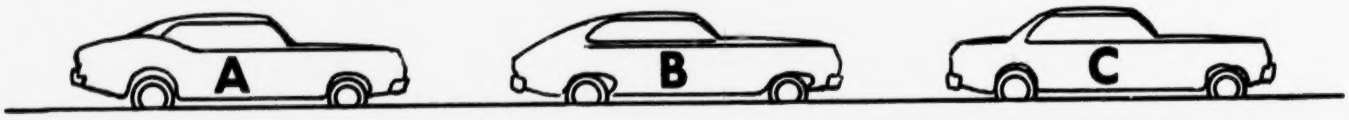
______________________________________________________________

______________________________________________________________

See if you can now find the number of times your heart beats in a week.

______________________________________________________________

3. Here is a picture of 3 standard cars parked along one side of a block:



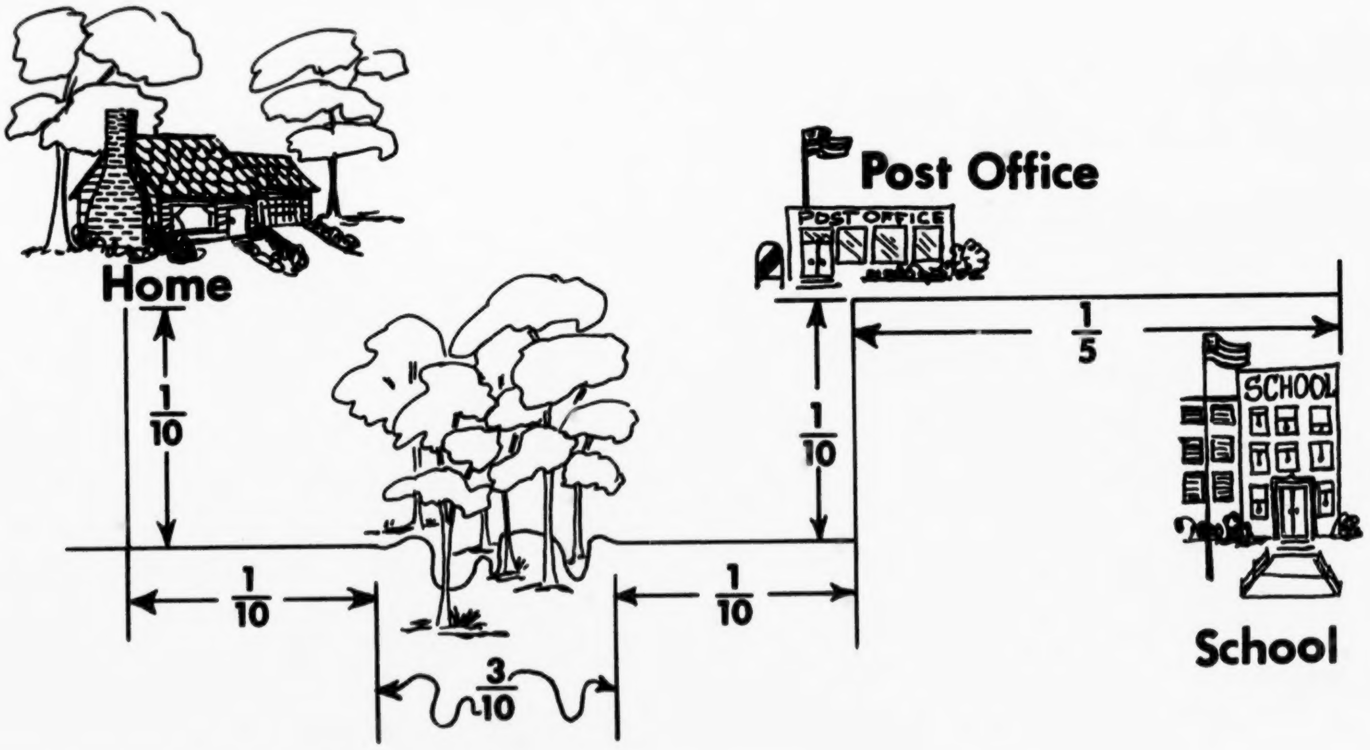
If we allow 4 feet between Car A and Car B and 4 feet also between Car B and Car C, about how many feet is it from the front of Car A to the back of Car C? _____
What would your answer be if these cars were all small cars? _____
What would your answer be if Car A were a standard car and the other 2 were small cars? _____
Would it make any difference in your answer if the standard car were parked between the 2 small cars? _____

[*Continued*]

*Continued* Name ______________________

4. Use the number of feet in the length of 1 block from table 3. About how many standard cars can be parked along one side of the block if we allow 4 feet between cars? ______
If police regulations require that we cannot park within 25 feet of a corner, how many standard-size cars can you park along 1 side of the block? ______

5. Here is a map of the walk Sam takes each day in going to and from school. All the measurements are in fractions of a mile.


Post Office
POST OFFICE
Home
$\frac{1}{10}$
$\frac{1}{10}$
$\frac{1}{5}$
SCHOOL
$\frac{1}{10}$
$\frac{1}{10}$
$\frac{3}{10}$
School

Does Sam live more or less than a mile from school? ______

How long does it take the average committee member to walk a mile? ______

How long do you think it will take Sam to walk home from school? ______

When Sam starts from home how long will it take him to get from his house to the other side of the woods? ______

Problem 1 Name ______________________________

See if you can answer this question: "I am thinking of three consecutive even integers. The sum of the first and third of these integers is 28. What are the integers?"

Write down a few guesses. ______________________________________________

Check your guesses to see if the sum of the first and third numbers is 28.

If you guessed the correct integers, circle the correct guess. Congratulations!

If your guesses didn't give the integers you were looking for, see if completing the table below will help you to find them.

Keep Going!

| 1st Number | 2nd Number | 3rd Number | Sum of 1st and 3rd |
|---|---|---|---|
| 2 | 4 | 6 | 8 |
| 4 | 6 | 8 | 12 |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

What are the three integers you were looking for? ______________

You are finished with Problem 1. Go on to Problem 2 now.

Problem 2 Name ______________________________

See if you can solve this tricky problem: The perimeter of a rectangle is 34 feet. Its length is 5 feet more than its width. What are the length and width of this rectangle?

Make a few guesses: _____ for the length and _____ for the width

or

_____ feet long and _____ feet wide

Check your guesses to see if the length is 5 feet more than the width.

Would the perimeter in your guesses be 34 feet at the same time the length is 5 feet more than the width? _____

If you have guessed the correct answers, circle them. You are a mighty good guesser. You are finished with Problem 2 and may go on to Problem 3 right now.

If you have not guessed the answers, see if filling in the table below might help you.

Keep Going!

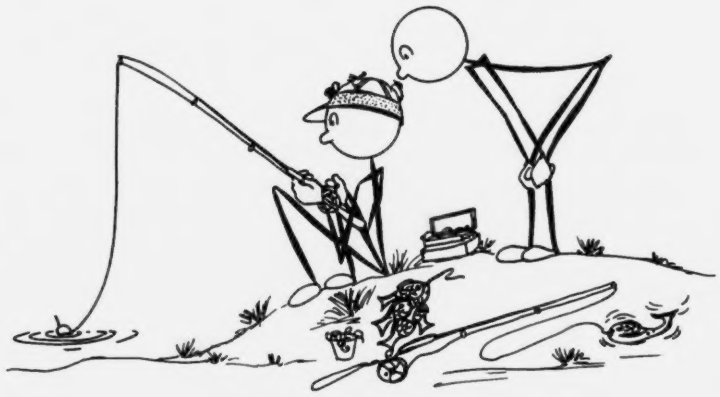
| Number of Feet in Length | Number of Feet in Width | Number of Feet in Perimeter |
|---|---|---|
| 6 | 1 | 14 |
| 7 | 2 | 18 |
| | | |
| | | |
| | | |
| | | |

What are the length and width you were looking for? _______________

USING TABLES TO SOLVE DIFFICULT PROBLEMS UNIT 8, EXPERIENCE 5

Problem 3 Name ______________________________

Suppose two boys, Tom and Sam, went fishing. Together they caught 6 fish. How many did Tom catch, and how many did Sam catch? See if this table, which has been started for you, helps to organize your answers.



Keep Going!

| Number of Fish Tom Caught | Number of Fish Sam Caught | Total Number of Fish Caught |
|---|---|---|
| 0 | 6 | 6 |
| 1 | 5 | 6 |
| | | |
| | | |
| | | |
| | | |
| | | |

How many pairs of answers are correct? ______

Suppose Tom caught twice as many fish as Sam. How many fish did Tom catch? ______

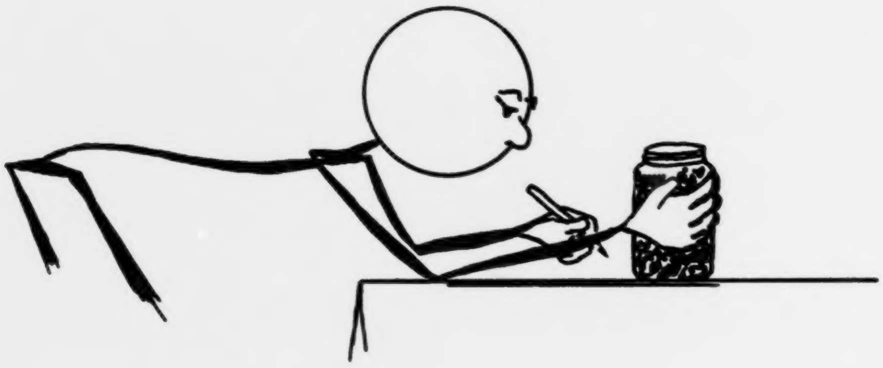
Suppose Sam's mother said: "If Sam had caught 4 more fish, he would have caught the same number of fish as Tom." How many did Sam catch? ______

Stumper 1 Name ____________________

The equipment you may use is pencil and paper, a pint jar, and a bag of lima beans. Use any or all of it to help you solve this stumper:

The number of bugs in a bottle doubles every hour. The bottle is full of bugs at the end of one day. When was the bottle half full of bugs?

Solution:



Stumper 2 Name ________________________________

The equipment you may use is pencil and paper, a large sheet of newspaper, and a pair of scissors. Use any or all of it to help you solve this stumper:

A sheet of newspaper is approximately .003 of an inch thick. Suppose it were possible to cut the sheet in half and place the 2 pieces together, one on top of the other, then cut them in half again and place the 4 pieces together in a pile, then cut them in half a third time and place the 8 pieces in a pile, and so on until the paper had been halved 25 times. How high would you guess the final pile would be? Check one of the following:

1 foot high ______ 1 mile high ______

100 feet high ______ More than 1 mile high ______

Solution:

School Supplement

Stumper 3 Name ______________________________

The equipment you may use is pencil and paper and a calendar page that shows one month. Use any or all of it to help you solve this stumper:

A woman was once told by her lawyer that she could pay her taxes in either of two ways. She could pay $100 a week for 3 weeks or she could pay 1¢ the first day, 2¢ the second day, 4¢ the third day, 8¢ the fourth day, and so on for 3 weeks, each day's taxes being twice that of the day before. Which way of paying her taxes should the woman choose?

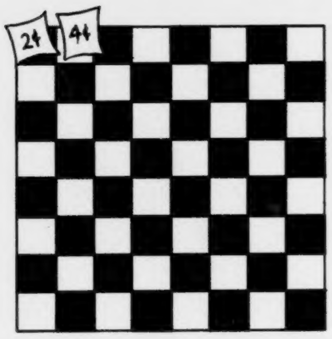
Solution:

Stumper 4 Name ____________________________

The equipment you may use is pencil and paper, a checkerboard, and 64 squares of paper. You may use any or all of this to help you solve this stumper:

Suppose a friend gives you this choice: "I will give you $1 for each square on the checkerboard, or I will give you 2¢ on the first square, 4¢ on the second square, 8¢ on the third square, 16¢ on the fourth square, and so on through the sixty-fourth square on the board." Which choice would you take?

Solution:



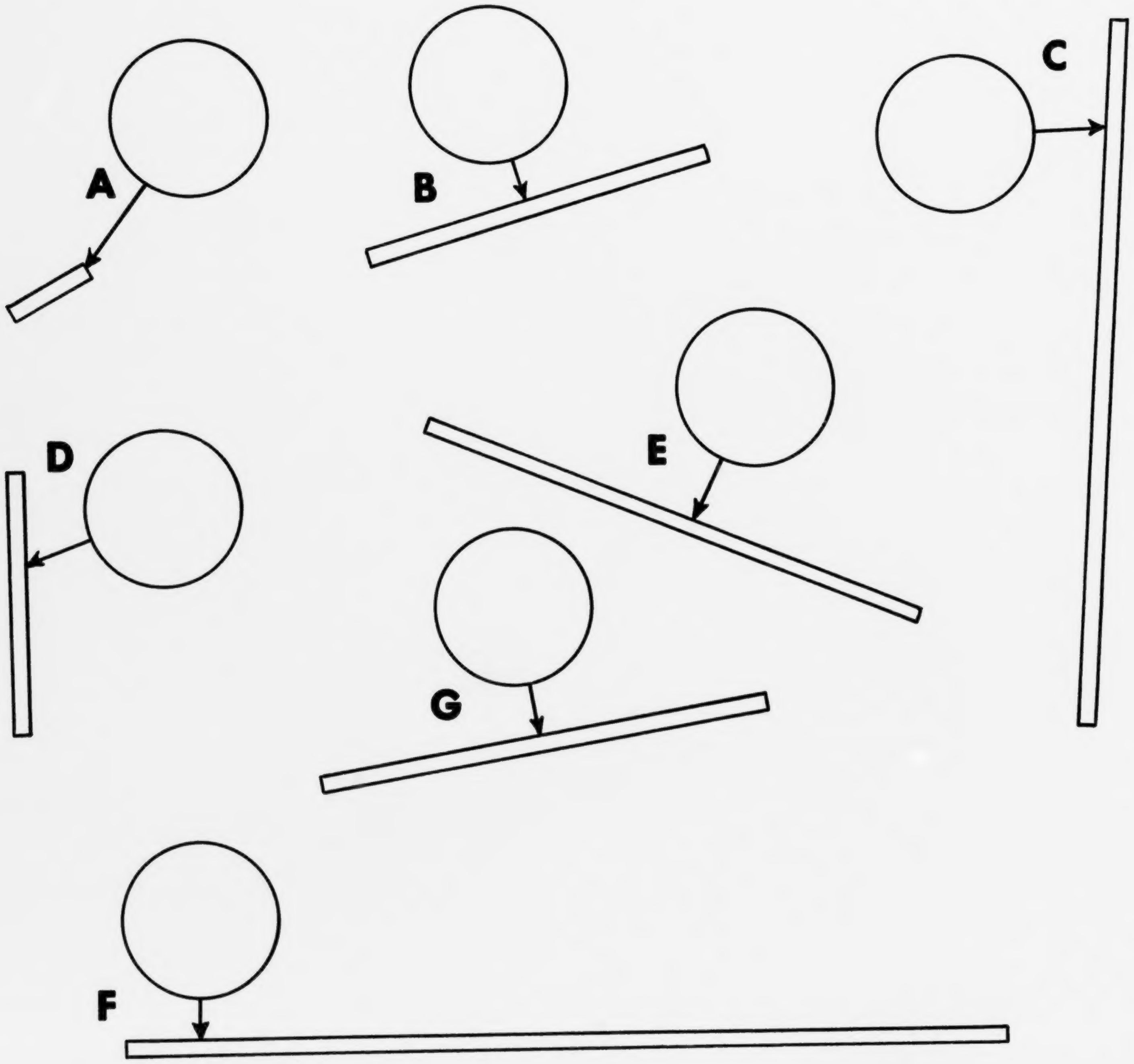
## FIND THE IMAGE

UNIT 9, EXPERIENCE 1

Station A

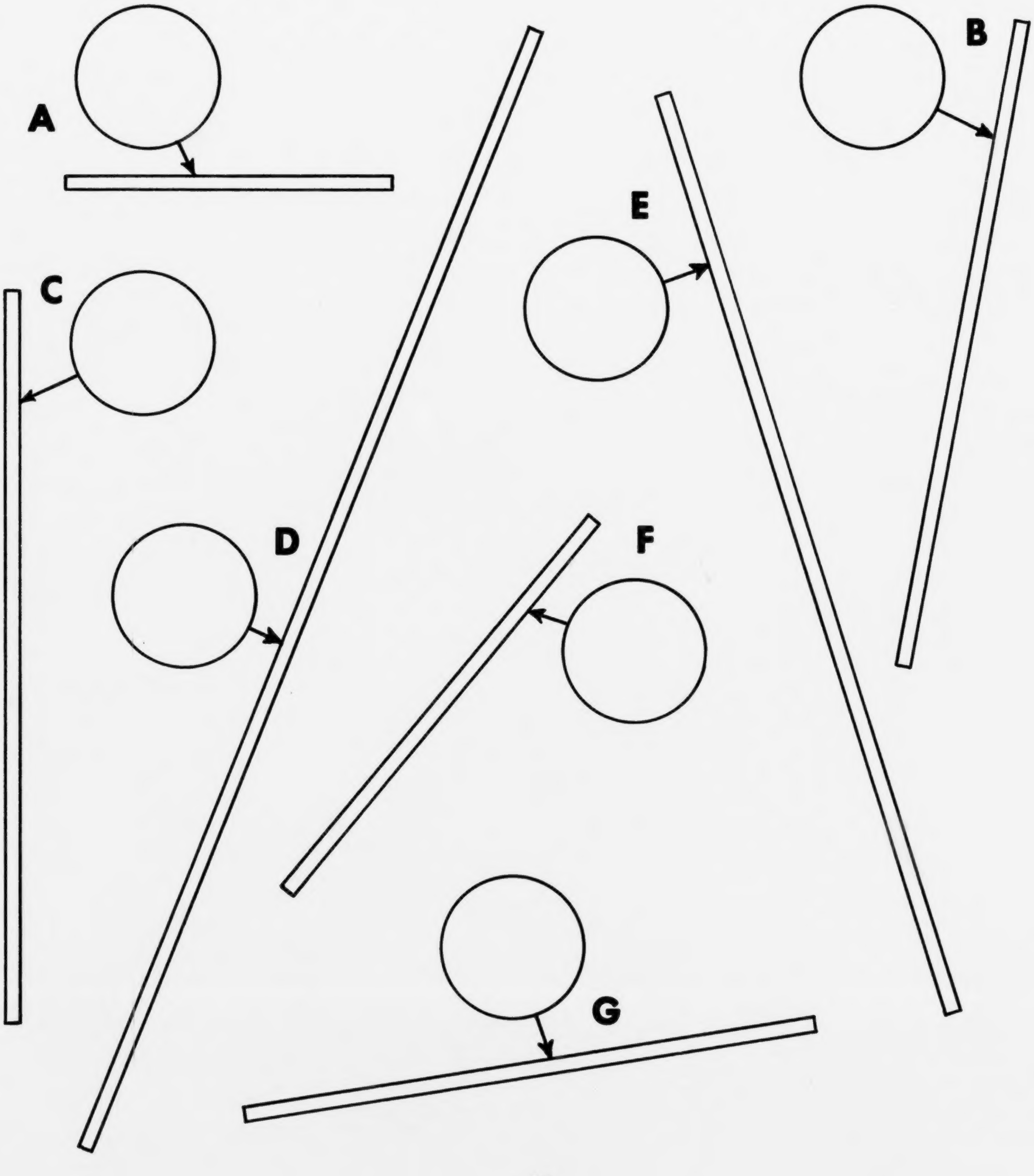
Name ______________________________

Each straw in one envelope has an image on this sheet if the object-image ratio is 1:2. Write the color of the straw or the numeral written on it in the circular region nearest the image.



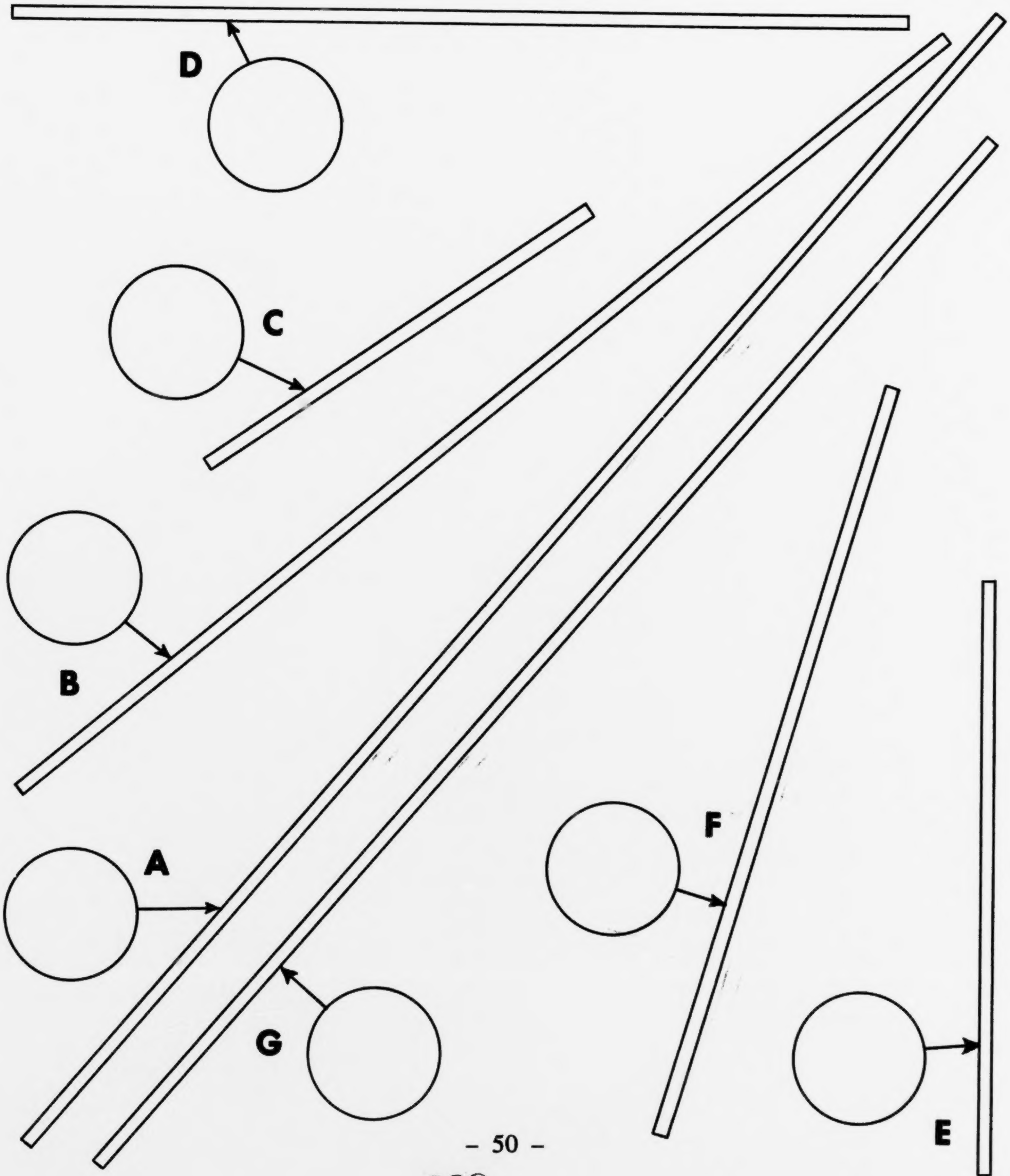
Station B Name ______________________________

Each straw in one envelope has an image on this sheet if the object-image ratio is 1:3. Write the color of the straw or the numeral written on it in the circular region nearest the image.



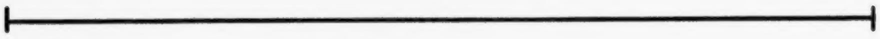
Station C Name ______________________________

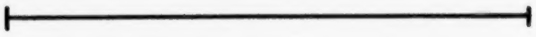
Each straw in one envelope has an image on this sheet if the object-image ratio is 1:4. Write the color of the straw or the numeral written on it in the circular region nearest the image.

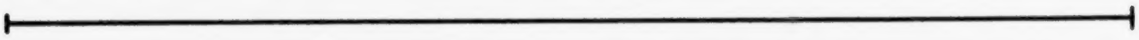


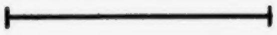
### *Teacher Materials*

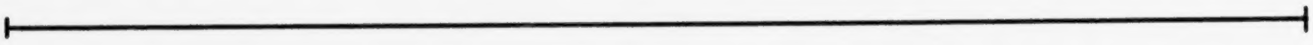
If you do not have access to equipment for duplicating this as a transparency, you can make a duplicate by tracing this sheet. Accuracy is important here so that students are not confused by a segment of length 15.1 cm when the intended length is 15 cm.





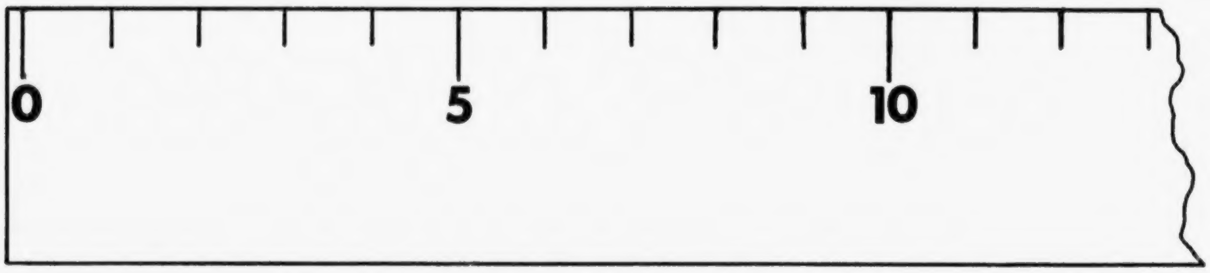


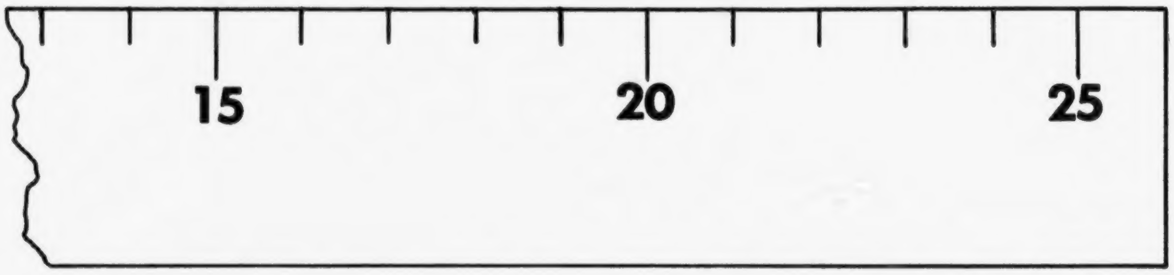


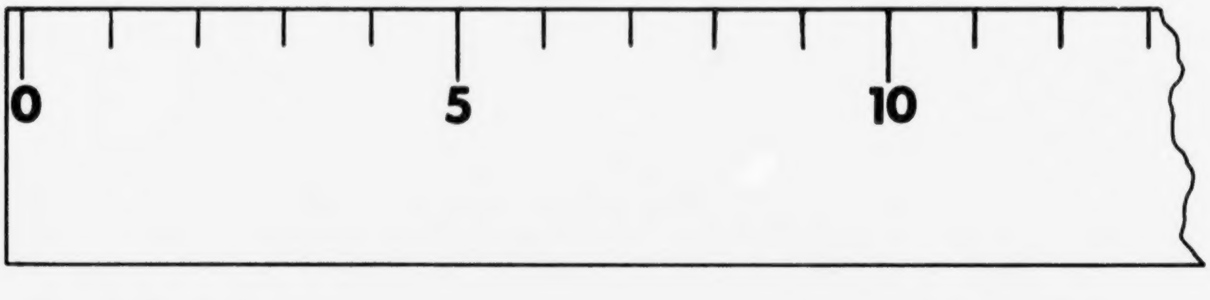


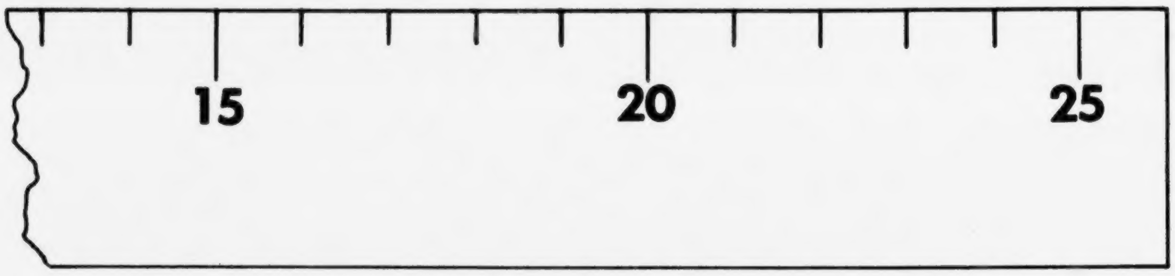
### *Teacher Materials*

Centimeter Rulers





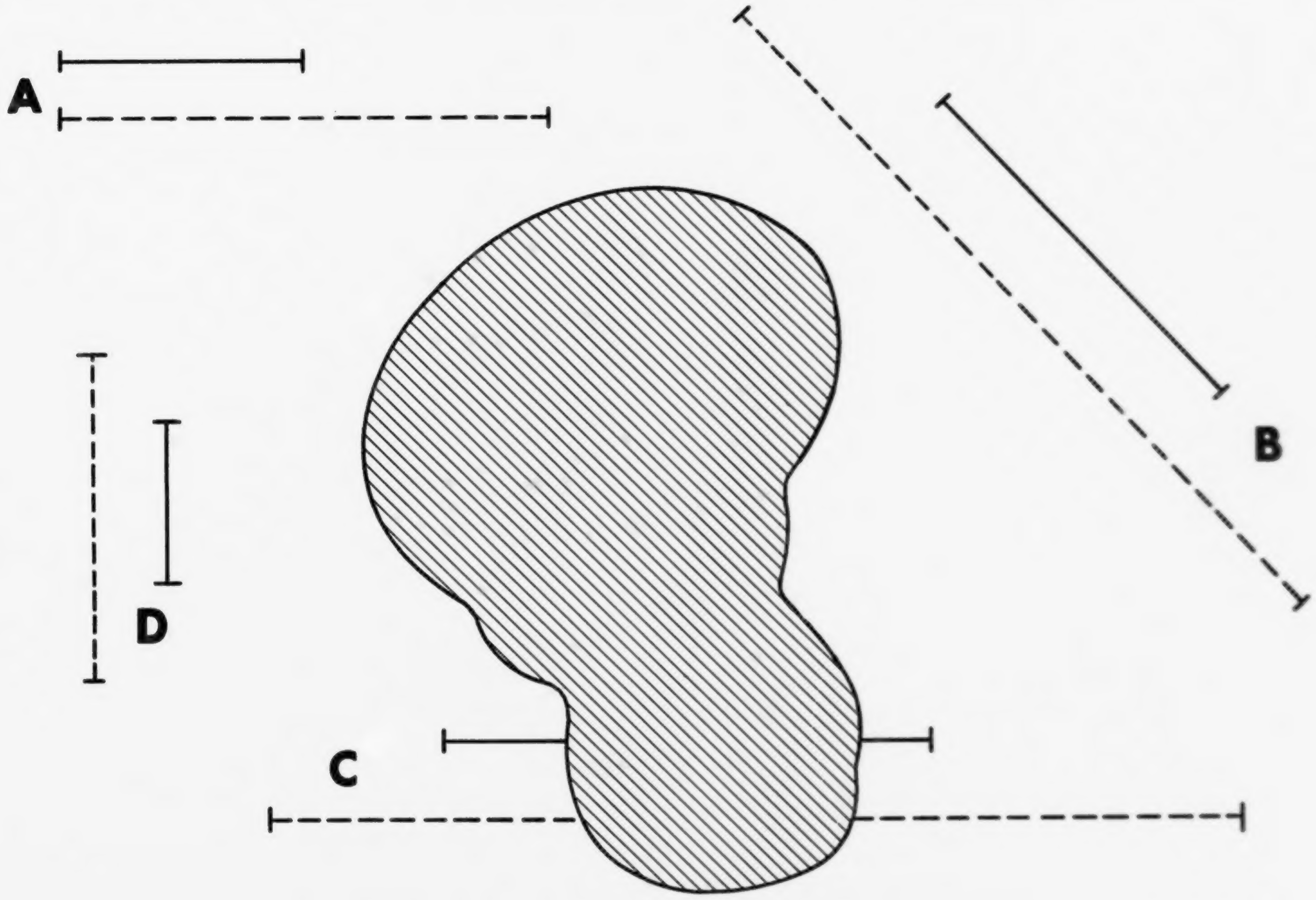




Activity 1 Name ______________________

Measure the lengths of each pair of segments, in centimeters, and record the lengths in the table. Determine the object-image ratio used in all the pairs and write it in the table.

Draw the missing parts of all segments which are covered by the blob.

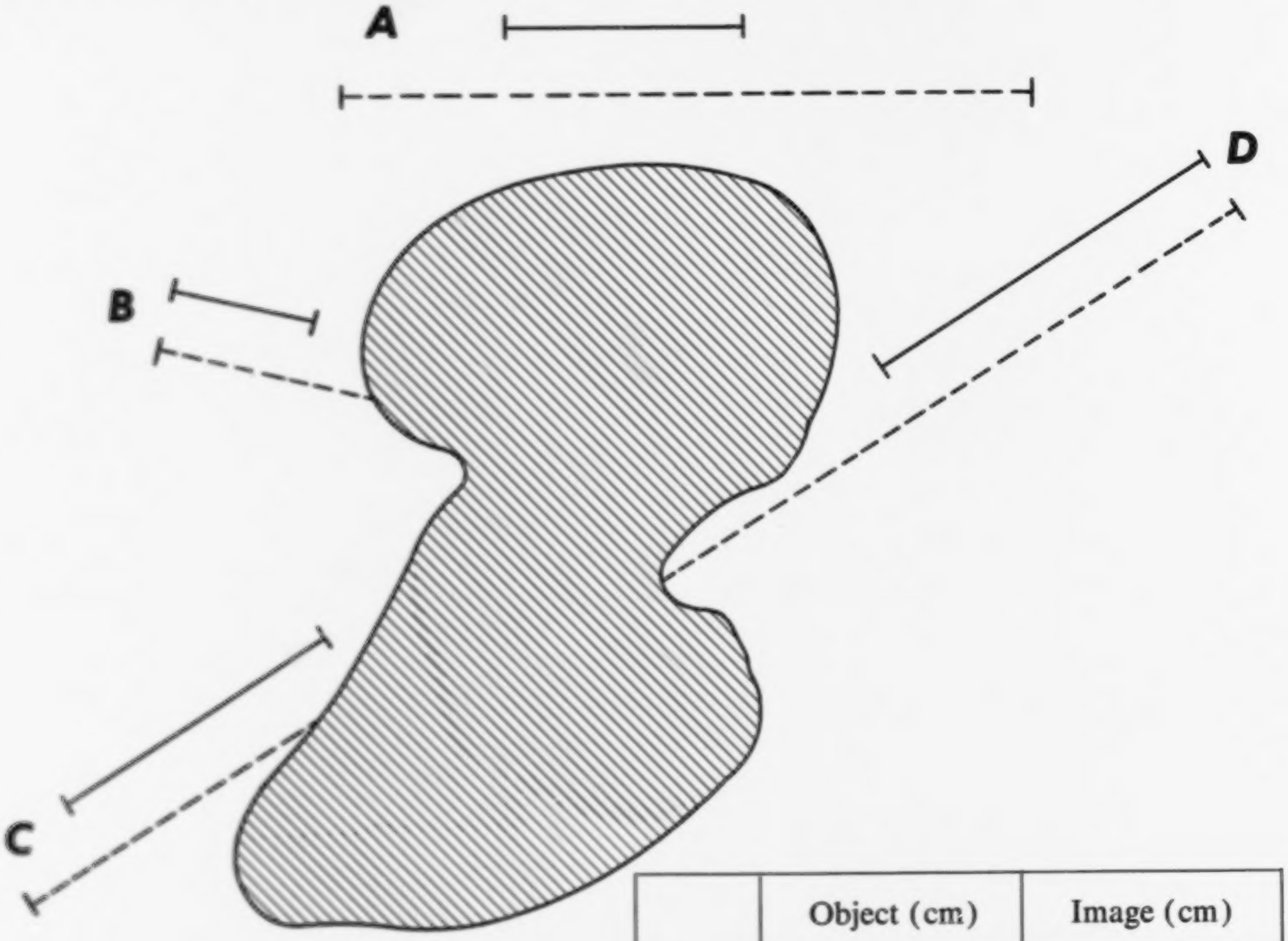


| | Object (cm) | Image (cm) |
|---|---|---|
| *A* | | |
| *B* | | |
| *C* | | |
| *D* | | |
| Ratio | : | |

Activity 2 Name ______________________

Measure the lengths of each pair of segments, in centimeters, and record the lengths in the table. Determine the object-image ratio used in all the pairs and write it in the table.

Draw the missing parts of all segments which are covered by the blob.

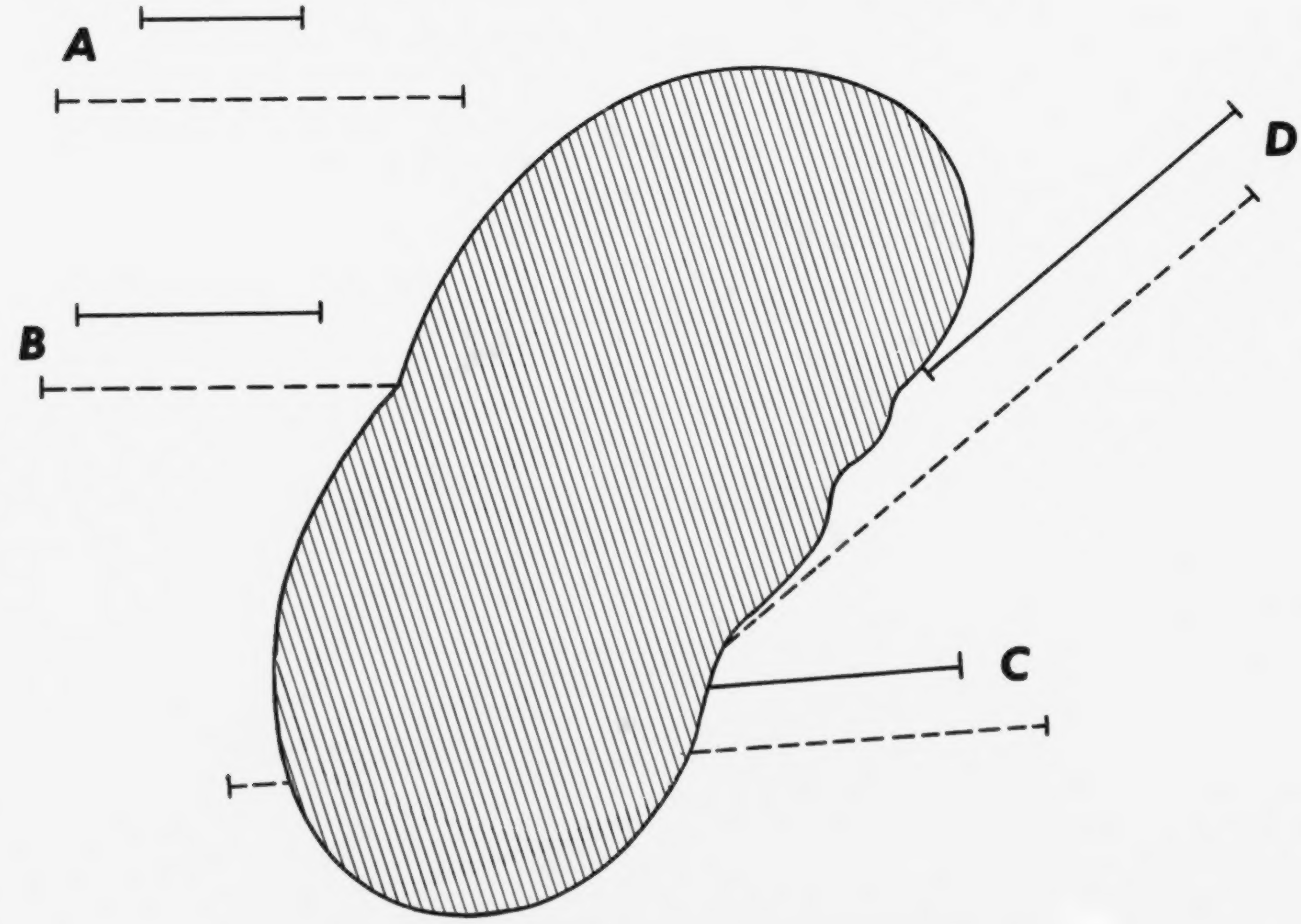


| | Object (cm) | Image (cm) |
|---|---|---|
| *A* | | |
| *B* | | |
| *C* | | |
| *D* | | |
| Ratio | : | |

Activity 3 Name ______________________________

Measure the lengths of each pair of segments, in centimeters, and record the lengths in the table. Determine the object-image ratio used in all the pairs and write it in the table.

Draw the missing parts of all segments which are covered by the blob.

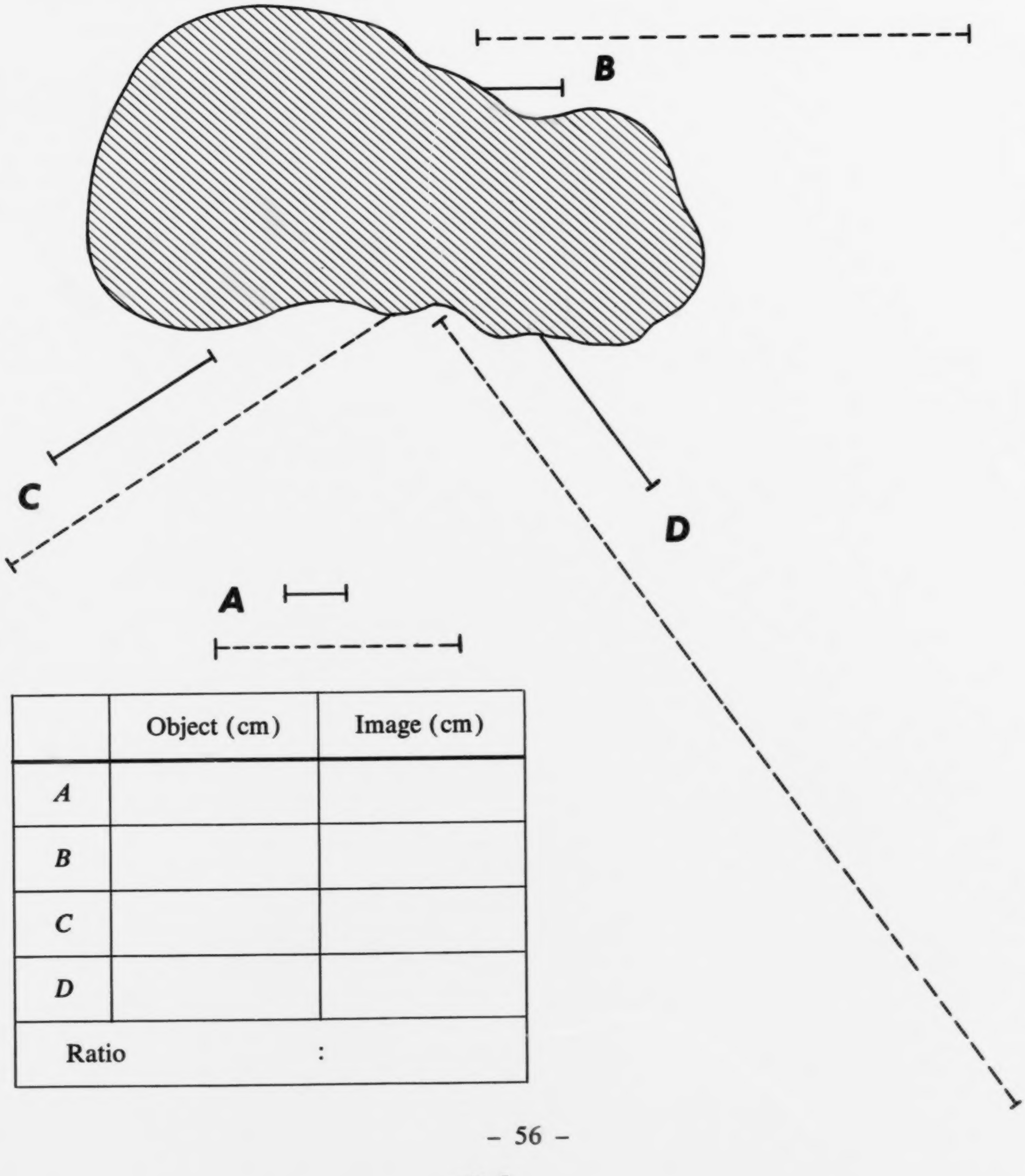


| | Object (cm) | Image (cm) |
|---|---|---|
| *A* | | |
| *B* | | |
| *C* | | |
| *D* | | |
| Ratio | : | |

Activity 4 Name ______________________________

Measure the lengths of each pair of segments, in centimeters, and record the measurements in the table. Determine the object-image ratio used in all the pairs and write it in the table.

Draw the missing parts of all segments which are covered by the blob.



| | Object (cm) | Image (cm) |
|---|---|---|
| *A* | | |
| *B* | | |
| *C* | | |
| *D* | | |
| Ratio | : | |

## FIND THE SCALE

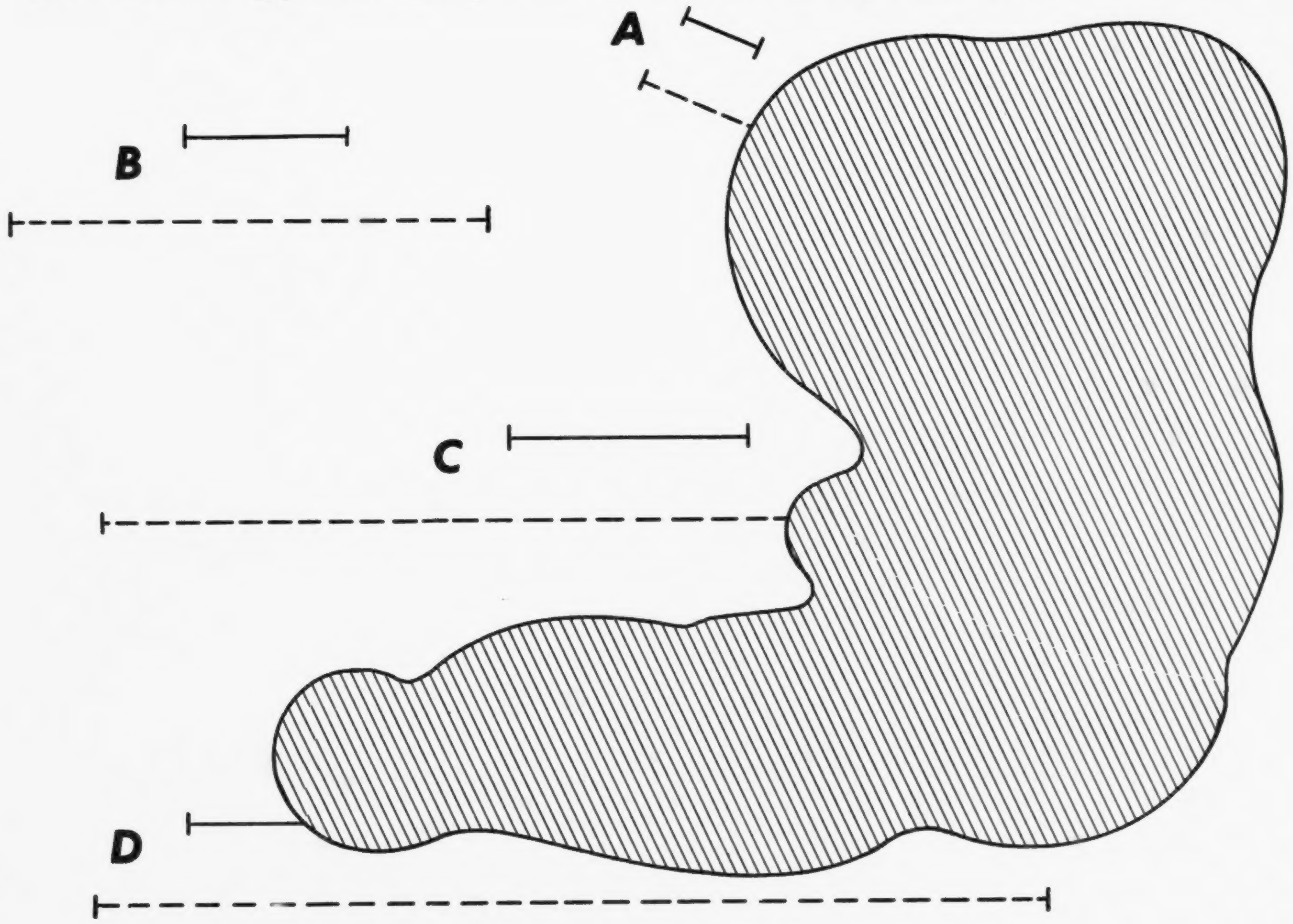
## UNIT 9, EXPERIENCE 3

Activity 5

Name ______________________________

Measure the lengths of each pair of segments, in centimeters, and record the measurements in the table. Determine the object-image ratio used in all the pairs and write it in the table.

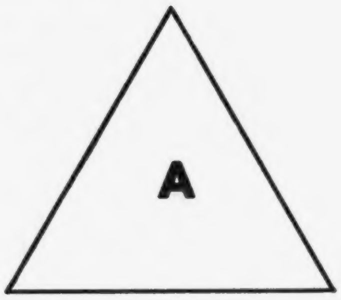
Draw the missing parts of all segments which are covered by the blob.

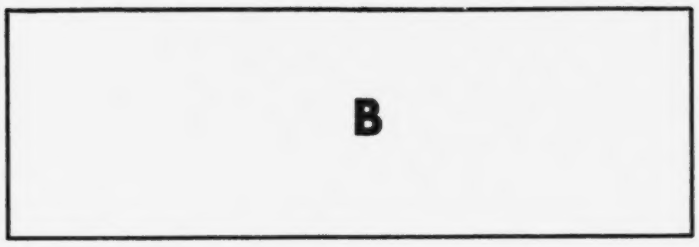


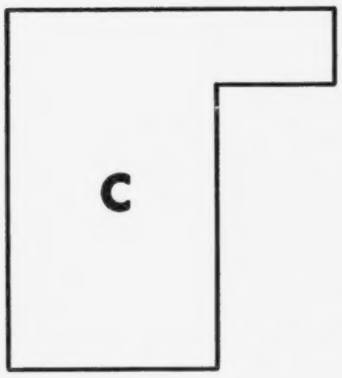
| | Object (cm) | Image (cm) |
|---|---|---|
| *A* | | |
| *B* | | |
| *C* | | |
| *D* | | |
| Ratio | : | |

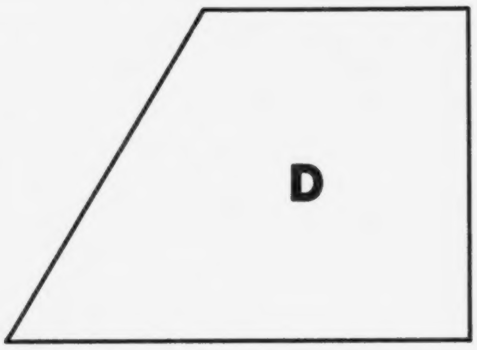
**Teacher Materials**

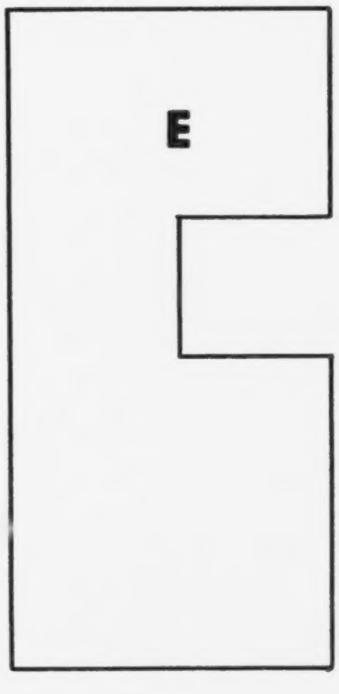
Guides for Large Images

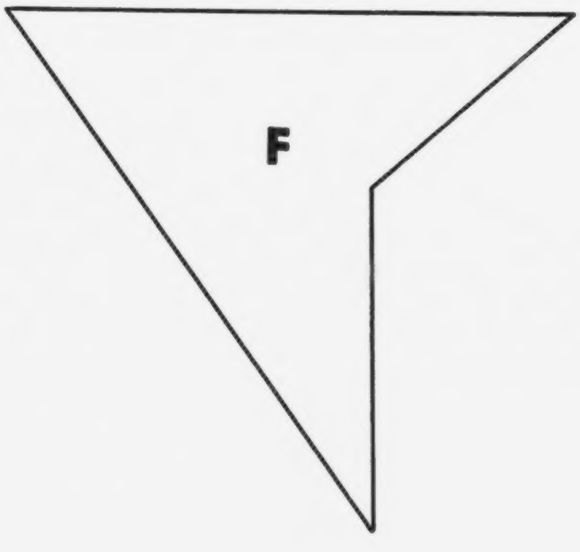






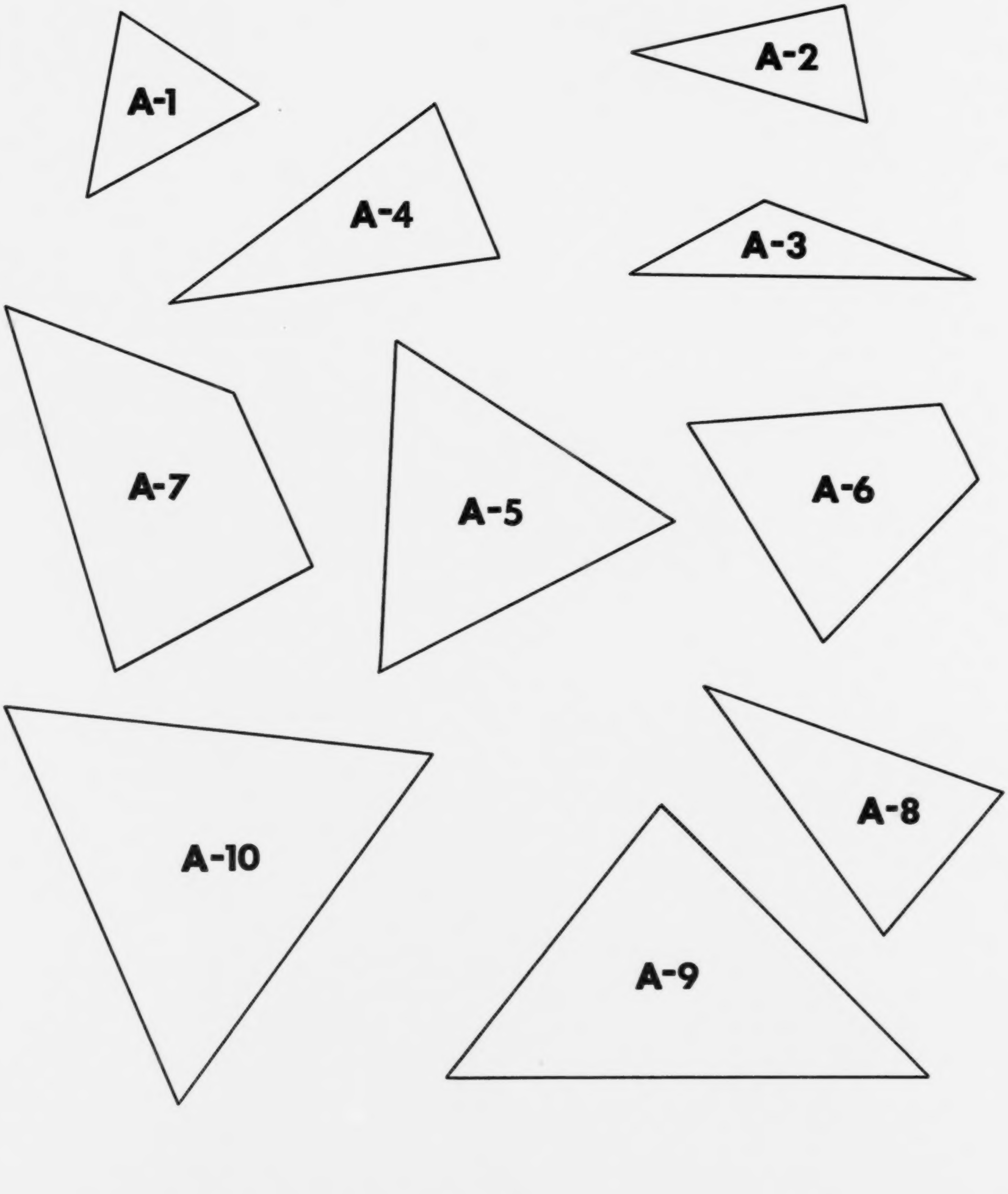






**Teacher Materials**

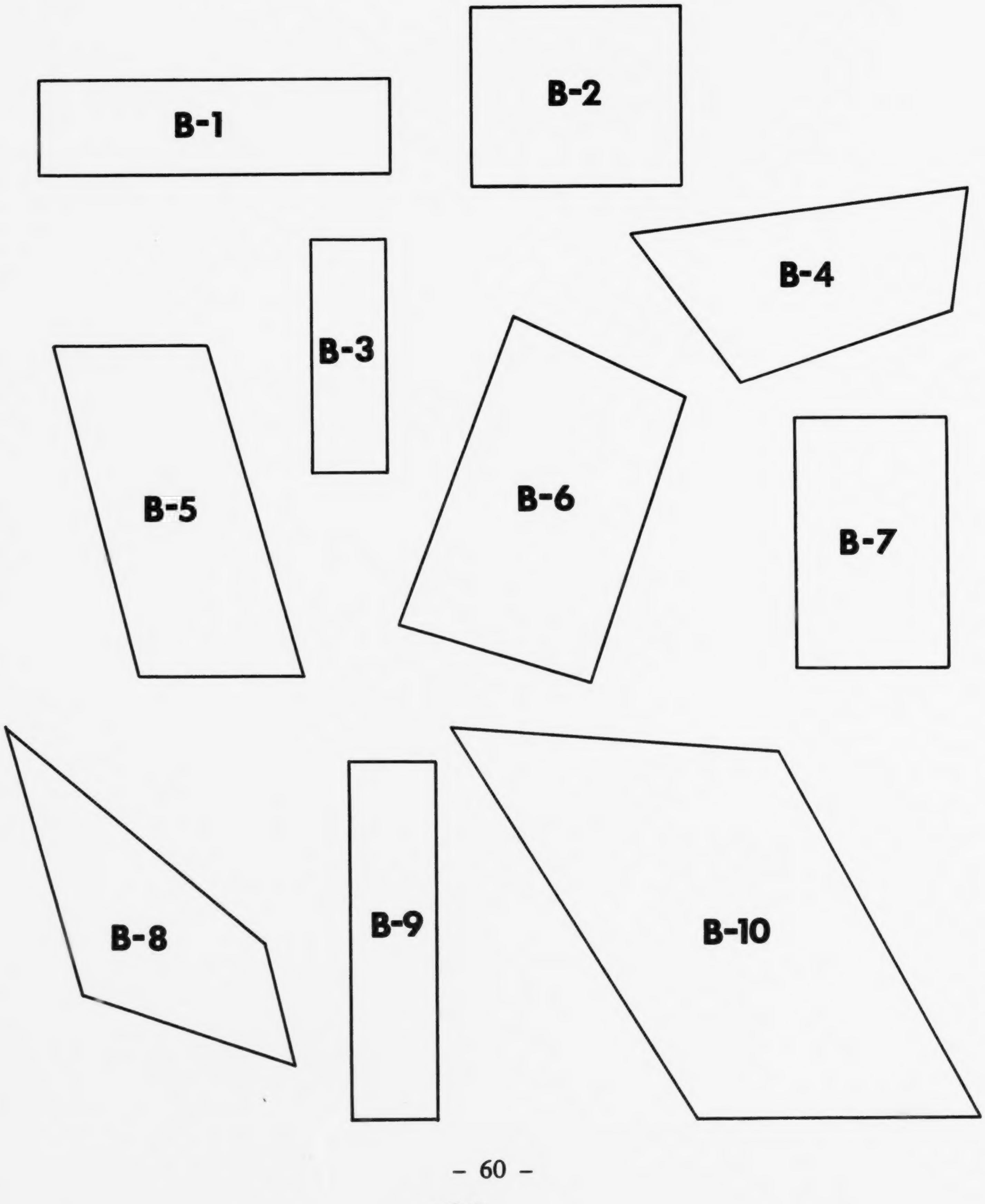
Guides for Objects


A-1
A-2
A-4
A-3
A-7
A-5
A-6
A-10
A-8
A-9

]

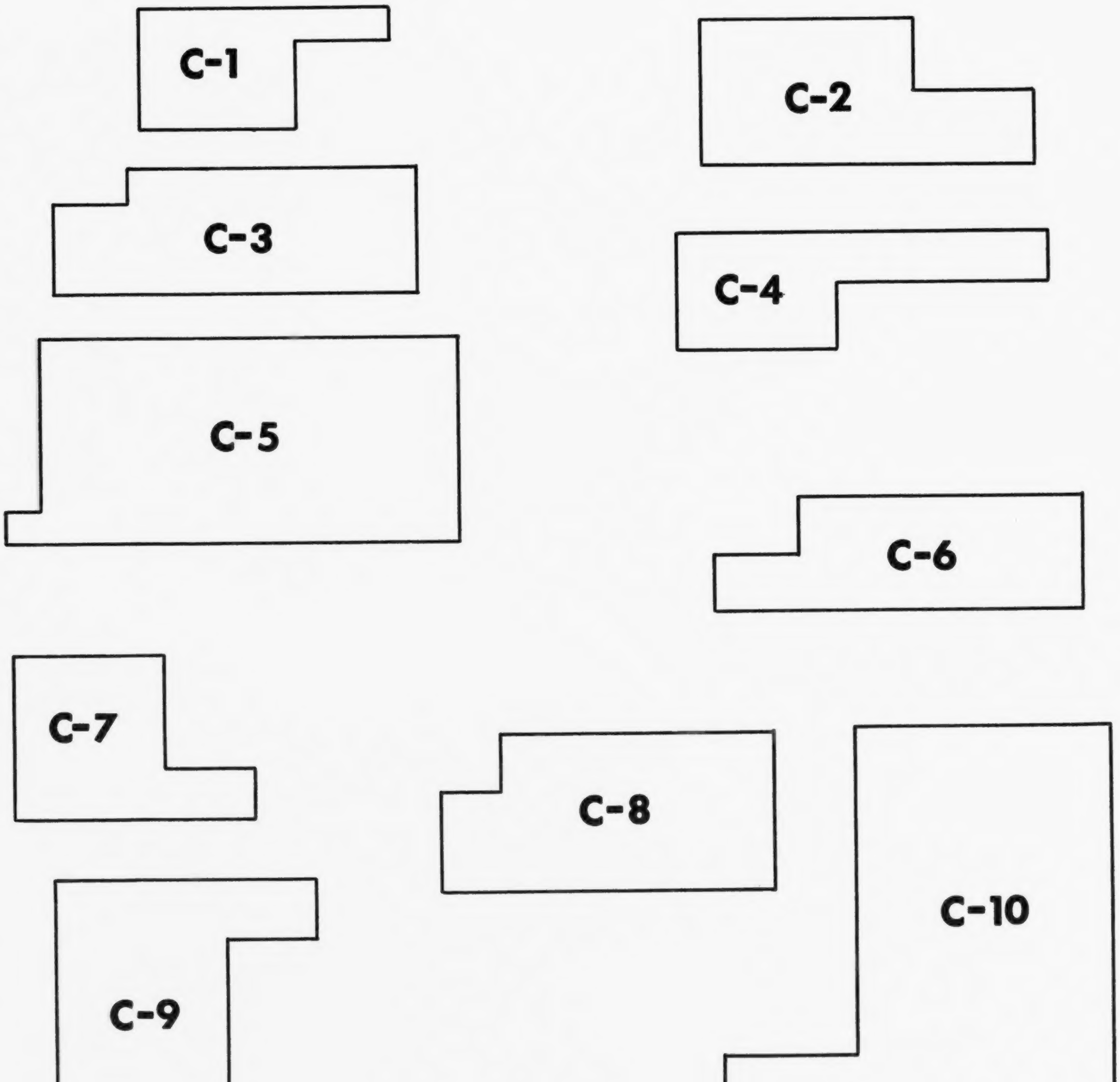
### *Teacher Materials*

Guides for Objects


B-1
B-2
B-3
B-4
B-5
B-6
B-7
B-8
B-9
B-10

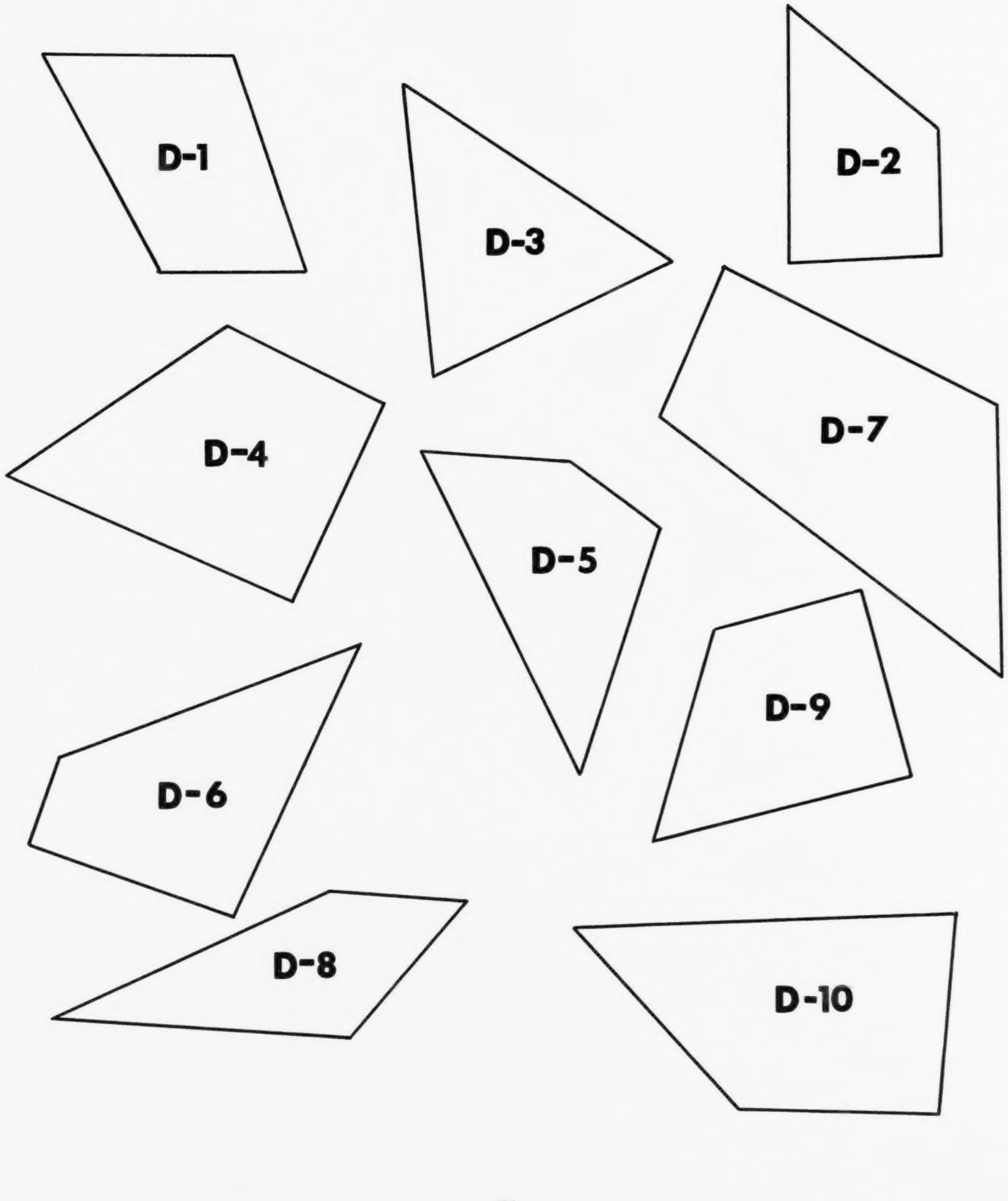
**Teacher Materials**

Guides for Objects


C-1
C-2
C-3
C-4
C-5
C-6
C-7
C-8
C-10
C-9

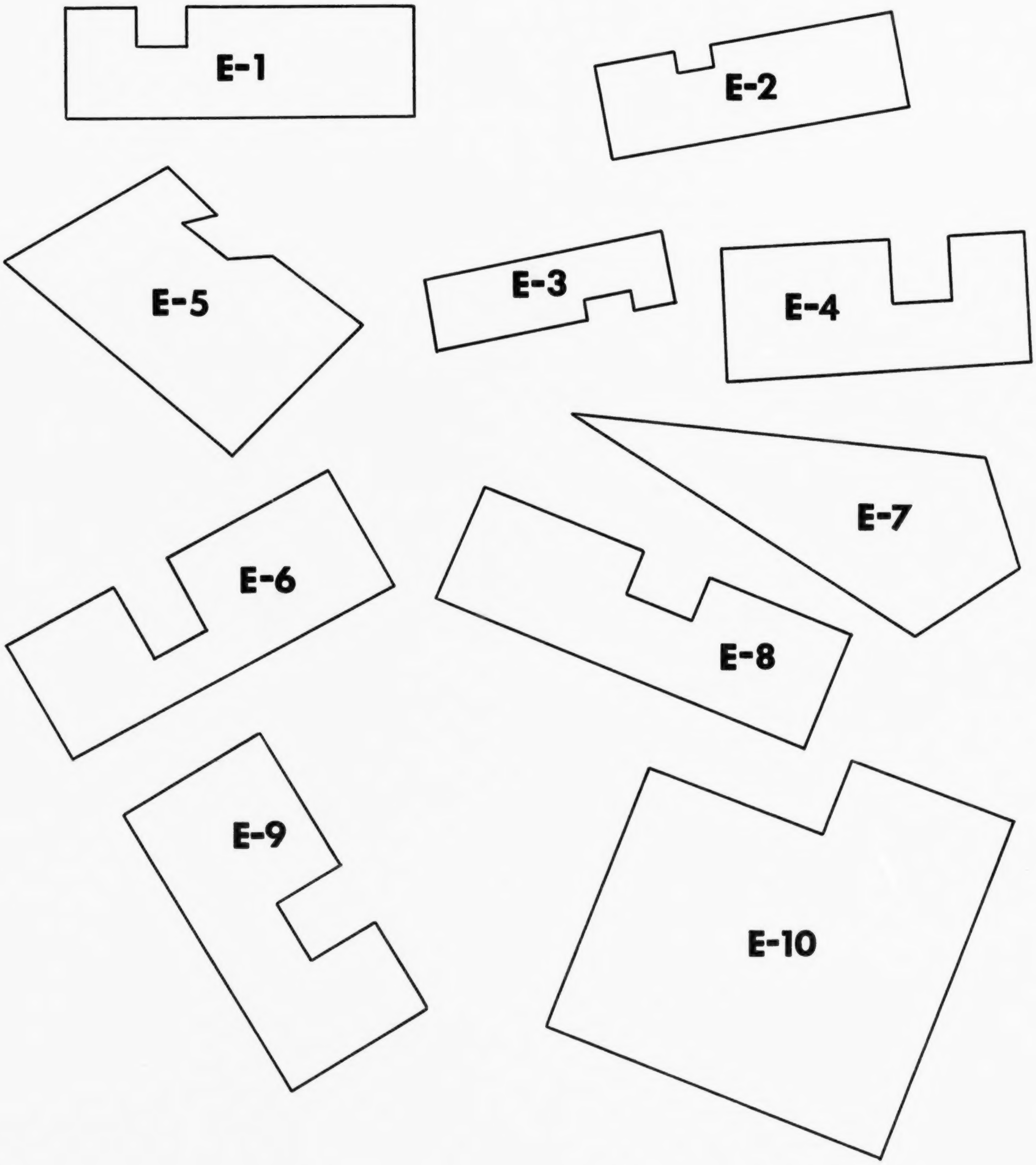
**Teacher Materials**

Guides for Objects


D-1
D-2
D-3
D-4
D-5
D-6
D-7
D-8
D-9
D-10

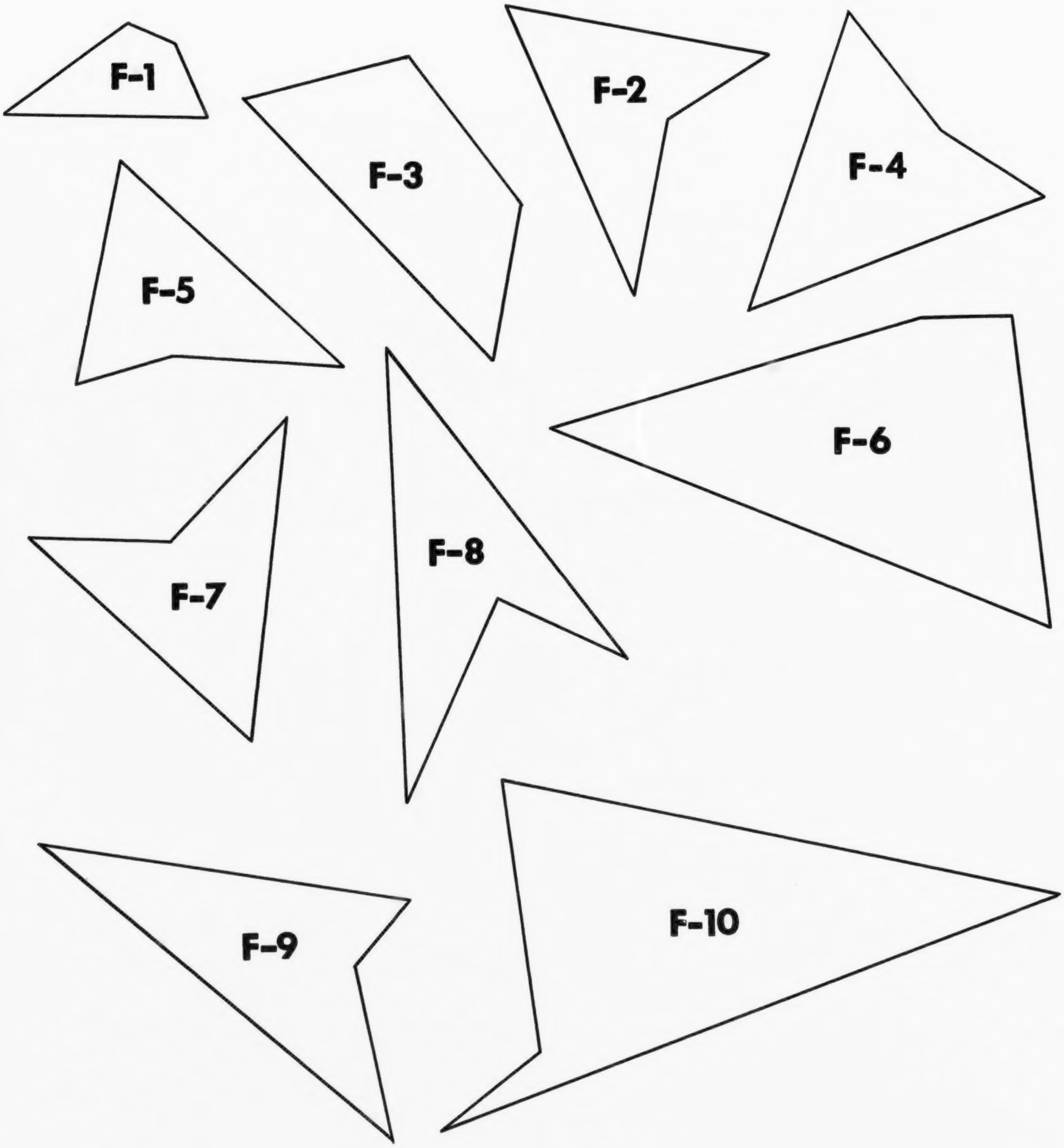
### *Teacher Materials*

Guides for Objects


E-1
E-2
E-5
E-3
E-4
E-7
E-6
E-8
E-9
E-10

***Teacher Materials***

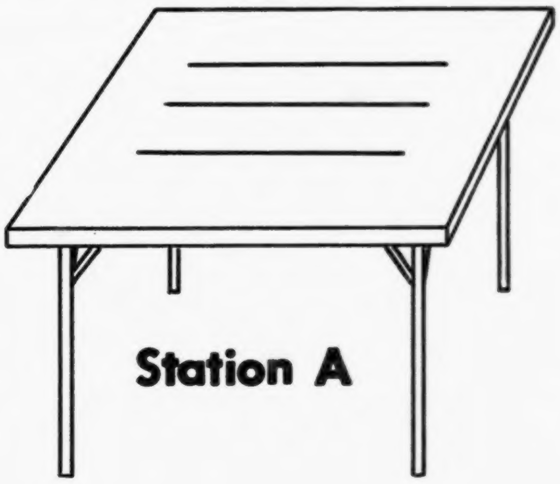
Guides for Objects

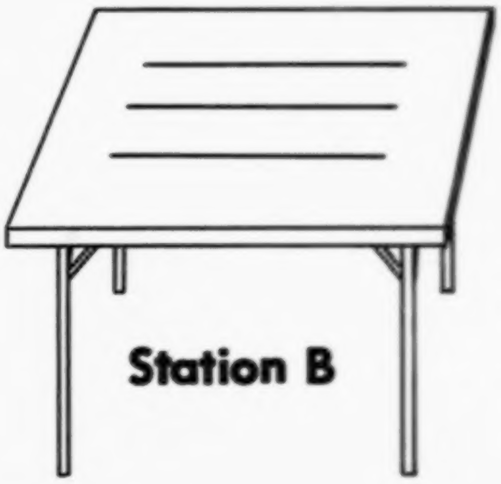

F-1
F-2
F-3
F-4
F-5
F-6
F-7
F-8
F-9
F-10

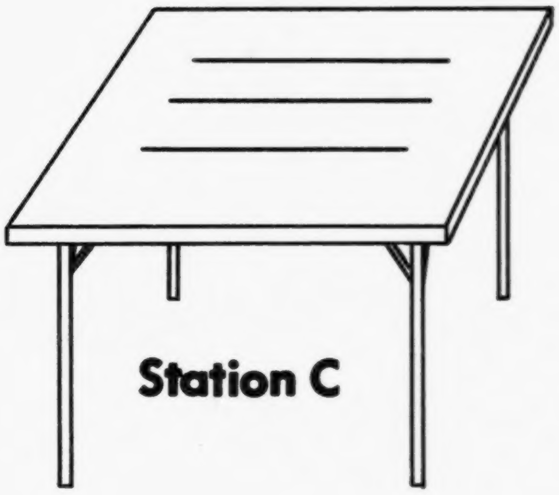
RECOGNIZE LIKE SHAPES UNIT 9, EXPERIENCE 4

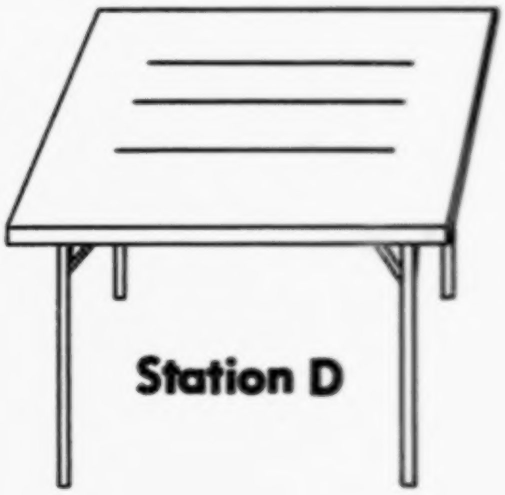
Name ______________________________

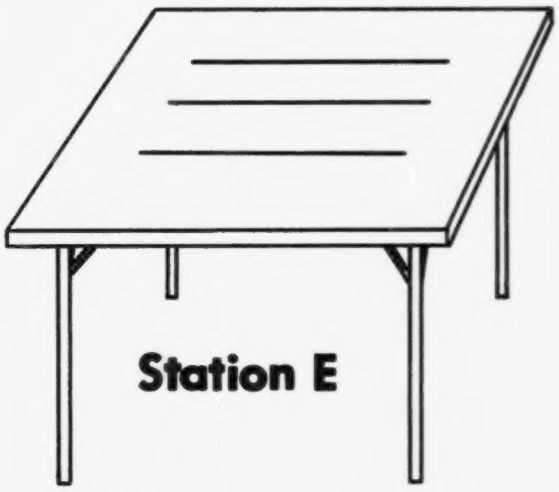
List the number or numbers of objects at each station that have the same shape as the large image.

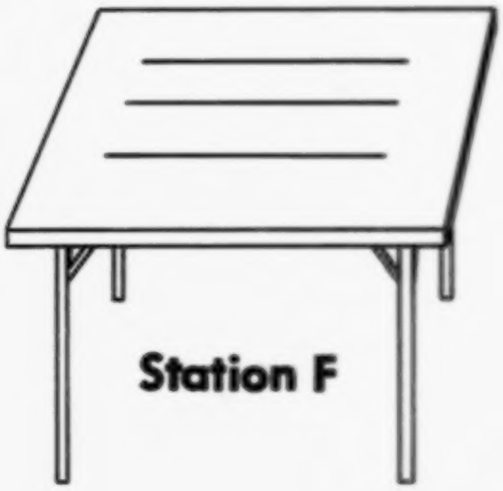






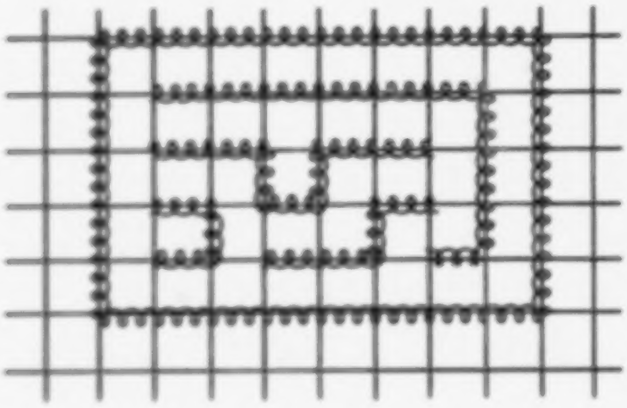






Activity 1 Name ______________________________

Using graph paper, make an image of this object which has line segments 3 times as long.

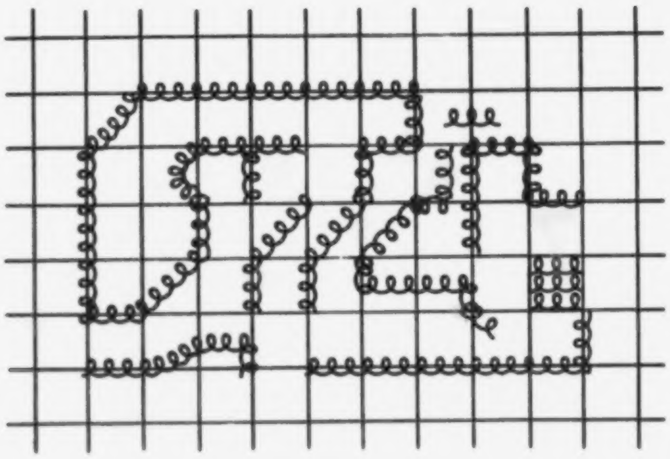


Place your image in the space below.

ENLARGE A PRINTED CIRCUIT UNIT 9, EXPERIENCE 5

Activity 2 Name ______________________________

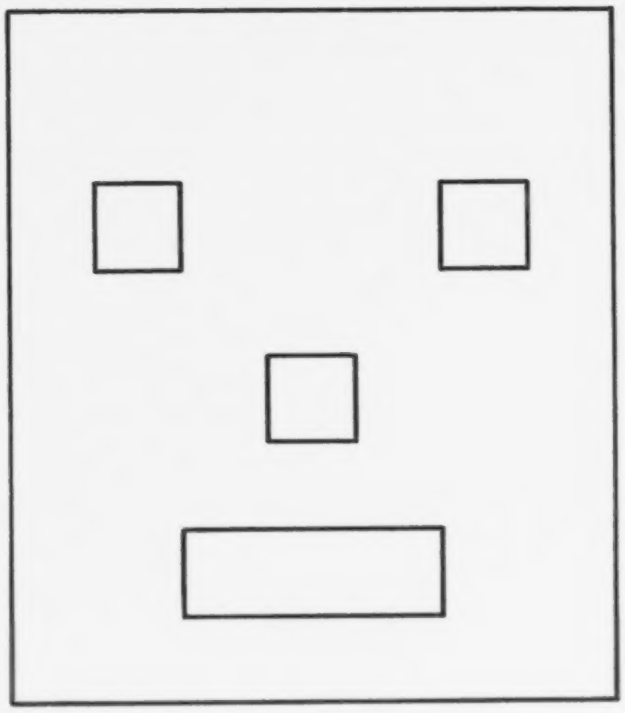
Using graph paper, make an image of this object in which line segments and curves are 2 times as long as they are here.



Place the image in the space below.

Name ________________________________

Construct the figure below on graph paper, using the ratio 1 centimeter: $\frac{1}{4}$ inch.



Place your answer in the space below.

# MAKING STRAIGHT-LINE GRAPHS OF PHYSICAL DATA

UNIT 10, EXPERIENCE 1

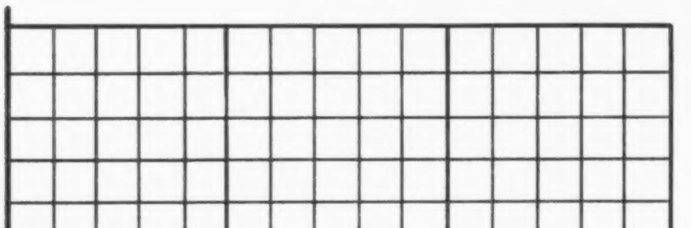
Station A

Name ______________________________

At this station you will find a coffee measure, a box of rice, a tall jar, and a ruler.

1. Pour 3 level measures of rice in the jar.
2. Measure the height of the rice in the jar to the nearest $\frac{1}{8}$ inch.
3. Enter the height in the table opposite the "3."
4. Repeat the above three steps for 5 measures, 7 measures, and 9 measures.
5. Label the axes of your graph, choose and mark the scale used on each axis, and plot the four points for the number pairs in the table.
6. Connect the points, using a straightedge. If they do not lie on a straight line, repeat the steps and check the measurements.
7. If you have not already visited four stations, go on to Station B.

| Measures | Inches |
|---|---|
| 3 | |
| 5 | |
| 7 | |
| 9 | |



## MAKING STRAIGHT-LINE GRAPHS OF PHYSICAL DATA

## UNIT 10, EXPERIENCE 1

Station B

Name ______________________________

At this station you will find a can with a mark on it and a line marked on the floor.

1. Place the can so the mark on it is directly on the mark on the floor.
2. Roll the can 1 roll (1 revolution) on the floor until the mark on the can touches the floor. Mark the floor at this spot.
3. Measure the distance to the nearest $\frac{1}{8}$ inch between the two marks on the floor.
4. Enter the distance in the table opposite the "1."
5. Repeat the above steps for 2 rolls, 3 rolls, and 4 rolls.
6. Label the axes of your graph, indicate the scales, and plot the points on the graph below. Use a straightedge to connect the points.
7. If you have not already visited four stations, go on to Station C.

| Rolls | Inches |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |

# MAKING STRAIGHT-LINE GRAPHS OF PHYSICAL DATA

## UNIT 10, EXPERIENCE 1

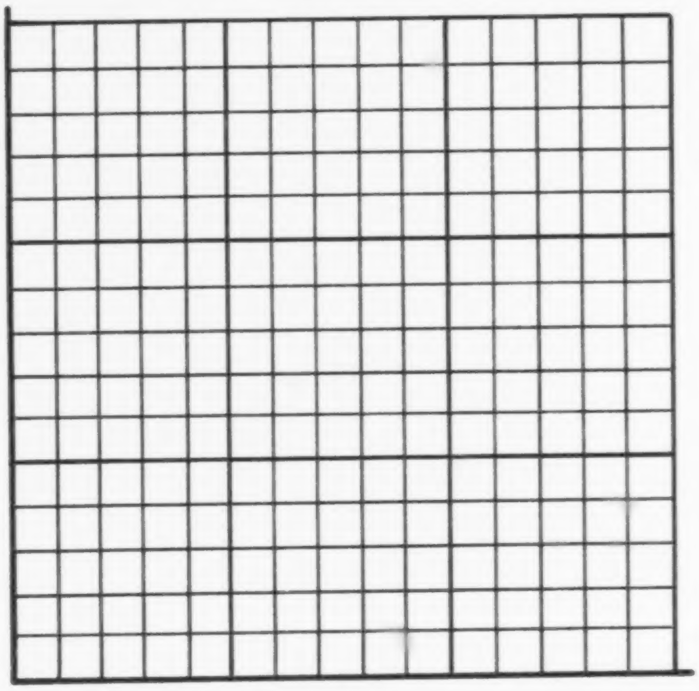
Station C

Name ______________________________

At this station you will find a bicycle placed upside down on its seat and handlebars.

1. Put the pedals in straight up-and-down position and mark the rear tire at the point directly opposite the bike's rear fork. Move the pedals slowly for safety.
2. Turn the pedals slowly 1 complete turn and count how many turns are made by the rear wheel. The mark on the rear wheel will help in counting, and fractions of a turn of the rear wheel can be counted by counting the spokes. For example, if there are 24 spokes, each one is $\frac{1}{24}$ of a turn.
3. Enter the number of turns of the wheel in the table opposite the "1."
4. Repeat the steps for 3 turns, 5 turns, and 7 turns of the pedals.
5. Label the axes of your graph, indicate the scales, and plot the points. Connect them with a straight-line segment.
6. If you have not already visited four stations, go on to Station D.

| Pedals | Wheel |
|---|---|
| 1 | |
| 3 | |
| 5 | |
| 7 | |



## MAKING STRAIGHT-LINE GRAPHS OF PHYSICAL DATA

## UNIT 10, EXPERIENCE 1

Station D

Name ____________________

At this station you will find a balance, a box of 20 pennies, and a supply of paper clips.

1. After balancing the stick place 5 pennies in one cup and balance by putting paper clips in the other cup. Count how many paper clips are used.
2. Enter the number of paper clips in the table opposite the "5."
3. Repeat the steps for 10 pennies, 15 pennies, and 20 pennies.
4. Label the axes of your graph, indicate the scale, plot the points, and connect them with a straight-line segment.
5. If you have not already visited four stations, go on to Station A.

| Pennies | Clips |
|---|---|
| 5 | |
| 10 | |
| 15 | |
| 20 | |

## MAKING CURVED-LINE GRAPHS OF PHYSICAL DATA

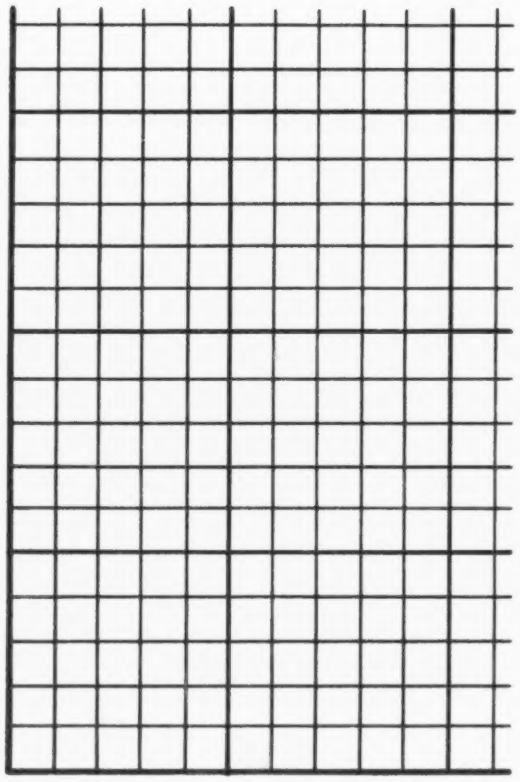
UNIT 10, EXPERIENCE 2

Station A

Name ______________________________

At this station you will find some sheets of newspaper and a pair of scissors. Select one of your group to carry out the steps of the experiment, and all record the data on your sheets.

1. Take a sheet of newspaper and cut it in half. Count the number of times it was cut (1) and the number of pieces there are (2).
2. Record the number pair in the table below.
3. On your graph label the horizontal axis "Number of Cuts" and the vertical axis "Number of Pieces."
4. Mark on the graph the point that represents 1 cut and 2 pieces.
5. Holding the 2 pieces together, cut through both pieces. You have now made 2 cuts and have 4 pieces. Record the number pair in the table and mark the point on the graph.
6. Repeat for 3 cuts through the 4 pieces from Step 5.
7. Repeat for 4 cuts through the pieces from Step 6.
8. Connect the dots with a smooth curve.
9. If you have not already visited four stations, go to Station B.



| Cuts | Pieces |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |

# MAKING CURVED-LINE GRAPHS OF PHYSICAL DATA

UNIT 10, EXPERIENCE 2

Station B

Name ______________________________

At this station you will find a pendulum device and a stopwatch. Select one of your group to carry out the steps of the experiment, and all record the data on your sheets.

1. Hold or tie the pendulum so it is exactly 1 foot long from the top to the center of the weight.
2. Swing the bob for 10 complete swings and count the time to the nearest second. Remember that back and forth counts as 1 complete swing.
3. Enter the time in the table below opposite the "1."
4. On your graph label the horizontal axis "Length in Feet" and the vertical axis "Time in Seconds."
5. Place a dot on the graph which represents your result for the 1-foot length.
6. Now make the pendulum 2 feet long and again time 10 swings. Record and graph your result.
7. Repeat for a pendulum 3 feet long and one that is 4 feet long.
8. Connect the dots with a smooth curve.
9. If you have not already visited four stations, go on to Station C.

| Feet | Seconds |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |

# MAKING CURVED-LINE GRAPHS OF PHYSICAL DATA

UNIT 10, EXPERIENCE 2

Station C

Name ______________________

At this station you will find four squares of paper and a ruler. Select one of your group to carry out the steps of the experiment, and all record the data on your sheets.

1. Measure the length of the diagonal of the smallest square. Compute the area of this square from this measurement, using the formula
$$A = \frac{1}{2}d^2.$$
2. Enter the length of the diagonal and the corresponding area of the square in the table.
3. On your graph label the horizontal axis "Length of Diagonal in Inches" and the vertical axis "Area in Square Inches."
4. Place the point on the graph which shows the result for Step 1.
5. Repeat Steps 1, 2, and 4 for the other three squares.
6. Connect the dots with a smooth curve.
7. If you have not already visited four stations, go on to Station D.

| Diagonal in Inches | Square Inches |
|---|---|
| | |
| | |
| | |
| | |

MAKING CURVED-LINE GRAPHS OF PHYSICAL DATA

UNIT 10, EXPERIENCE 2

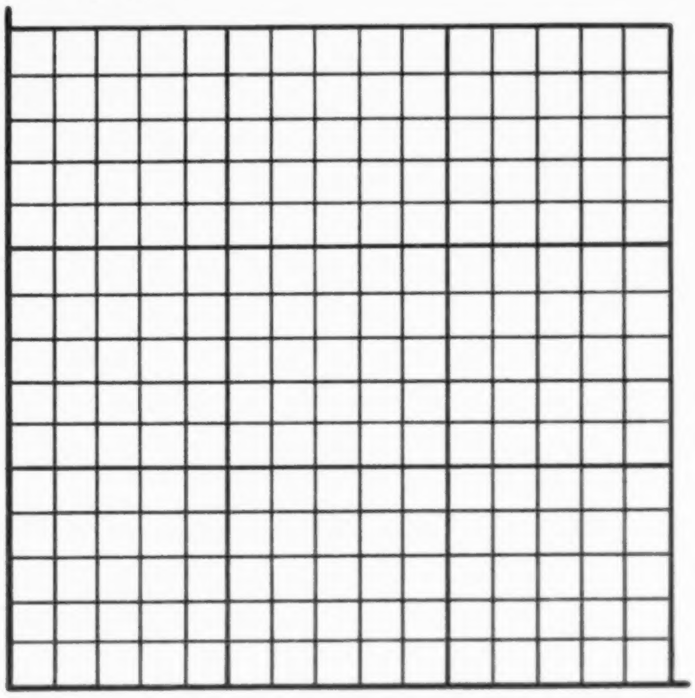
Station D

Name ______________________________

At this station you will find a balancing device and a supply of 40 pennies. Select one of your group to carry out the steps of the experiment, and all record the data on your sheets.

1. Place 10 pennies in one cup and 15 pennies in the other cup. Adjust the position of the stick on the fulcrum until the device is in balance again.
2. Measure the distance from the fulcrum to the heavier cup.
3. Enter the distance in the table below, opposite "15."
4. On your graph label the horizontal axis "Number of Pennies" and the vertical axis "Distance in Inches."
5. Place a point on the graph which indicates the distance that 15 pennies must be from the fulcrum to balance 10 pennies.
6. Place 5 more pennies in the heavier cup (for a total of 20), and adjust until the device balances. Measure the distance from the fulcrum to the heavier cup. Enter this in the table.
7. Place a point on the graph which indicates the distance that 20 pennies must be from the fulcrum to balance 10.
8. Repeat with totals of 25 and 30 pennies in the heavier cup. Make the appropriate entries in the table and place the appropriate points on the graph.
9. Connect the dots with a smooth curve.
10. If you have not already visited four stations, go on to Station A.

| Pennies | Distance |
|---|---|
| 15 | |
| 20 | |
| 25 | |
| 30 | |



## USING GRAPHS TO DETERMINE ADDITIONAL RESULTS IN EXPERIMENTS

## UNIT 10, EXPERIENCE 3

Name ______________________________

Use your graphs from Experiences 1 and 2, as indicated, to answer the questions below.

### Experience 1, Station D

1. How many pennies would it take to balance 120 paper clips? _____
2. If 30 pennies were placed in one end, how many paper clips would it take to balance them? _____
3. How many paper clips does 1 penny weigh? _____
4. How many pennies does 1 paper clip weigh? _____

### Experience 2, Station B

5. Suppose the pendulum is 2 feet, 3 inches long. How long will 10 complete swings take? _____
6. How long will 10 swings take if the pendulum is $3\frac{1}{2}$ feet long? _____
7. What length pendulum will swing 10 times in 25 seconds? _____
8. What length pendulum would make a good clock and make 1 complete swing every second? _____

### Experience 2, Station C

9. What area does a square have if its diagonal is 4 inches long? _____
10. What is the area if the diagonal is $6\frac{1}{2}$ inches long? _____
11. What is the diagonal of a square whose area is 30 square inches? _____
12. What is the diagonal of a square whose area is 10 square inches? _____

### Experience 2, Station D

13. How many pieces of paper would there be if you made 6 cuts? _____
14. What if you made 10 cuts? If you want to have some fun, try doing this with a fresh sheet of newspaper. Don't be surprised if you have some trouble. _____
15. How many cuts do you think it would take to make 256 pieces? _____

CHOOSING THE GRAPH TO FIT
DATA FOR A PHYSICAL SITUATION

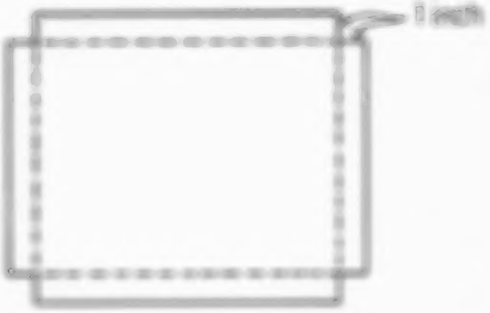
UNIT 10, EXPERIENCE 4

Station A

Name ______________________________

At this station you will find a supply of $8\frac{1}{2}$-by-11-inch paper, rulers, and scissors. You will also have four graphs to choose from.

1. Cut a 1-inch square from each corner of a piece of paper, like this:



2. Fold on the dotted lines so that a box is formed.
3. Compute the volume of the box by measuring length, height, and width and multiplying according to the formula below:

$$V = l \times w \times h.$$

4. Enter the volume in the table below, opposite the "1."
5. Repeat these four steps for $1\frac{1}{2}$-inch, 2-inch, and 3-inch squares.

| Side of Square | Volume |
|---|---|
| 1 | |
| $1\frac{1}{2}$ | |
| 2 | |
| 3 | |

Show your calculations here.

6. Pick the graph that best represents the various situations after removing squares of 1, $1\frac{1}{2}$, 2, and 3 inches.

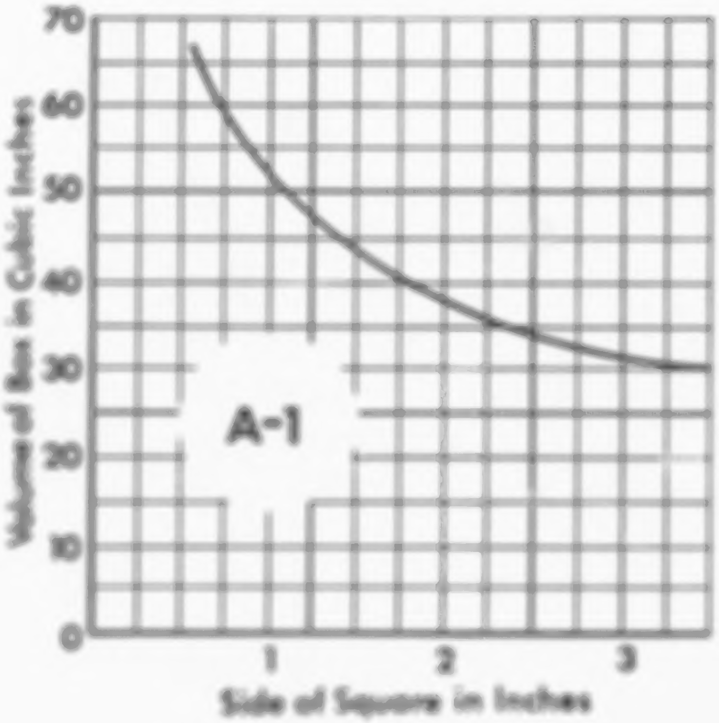
The graph that best represents the data is Graph ______.

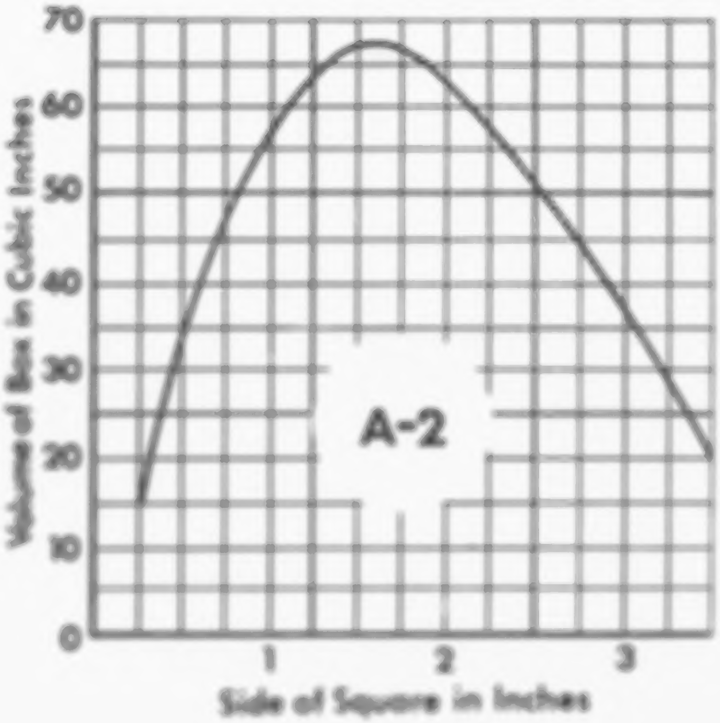
CHOOSING THE GRAPH TO FIT
DATA FOR A PHYSICAL SITUATION

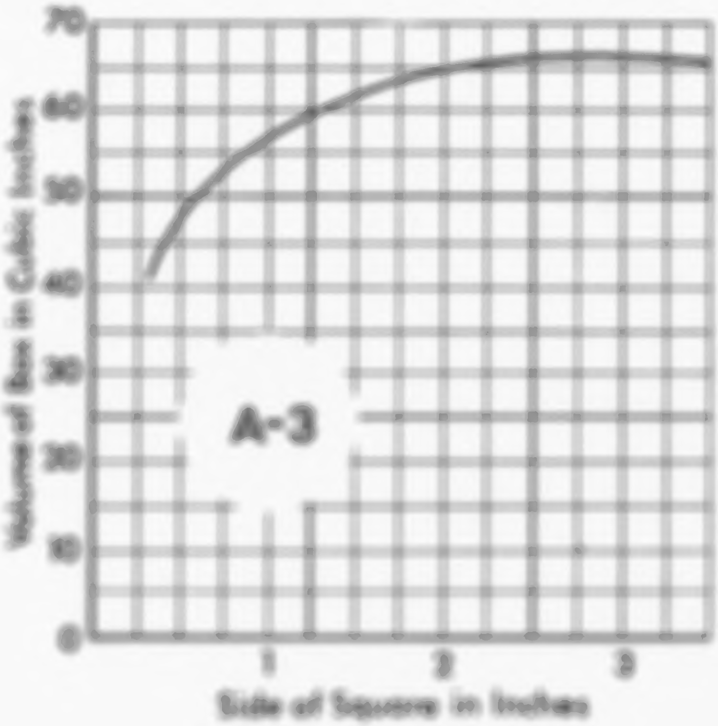
UNIT 10, EXPERIENCE 4

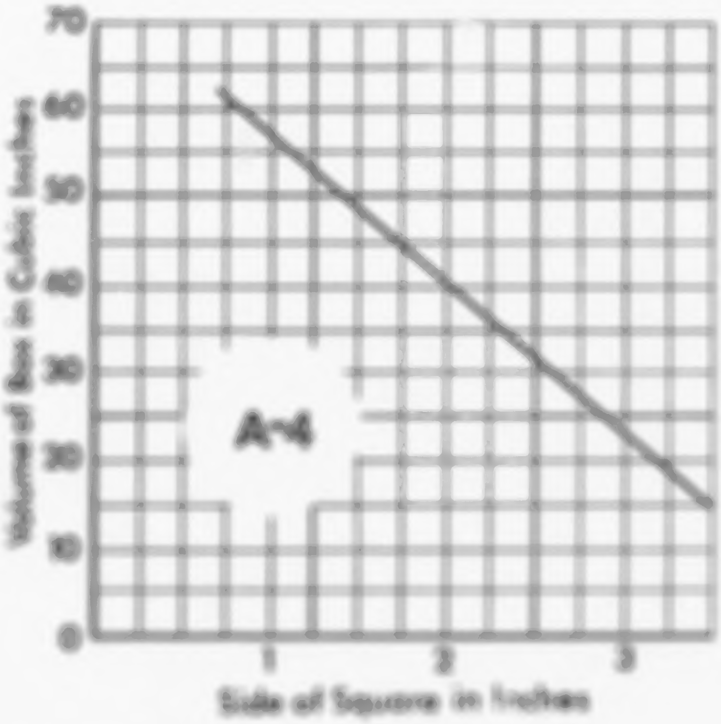
Graphs for Station A

Name ____________________________









CHOOSING THE GRAPH TO FIT
DATA FOR A PHYSICAL SITUATION

UNIT 10, EXPERIENCE 4

Station B

Name ____________________________________

1. Suppose you drive from Chicago to New York by a route 800 miles long. If your average speed is 30 miles per hour, how many hours will the trip take? ______

2. Supposing your average speed is 40 miles per hour, how long will the trip take in hours? ______

3. Repeat for 50 miles per hour. ______ For 60 miles per hour. ______

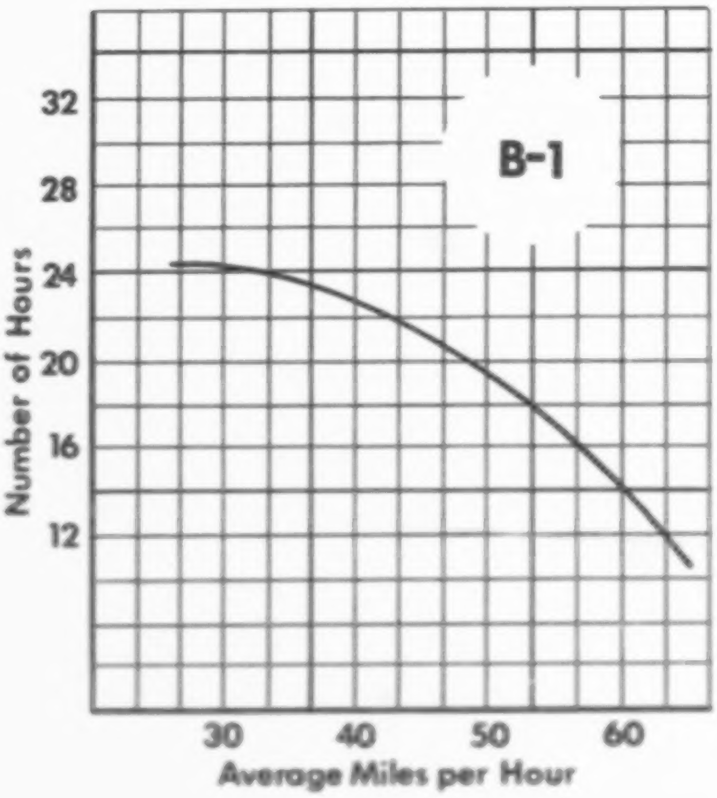
4. Select the graph that best describes the relation of speed and time for this trip.

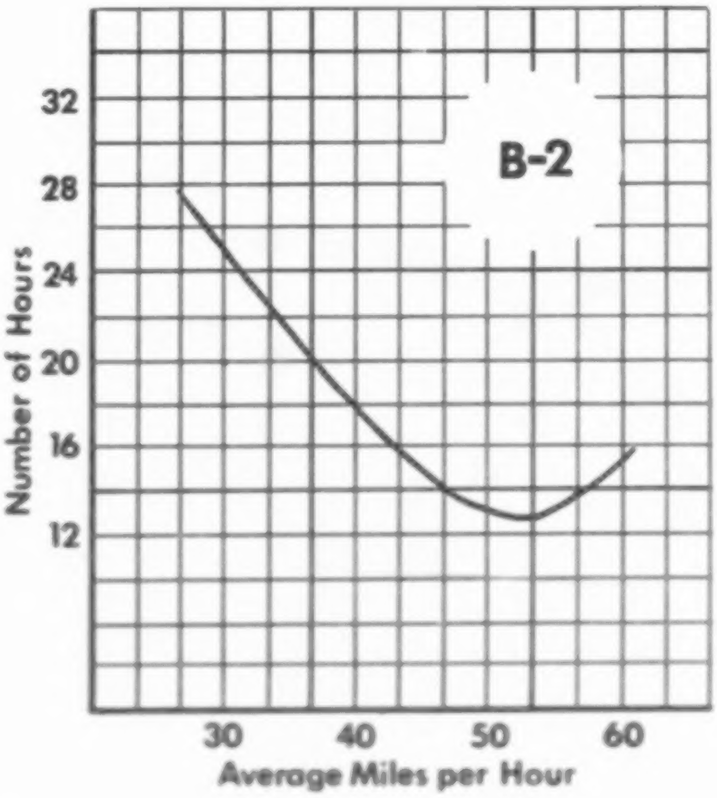
The graph that best represents the data is Graph ______.

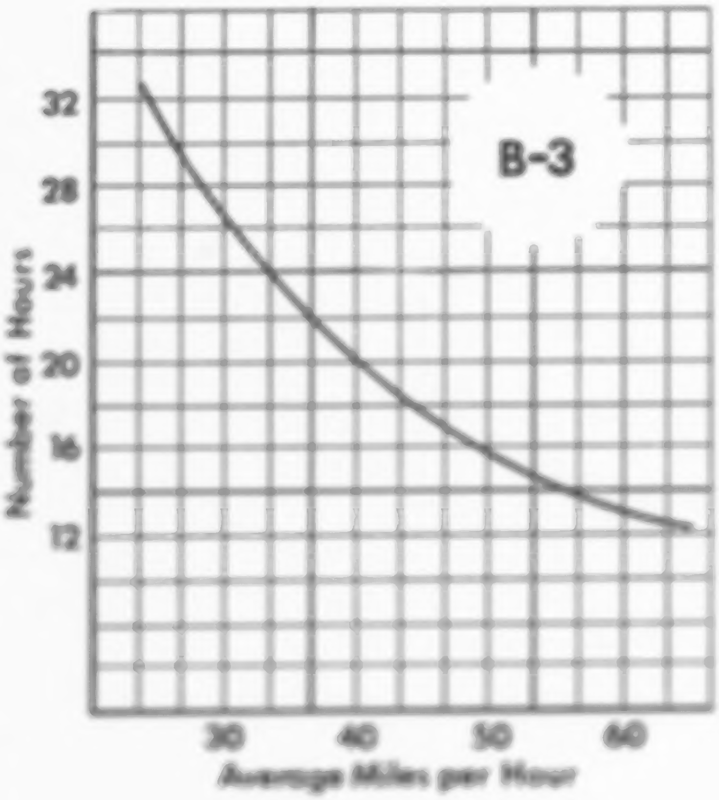
## CHOOSING THE GRAPH TO FIT DATA FOR A PHYSICAL SITUATION

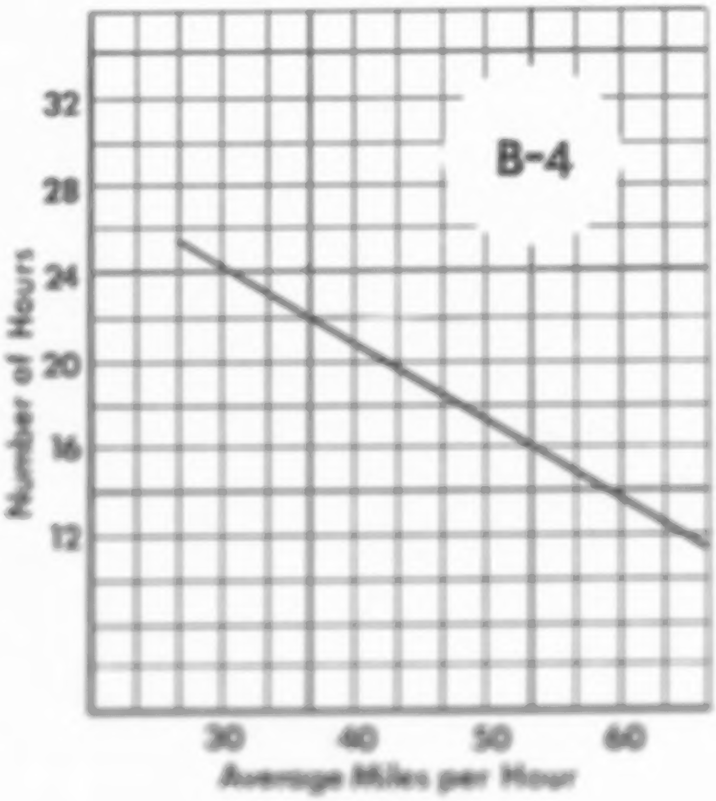
Graphs for Station B

Name ______________________________









## CHOOSING THE GRAPH TO FIT DATA FOR A PHYSICAL SITUATION

UNIT 10, EXPERIENCE 4

Station C

Name ________________________________

At this station you will find several circular objects.

1. Measure the diameter of the smallest circular object. Remember that the radius is one-half of the diameter.
2. Compute the area, using the formula that area is equal to $\pi$ times the square of the radius and remembering that $\pi$ is approximately $3\frac{1}{7}$:

$$A = 3\frac{1}{7}\,(r^2).$$

3. Repeat this for the other objects. Fill in the table below.

| Diameter in Inches | Square Inches |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |

4. Pick the graph that best shows the relationship between diameter and area for circles.

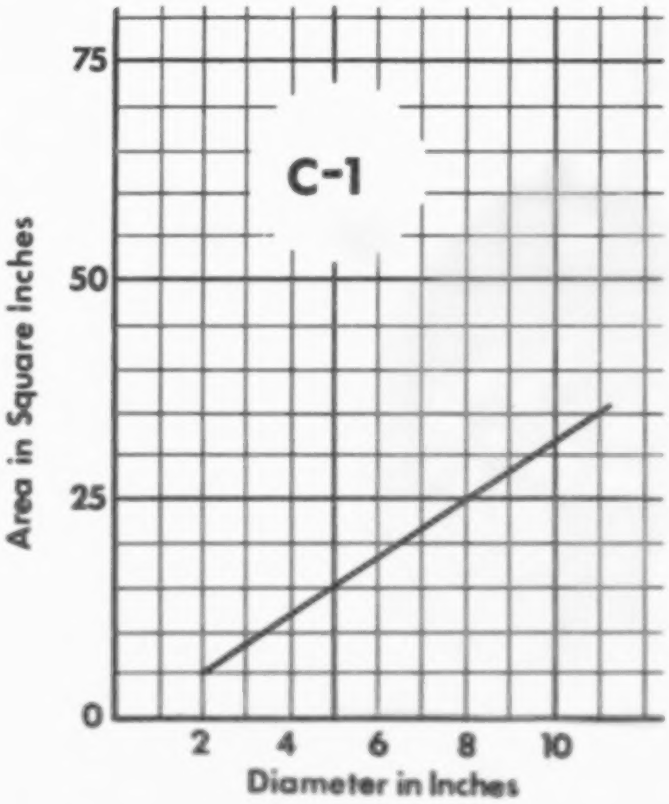
The graph that best represents the data is Graph ______.

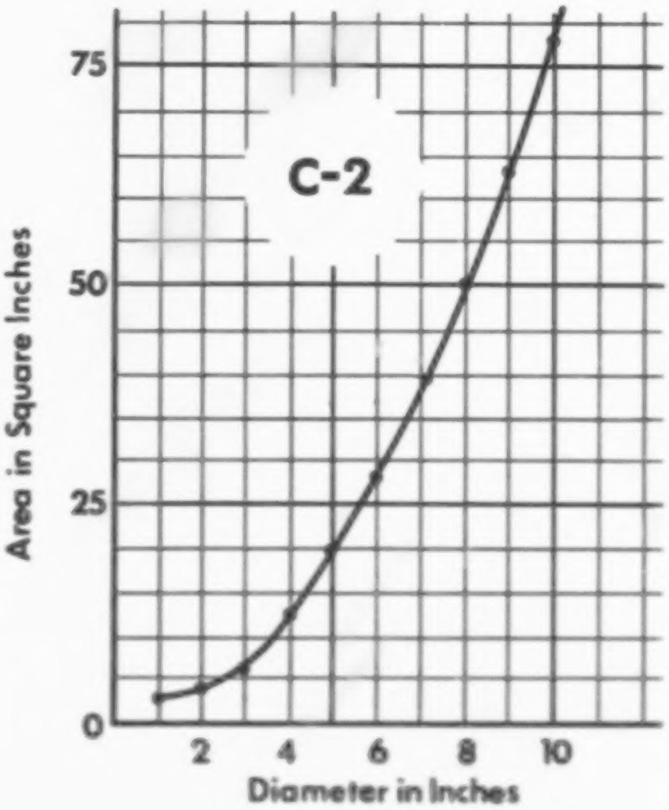
CHOOSING THE GRAPH TO FIT
DATA FOR A PHYSICAL SITUATION

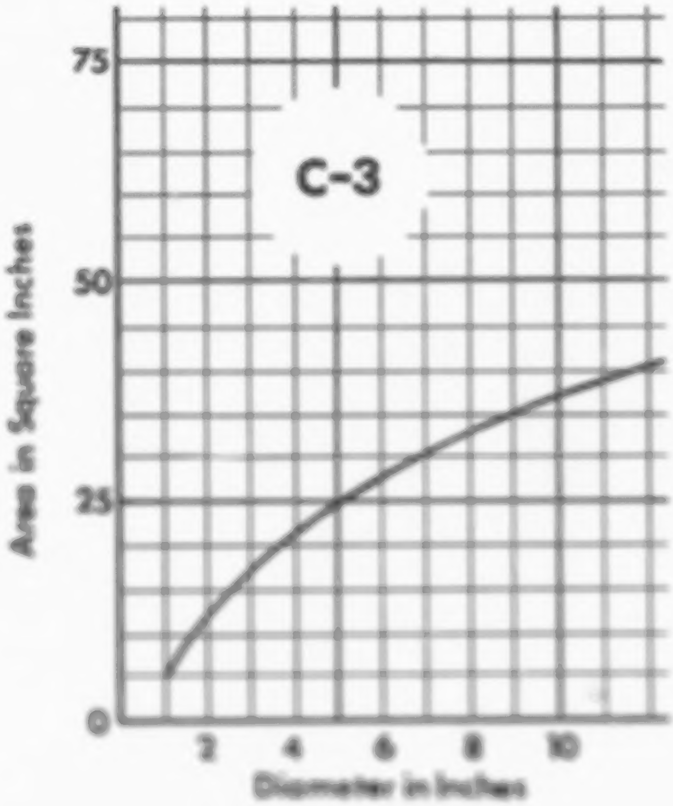
UNIT 10, EXPERIENCE 4

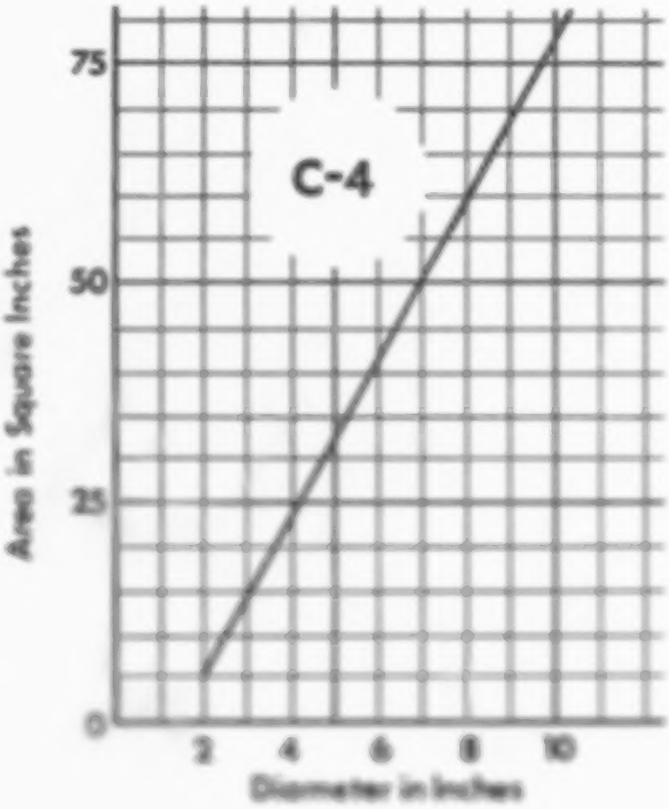
Graphs for Station C

Name ______________________________









## DATA FROM THE CLASS

## UNIT 11, EXPERIENCE 1

Activity 1

Name ______________________________

The students in Classroom 204 turned in a record of their birthdays. Here they are:

Jan. 20
Aug. 15
Apr. 12
Sept. 10
Feb. 15
Oct. 20
Nov. 12
Apr. 15
Jan. 9
Sept. 22
Feb. 14
Aug. 18
Mar. 20
Oct. 10
June 9
Aug. 16
Jan. 2
Aug. 14
June 9
Nov. 11
Feb. 12
Dec. 20
Jan. 15
Oct. 14
Jan. 20

Use this information to complete the frequency tables below. Then answer the questions.

| Month | Tally | Frequency |
|---|---|---|
| Jan. | | |
| Feb. | | |
| Mar. | | |
| Apr. | | |
| May | | |
| June | | |
| July | | |
| Aug. | | |
| Sept. | | |
| Oct. | | |
| Nov. | | |
| Dec. | | |

| Day | Tally | Frequency | Day | Tally | Frequency | Day | Tally | Frequency |
|---|---|---|---|---|---|---|---|---|
| 1 | | | 11 | | | 21 | | |
| 2 | | | 12 | | | 22 | | |
| 3 | | | 13 | | | 23 | | |
| 4 | | | 14 | | | 24 | | |
| 5 | | | 15 | | | 25 | | |
| 6 | | | 16 | | | 26 | | |
| 7 | | | 17 | | | 27 | | |
| 8 | | | 18 | | | 28 | | |
| 9 | | | 19 | | | 29 | | |
| 10 | | | 20 | | | 30 | | |

What month has the most births? ______ How many? ______

Do any months have no births? ______ If so, which one(s)? ______________________

How many months have at least 3 births? ______

How many have less than 2? ______

On what day of the month did most births occur? ______

On what days were there no births? ______________

________________________________________

________________________________________

Activity 2 Name ______________________________

The data below came from a survey of the number of brothers and sisters of students in a certain class. "B" means brothers and "S" means sisters, so that 2B 3S means 2 brothers and 3 sisters.

2B 3S
3B 2S
1B 0S
4B 2S
1B 1S
3B 1S
2B 2S
1B 3S
2B 3S
0B 0S
2B 0S
0B 2S
2B 5S
4B 0S
0B 2S
2B 0S
3B 1S
3B 3S
2B 2S
0B 0S
3B 1S
4B 1S
2B 1S
2B 2S
1B 3S
1B 4S
3B 4S
1B 4S
2B 3S
0B 4S
6B 1S
1B 2S

Complete the frequency tables below and then answer the questions that follow.

| Number of Brothers | Tally | Frequency |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |

| Number of Sisters | Tally | Frequency |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |

| Total Number of Both | Tally | Frequency |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |

The greatest number of brothers anyone has is ______.

The greatest number of sisters is ______.

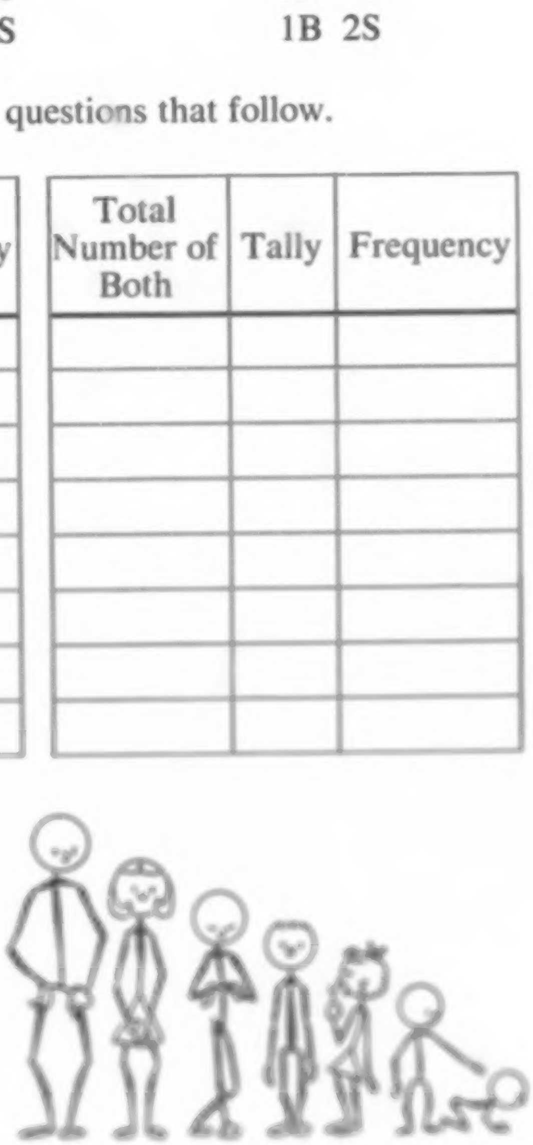
The least number of brothers is ______.

The least number of sisters is ______.

What is the most frequent number of brothers? ______

Of sisters? ______

Of brothers and sisters? ______



COLLECTING AND ORGANIZING YOUR OWN DATA

UNIT 11, EXPERIENCE 2

Activity 1, Station A

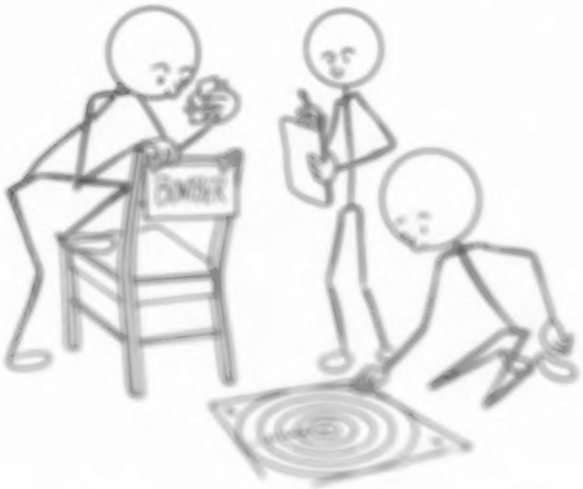
Name ______________________________

## *BOMBER OVER TARGET*

### Individual Data Sheet

| Target Area | Trial 1 | | Trial 2 | | Trial 3 | |
|---|---|---|---|---|---|---|
| | Tally | Score | Tally | Score | Tally | Score |
| 5 | | | | | | |
| 4 | | | | | | |
| 3 | | | | | | |
| 2 | | | | | | |
| 1 | | | | | | |
| 0 | | | | | | |
| Score Totals | | | | | | |

To obtain the score, multiply the number of the target area by the number of buttons that landed there. For example, if the frequency for Target Area 5 is 2, the score to be entered for that area would be 10.



Score for Trial 1 ______

Score for Trial 2 ______

Score for Trial 3 ______

Activity 1, Station A

Name ______________________

Name ______________________

Name ______________________

*BOMBER OVER TARGET*

Team Data Sheet

Summary of Scores

| Name | Trial 1 | Trial 2 | Trial 3 |
|---|---|---|---|
| | | | |
| | | | |
| | | | |

Distribution of Scores for the Team

| Scores | Tally | Frequency |
|---|---|---|
| 0–4 | | |
| 5–9 | | |
| 10–14 | | |
| 15–19 | | |
| 21–25 | | |

COLLECTING AND ORGANIZING YOUR OWN DATA

UNIT 11, EXPERIENCE 2

Activity 2, Station B

Name ______________________________

### *BUTTON TOSS*

### Individual Data Sheet

| Distance from Target, in Inches | Trial 1 | | Trial 2 | | Trial 3 | |
|---|---|---|---|---|---|---|
| | Tally | Score | Tally | Score | Tally | Score |
| 0–3 | | | | | | |
| 4–7 | | | | | | |
| 8–11 | | | | | | |
| 12–15 | | | | | | |
| 16–19 | | | | | | |
| 20–23 | | | | | | |
| 24–27 | | | | | | |
| 28–31 | | | | | | |
| 32–35 | | | | | | |
| 36–39 | | | | | | |
| 40 or More | | | | | | |
| Total Scores | | | | | | |

Score Key

| Distance, in Inches | Points | Distance, in Inches | Points |
|---|---|---|---|
| 0–3 | 10 | 24–27 | 4 |
| 4–7 | 9 | 28–31 | 3 |
| 8–11 | 8 | 32–35 | 2 |
| 12–15 | 7 | 36–39 | 1 |
| 16–19 | 6 | 40 or More | 0 |
| 20–23 | 5 | | |

Score for Trial 1 ______

Score for Trial 2 ______

Score for Trial 3 ______

COLLECTING AND ORGANIZING
YOUR OWN DATA

UNIT 11, EXPERIENCE 2

Activity 2, Station B

Name ______________________________

Name ______________________________

Name ______________________________

## *BUTTON TOSS*

### Team Data Sheet

Summary of Scores

| Team | Trial 1 | Trial 2 | Trial 3 |
|---|---|---|---|
| Team Member 1 | | | |
| Team Member 2 | | | |
| Team Member 3 | | | |

Distribution of Scores for the Team

| Scores | Tally | Frequency |
|---|---|---|
| 0-4 | | |
| 5-9 | | |
| 10-14 | | |
| 15-19 | | |
| 20-24 | | |
| 25-29 | | |
| 30-34 | | |
| 35-39 | | |
| 40-44 | | |
| 45-49 | | |
| 50 | | |

COLLECTING AND ORGANIZING
YOUR OWN DATA

UNIT 11, EXPERIENCE 2

Activity 3

Name ______________________________

## *VOWEL COUNT*

From your English classes, you are already familiar with the vowels *a, e, i, o,* and *u.* I'll bet you never expected to use them in math. Actually, all we're trying to do is find out which ones are used the most.

Take the clipping out of your "Vowel Count" envelope.

Look at the first word. Suppose it is "today." For that word you would put a tally in the *o* column and another tally in the *a* column. Do this for every word in the sentences you have. When you have finished, find the total number of tallies for each vowel and place that number in the last column. Then place your results in the proper place on the sheet for the class report.

| Vowel | Tally | Frequency |
|---|---|---|
| *a* | | |
| *e* | | |
| *i* | | |
| *o* | | |
| *u* | | |

The quick brown fox jumped ..
Vowel Count Envelope

COLLECTING AND ORGANIZING YOUR OWN DATA

UNIT 11, EXPERIENCE 2

Activity 4

Name ______________________________

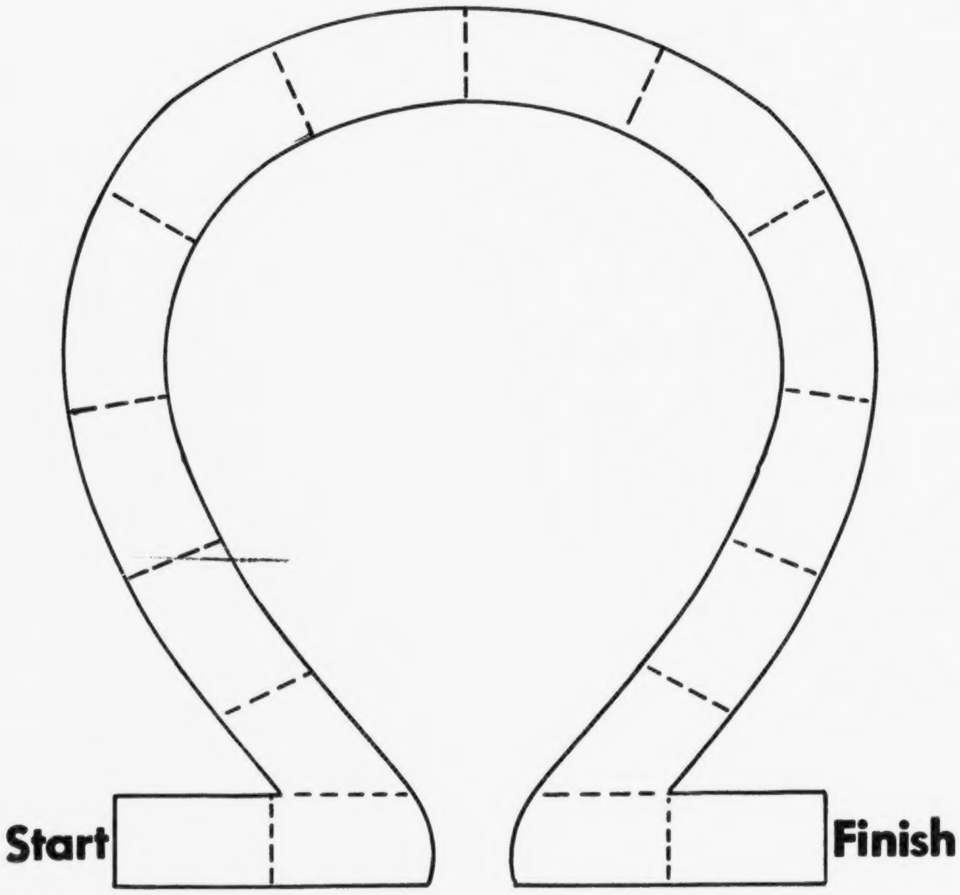
### *HORSESHOE GAME*

The object of this game is to move your marker from start to finish in as few moves as possible.

Place your marker at the starting line. Toss the die. Move the marker forward along the horseshoe as many spaces as the top of the die indicates, and record a tally mark in the Tally of Tosses column on your Individual Data Sheet for this activity. Continue tossing the die, moving forward the indicated number of spaces, and recording a tally mark for each toss.

You must cross the finish line on an exact count. If your die toss results in a number more than the number of spaces that remain you must keep your marker where it is, record a tally for the toss, and toss again (with a tally each time) until you get a number that you can use.

You may continue to play the game until the warning signal is given by your teacher. Then complete the frequency table on the Individual Data Sheet and record your data on the table for the class.



Activity 4

Name ______________________________

## *HORSESHOE GAME*

### Individual Data Sheet

| Trial | Tally of Tosses | Frequency |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |
| 9 | | |
| 10 | | |
| 11 | | |
| 12 | | |
| 13 | | |
| 14 | | |
| 15 | | |
| 16 | | |
| 17 | | |
| 18 | | |
| 19 | | |
| 20 | | |

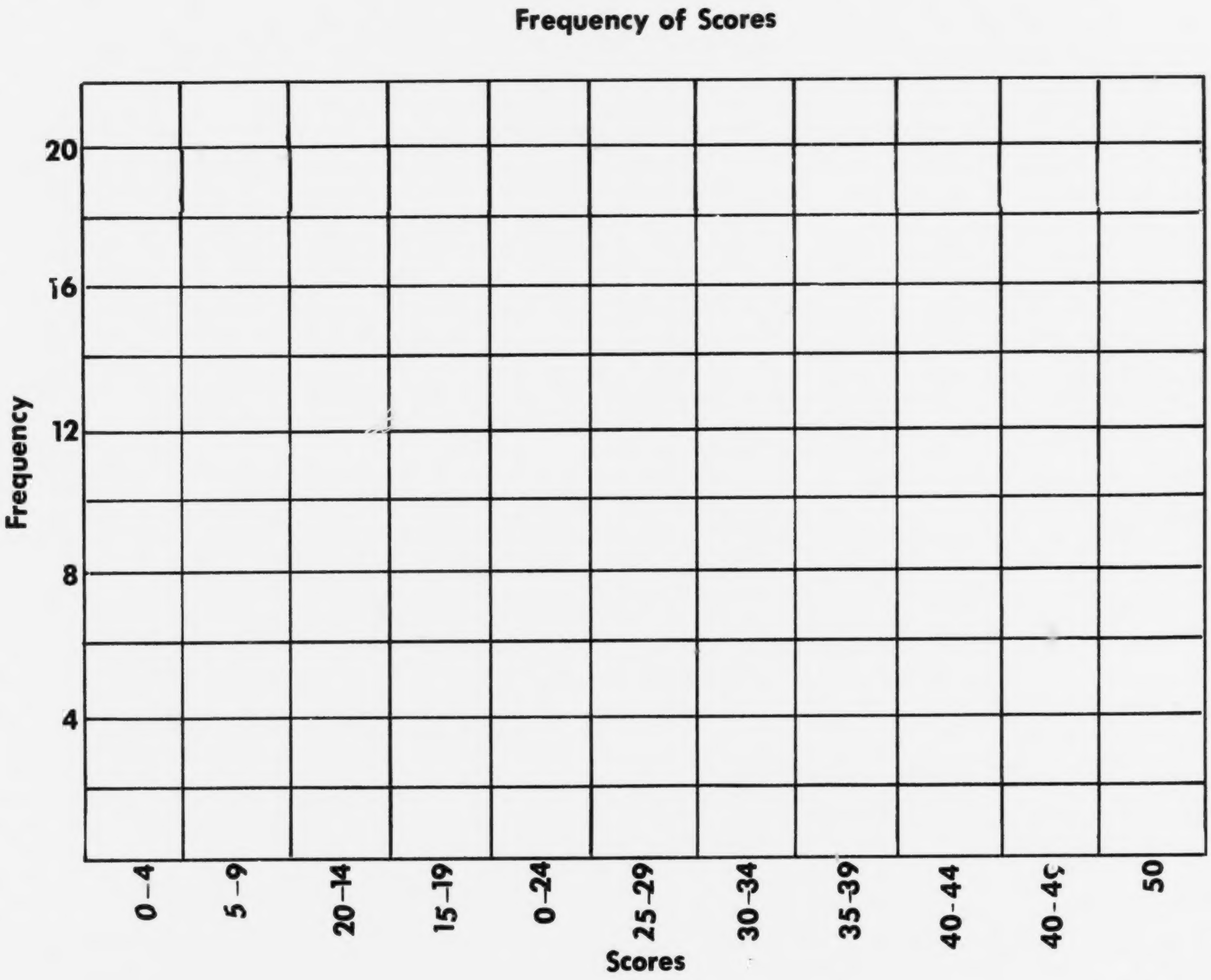
Summary of Trials

| Number of Tosses | Tally | Frequency |
|---|---|---|
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |
| 9 | | |
| 10 | | |
| 11 | | |
| 12 | | |
| 13 | | |
| 14 or More | | |

GRAPHING AND INTERPRETING DATA UNIT 11, EXPERIENCE 3

Activity 1 Name ______________________________

Follow the directions given in class to construct a bar graph to represent the class data from the Bomber over Target game.



Activity 2, Station A

Name ____________________

### *LETTER COUNT*

Take a paragraph out of one of the Letter Count envelopes.

Look at the first word. Suppose it is the word *when. When* is a 4-letter word, so you should put a tally mark after the 4 in the Tally column of the table below. Use hyphenated words as one word.

When you have tallied the number of letters for each word in your paragraph, find the total number of tallies for each number of letters and put that number in the last column.

Individual Data Sheet

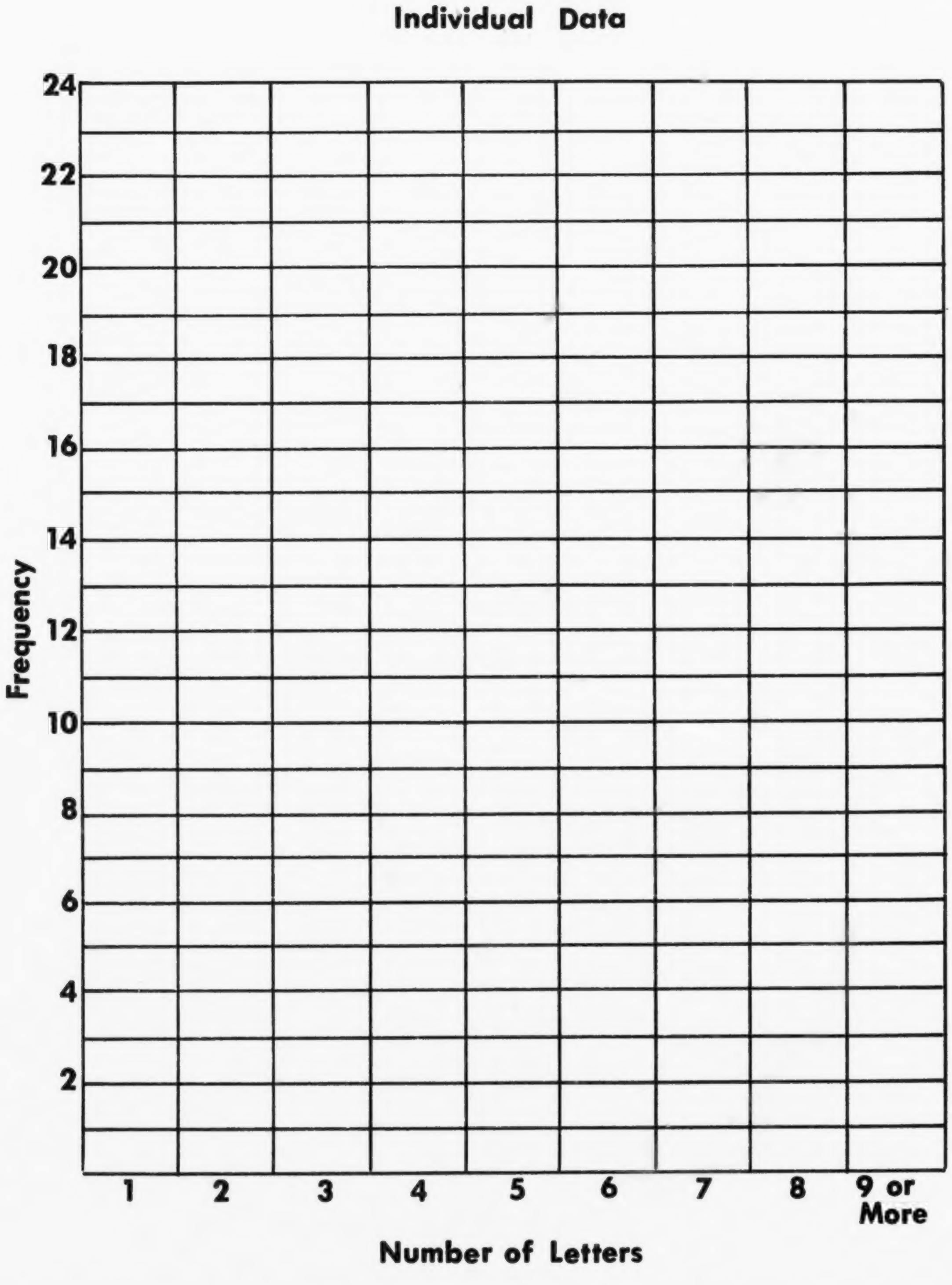
| Number of Letters in Word | Tally | Frequency |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |
| 9 or More | | |

Enter these results in your group data report, then go on to the next page.

[*Continued*]

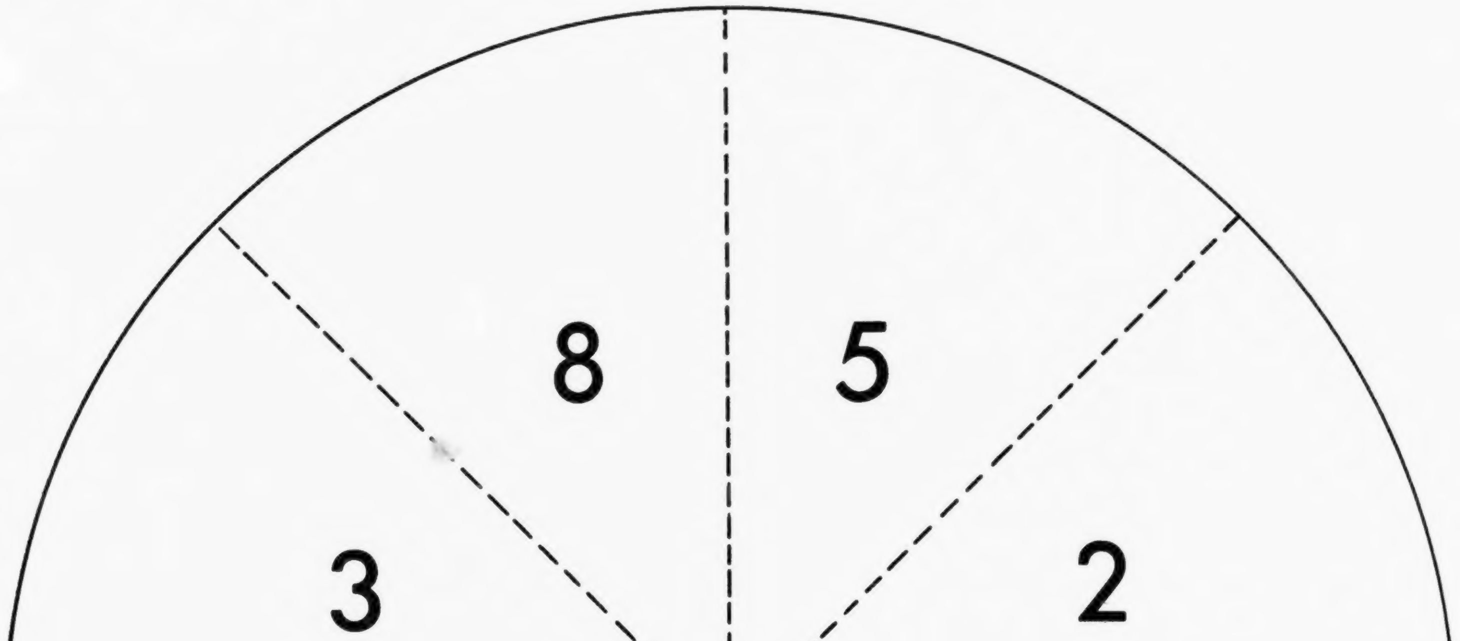
Activity 2–*Continued* Name ______________________________

Make a graph of the information in your Individual Data Sheet.



When your group data sheet has been completed, make a graph of the information. No form is provided.

***Teacher Materials***



Activity 3, Station B

Name ________________________________

### *SPINNER SUMS*

Pick up a spinner from Station B and take it to your desk. Keep it flat on your desk and spin the pointer once. Look at the number that the pointer indicates. If a pointer stops on a line, spin it again. Then spin it again to get a second number.

Find the sum of the numbers. Write the sum after Trial 1, below. Continue spinning the pointer and finding the sums until you have filled in all the spaces in the charts below.

| Trial | Sum |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| 13 | |
| 14 | |
| 15 | |
| 16 | |

| Trial | Sum |
|---|---|
| 17 | |
| 18 | |
| 19 | |
| 20 | |
| 21 | |
| 22 | |
| 23 | |
| 24 | |
| 25 | |
| 26 | |
| 27 | |
| 28 | |
| 29 | |
| 30 | |
| 31 | |
| 32 | |

| Trial | Sum |
|---|---|
| 33 | |
| 34 | |
| 35 | |
| 36 | |
| 37 | |
| 38 | |
| 39 | |
| 40 | |
| 41 | |
| 42 | |
| 43 | |
| 44 | |
| 45 | |
| 46 | |
| 47 | |
| 48 | |

[*Continued*]

Activity 3–*Continued* Name ______________________________

Look at your data on the preceding page. What is the smallest sum you got? Put it on the first line below. What is the largest sum? Put it on the bottom line. Fill the other sums in, in order from smallest to largest. Record tally marks after them to indicate the number of times you obtained each sum.

Individual Data Sheet

| | Sum | Tally | Frequency |
|---|---|---|---|
| Smallest Sum = | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| Largest Sum = | | | |

Record the above data on your Group Data Sheet.

[*Continued*]

Activity 3–*Continued*

Name ______________________________

Complete this chart just as you did when you organized your own data except that this time you should use the data reported by your group on the group data report form. When you have completed the chart, use the information to construct a graph in the grid provided on the next page.

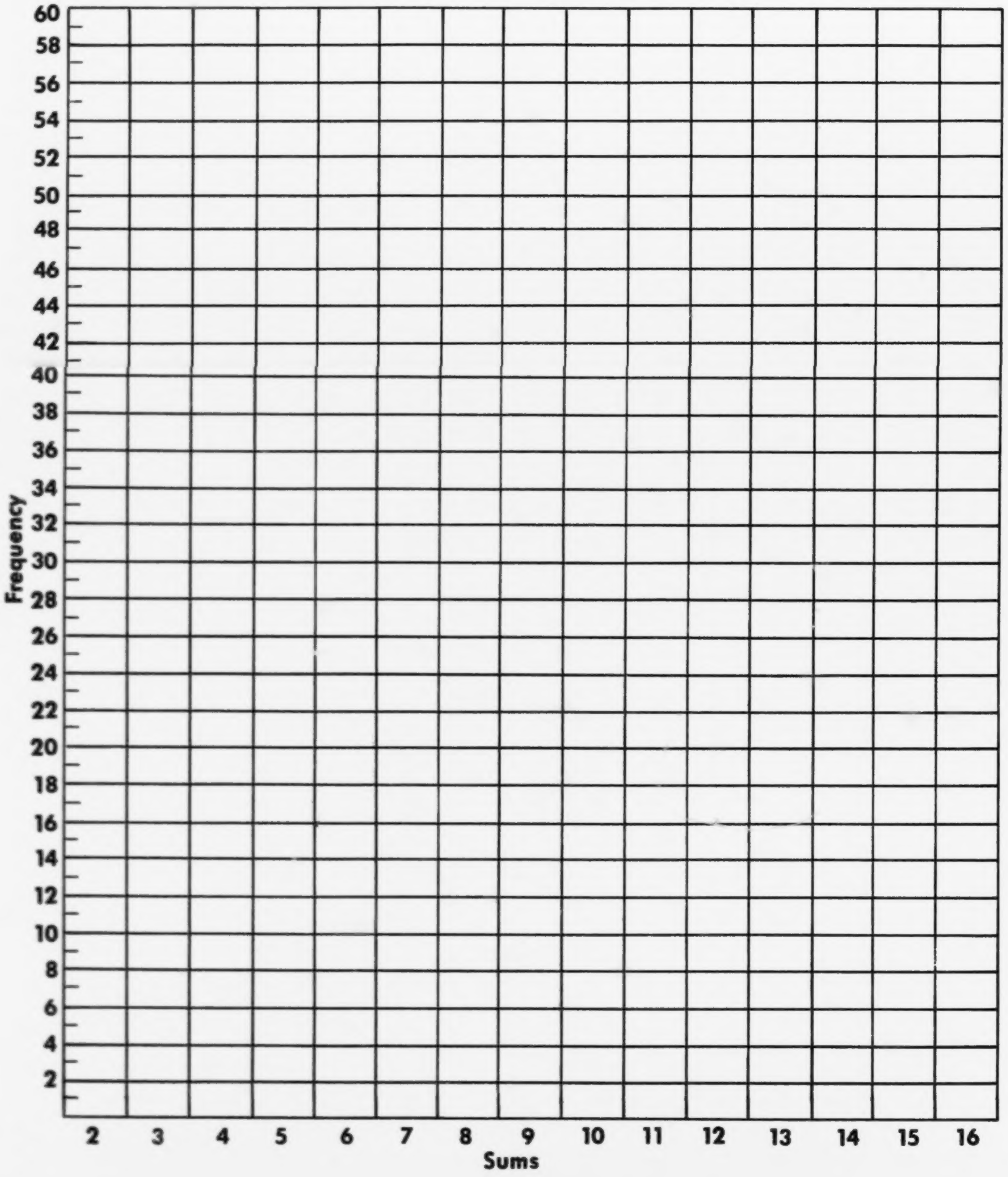
Group Data Sheet

| Sum | Frequency |
|---|---|
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| 13 | |
| 14 | |
| 15 | |
| 16 | |

[*Continued*]

Activity 3–*Continued* Name ______________________________

Graph of Group Data

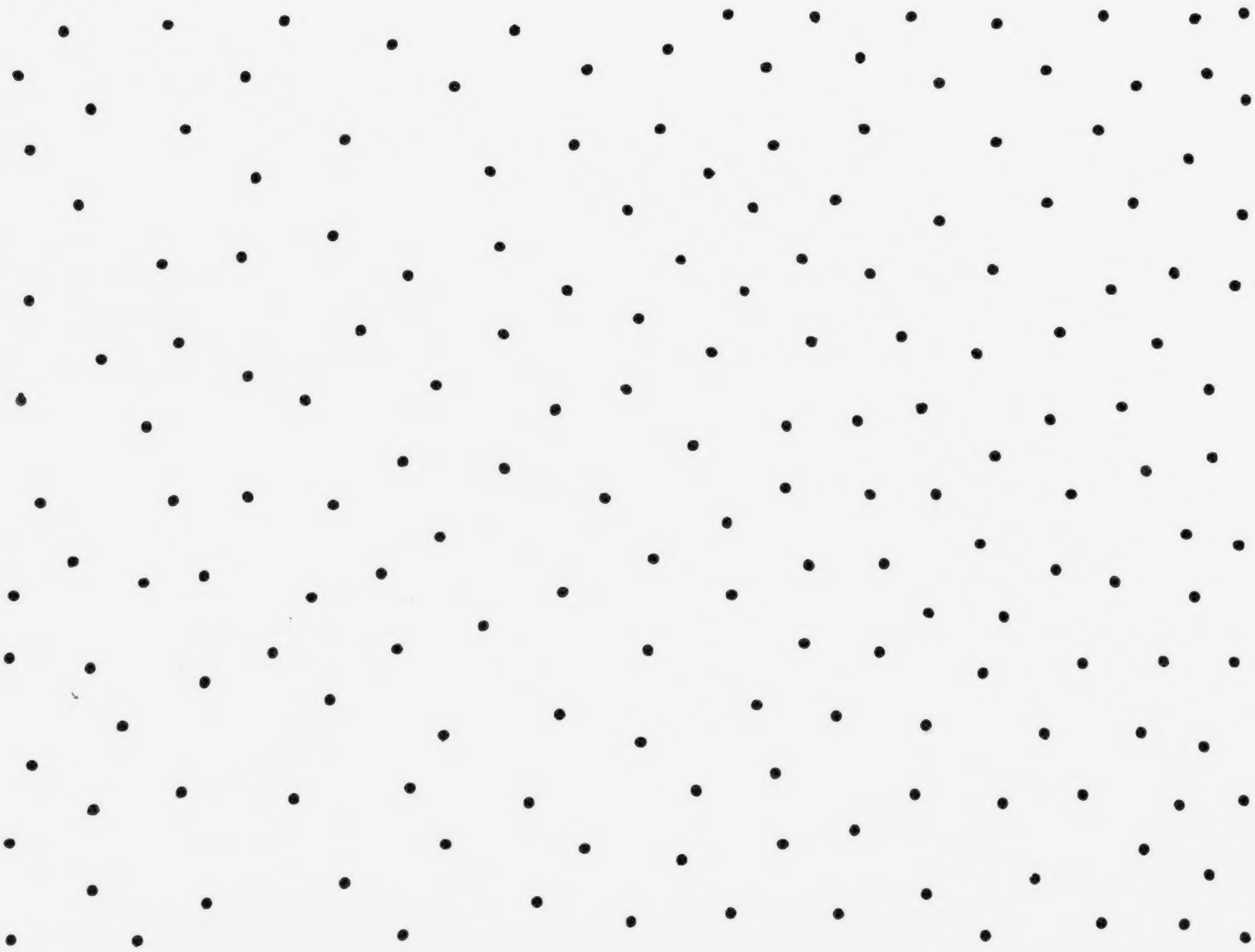


Activity 4 Name ______________________________

### *DOT COUNT*

Hold your hand over the bottom part of this sheet. Close your eyes. Drop your hand, flat, on the sheet. Trace around your hand with a pencil. Stop at the base of your palm. Then count the number of dots in the area your hand covered. To make counting easier, do it by areas. For example, count the dots in each of the fingers and then divide the palm into regions and count them.

After you have counted the dots, measure the distance from the tip of your longest finger to the base of your palm as you have traced your hand on the paper. Also measure the length of one of your shoes. Enter this information in the spaces provided below and on your group report sheet.



Number of dots inside sketch of hand ________

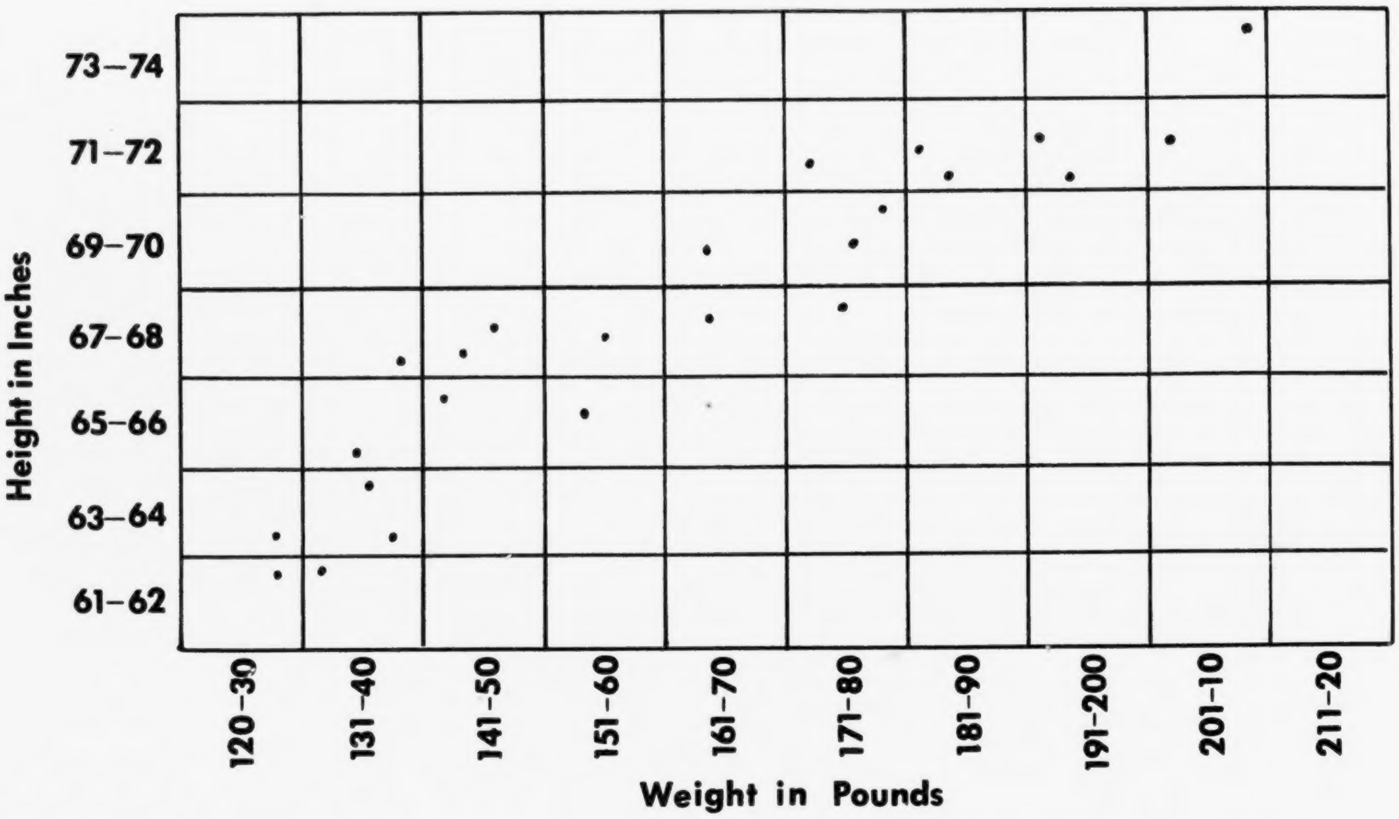
Length of hand to nearest $\frac{1}{4}$ inch ________ Length of shoe to nearest $\frac{1}{4}$ inch ________

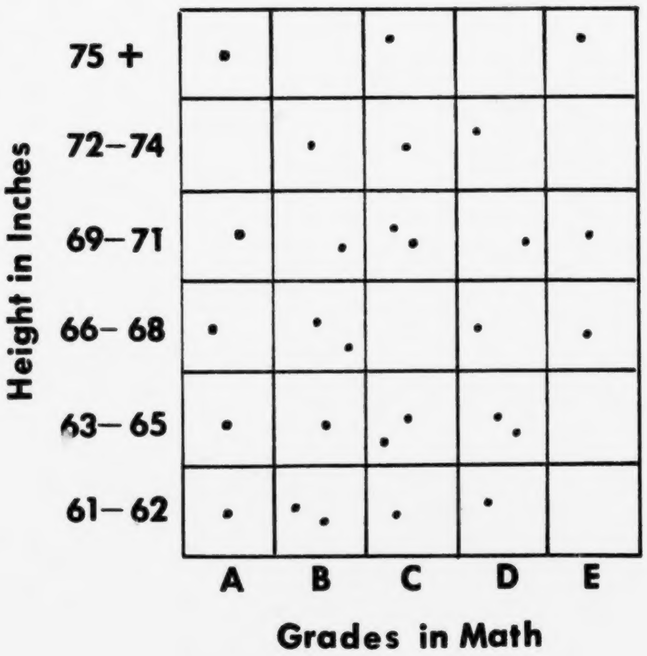
Activity 5 Name ______________________________

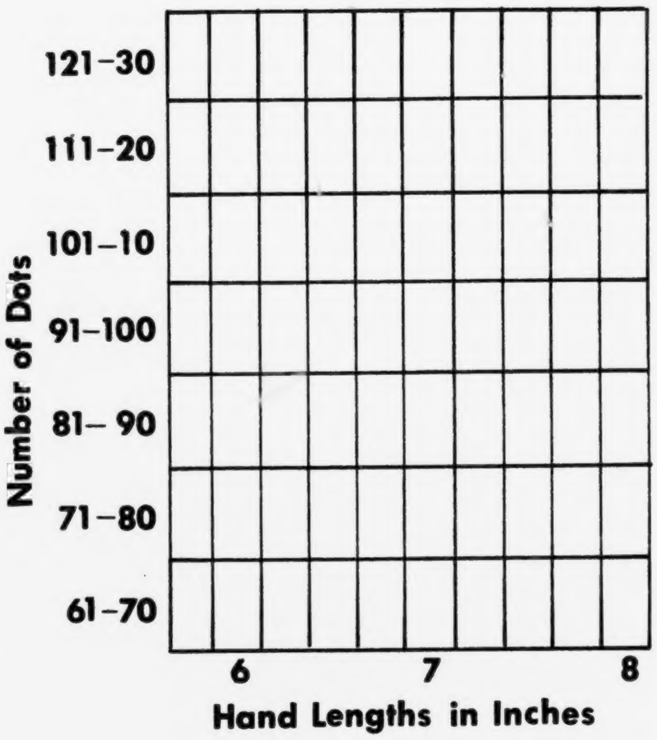
Study the tables and graphs you have made in the activities in this experience, then use them to help you answer the following questions:

1. How many different sums did you find, using the two spinners? _____
2. Did you get some of the sums more often than others? _____
3. What sum(s) did you get most often? _____
4. What sum(s) did you get least often? _____
5. Did you ever get a sum of 20? _____ If not, why not? ______________________
______________________________________________
6. How many letters were there in the shortest word you had? _____
7. How many letters were there in the longest word you had? _____
8. Do you think that you might have had different results if you had used a different magazine? _____
9. In general most words in the paragraph are 3 4 5 6 letters in length. Circle the number(s) that apply.
10. Do you think anyone else's Dot Count will be exactly the same as yours? _____
11. Do you think your Dot Count will be high, low, or in the middle of the data collected by the entire class? _____
12. Do you think that the size of your hand makes any difference in the number of your Dot Count? _____

***Teacher Materials***







MORE ON INTERPRETING DATA UNIT 11, EXPERIENCE 4

Activity 1 Name ______________________________

### BUTTON TOSS

Complete the Individual Data charts, then have one member of your group complete the Group Data Sheet on the next page.

Individual Data

Line A (2 ft.)

| Distance in Inches | Tally |  |
|---|---|---|
|  | 1 | 2 |
| 0–3 |  |  |
| 4–7 |  |  |
| 8–11 |  |  |
| 12–15 |  |  |
| 16–19 |  |  |
| 20–23 |  |  |
| 24–27 |  |  |
| 28–31 |  |  |
| 32–35 |  |  |
| 36–39 |  |  |
| 40 or More |  |  |

Scores ____ ____

Line B (4 ft.)

| Distance in Inches | Tally |  |
|---|---|---|
|  | 1 | 2 |
| 0–3 |  |  |
| 4–7 |  |  |
| 8–11 |  |  |
| 12–15 |  |  |
| 16–19 |  |  |
| 20–23 |  |  |
| 24–27 |  |  |
| 28–31 |  |  |
| 32–35 |  |  |
| 36–39 |  |  |
| 40 or More |  |  |

Scores ____ ____

Line C (6 ft.)

| Distance in Inches | Tally |  |
|---|---|---|
|  | 1 | 2 |
| 0–3 |  |  |
| 4–7 |  |  |
| 8–11 |  |  |
| 12–15 |  |  |
| 16–19 |  |  |
| 20–23 |  |  |
| 24–27 |  |  |
| 28–31 |  |  |
| 32–35 |  |  |
| 36–39 |  |  |
| 40 or More |  |  |

Scores ____ ____

Score Key

| Distance in Inches | Points | Distance in Inches | Points | Distance in Inches | Points |
|---|---|---|---|---|---|
| 0–3 | 10 | 16–19 | 6 | 32–35 | 2 |
| 4–7 | 9 | 20–23 | 5 | 36–39 | 1 |
| 8–11 | 8 | 24–27 | 4 | 40 or More | 0 |
| 12–15 | 7 | 28–31 | 3 |  |  |

[*Continued*]

Activity 1–*Continued*

Name ____________________ ____________

Line D (8 ft.)

| Distance in Inches | Tally | |
|---|---|---|
| | 1 | 2 |
| 0–3 | | |
| 4–7 | | |
| 8–11 | | |
| 12–15 | | |
| 16–19 | | |
| 20–23 | | |
| 24–27 | | |
| 28–31 | | |
| 32–35 | | |
| 36–39 | | |
| 40 or More | | |

Scores ____ ____

Line E (10 ft.)

| Distance in Inches | Tally | |
|---|---|---|
| | 1 | 2 |
| 0–3 | | |
| 4–7 | | |
| 8–11 | | |
| 12–15 | | |
| 16–19 | | |
| 20–23 | | |
| 24–27 | | |
| 28–31 | | |
| 32–35 | | |
| 36–39 | | |
| 40 or More | | |

Scores ____ ____

Line F (12 ft.)

| Distance in Inches | Tally | |
|---|---|---|
| | 1 | 2 |
| 0–3 | | |
| 4–7 | | |
| 8–11 | | |
| 12–15 | | |
| 16–19 | | |
| 20–23 | | |
| 24–27 | | |
| 28–31 | | |
| 32–35 | | |
| 36–39 | | |
| 40 or More | | |

Scores ____ ____

One of your group members should make a summary of the scores in the report below.

Group Data Sheet

| Line A | Line B | Line C | Line D | Line E | Line F |
|---|---|---|---|---|---|
| | | | | | |
| | | | | | |
| | | | | | |
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| | | | | | |

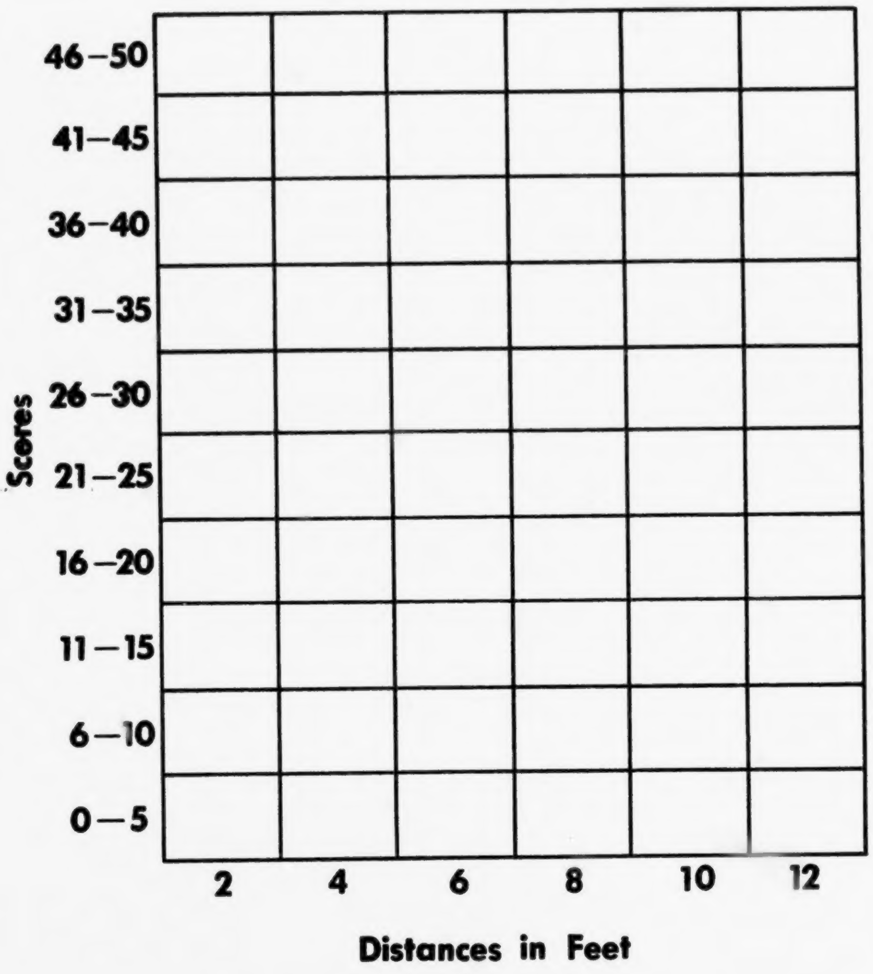
[*Continued*]

Activity 1–*Continued* Name ____________________

Complete the chart and the graph below.

Group Results

| Scores | |
|---|---|
| A Line<br>(2 ft.) | |
| B Line<br>(4 ft.) | |
| C Line<br>(6 ft.) | |
| D Line<br>(8 ft.) | |
| E Line<br>(10 ft) | |
| F Line<br>(12 ft.) | |



According to your group's graph, did there appear to be any relation between the distances you stood from the target line and the scores you made? ______

If so, what relation did you notice? ____________________

____________________

From which line were the highest scores made? ______

From which line were the lowest scores made? ______

MORE ON INTERPRETING DATA UNIT 11, EXPERIENCE 4

Activity 2 Name ______________________________

### *SPINNER RELATIONS*

Place your spinner flat on your desk. Spin the pointer once and record the number in the first column of table 1. Spin it again and record the number in the second column of the same table. Then, in table 2, record the number for Spin 1 and the sum of the numbers on the two spins.

Continue doing this, recording the results in the two tables, until you have completely filled in all the spaces. Then go on to the next page.

Table 1

| Numbers on Spinner | | Numbers on Spinner | |
|---|---|---|---|
| Spin 1 | Spin 2 | Spin 1 | Spin 2 |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
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| | | | |

Table 2

| Spin 1 and Sums | | Spin 1 and Sums | |
|---|---|---|---|
| Spin 1 | 1 and 2 | Spin 1 | 1 and 2 |
| | | | |
| | | | |
| | | | |
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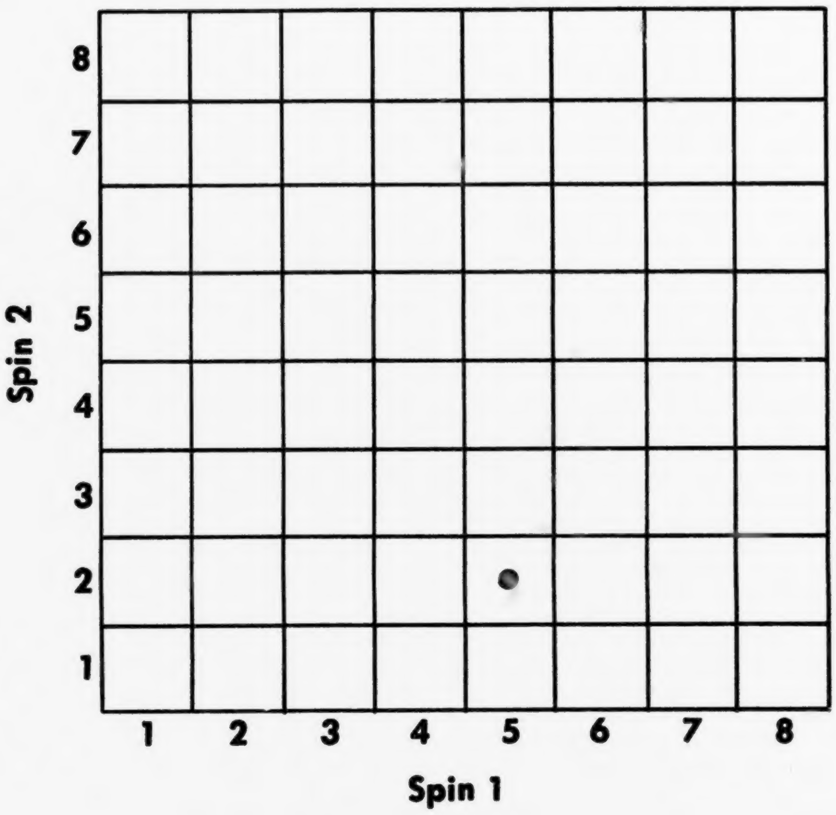
[*Continued*]

Activity 2–*Continued* Name ______________________________

Use the pairs of numbers that you collected for table 1 of Spinner Relations to construct the graph below.

For each pair of numbers there should be a dot on the graph. For example: If the first pair of numbers was a 5 on Spin 1 and a 2 on Spin 2, you should place a dot, as shown, in the space where the fifth vertical column and the second horizontal column intersect.



Answer the questions below, then go on to the next page.

Does your graph seem to indicate that there is any relation between the numbers on Spin 1 and Spin 2? ______

If so, what relation do you notice? ______________________________________________

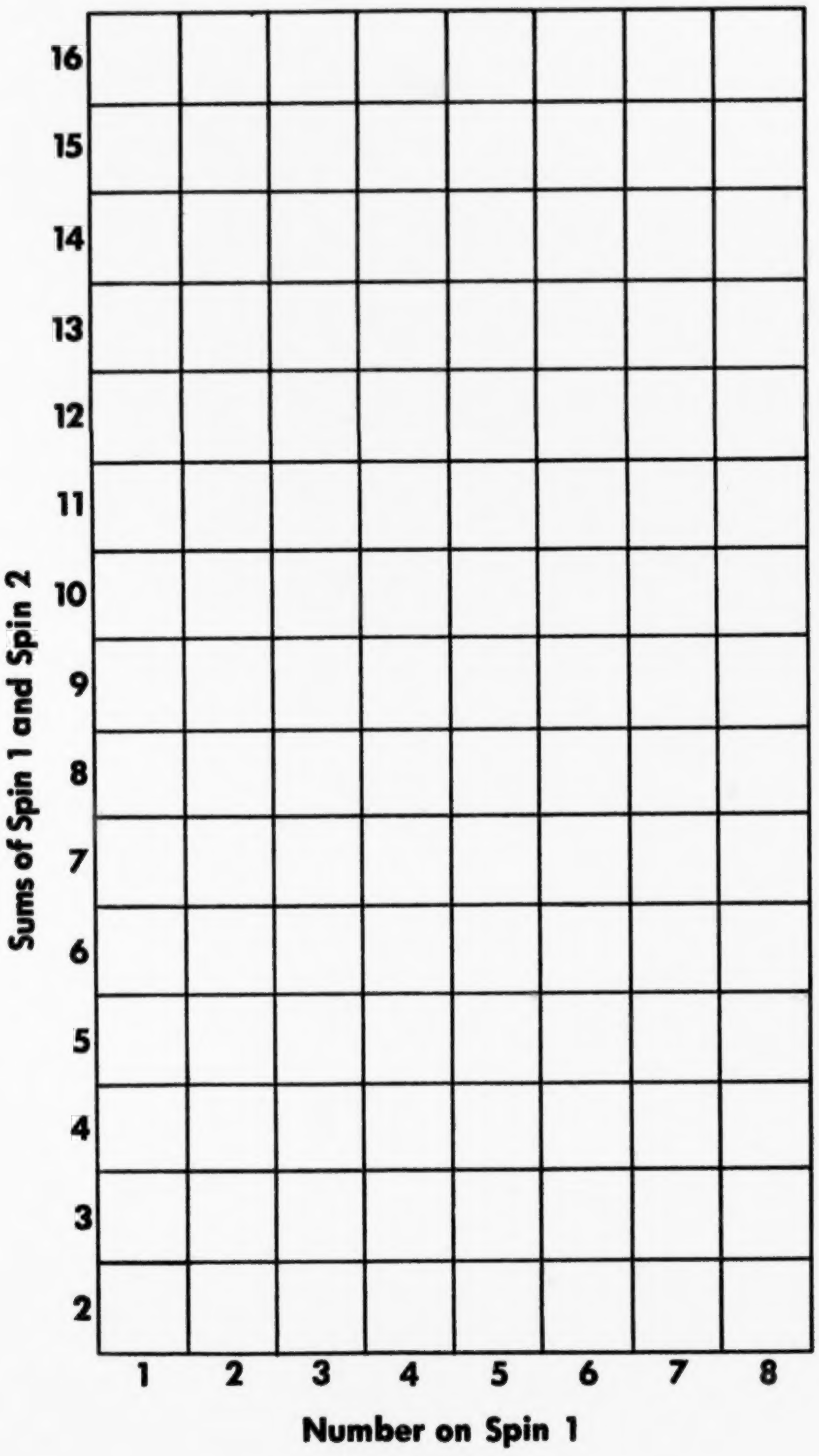
______________________________________________________________________

Did you expect to find a relation? ______

[*Continued*]

Activity 2–*Continued* Name ______________________________

Use the same procedure as before, but this time use the data from table 2 and look for a relation between the numbers on Spin 1 and the sum of the numbers on Spins 1 and 2.



Does your graph seem to indicate that there is any relation between the numbers on Spin 1 and the sums of the numbers of Spins 1 and 2? ______

If it does, what relation do you notice? ______________________

______________________________

______________________________

Did you expect to find a relation?

______

Do you think that there would be a relation between the numbers on Spin 2 and the sums of the numbers of Spins 1 and 2? ______

Activity 3 Name ________________________________

### *WORD LENGTHS AND NUMBER OF VOWELS*

How do you think the number of vowels a word has compares with the number of letters it has? The longer the word the more vowels? How about the words *a* and *check?* Both words have only 1 vowel, but one has 1 letter and the other has 5. Perhaps these are just exceptions?

Take a paragraph from the envelope. Look at the first word. Put the number of letters it has in the top space of the first column and the number of vowels in the top space of the second column. Make one entry in the chart for each word in your paragraph.

Make a graph of the data in the chart on the grid provided on the next page. There should be one dot in your graph for each pair of numbers that appears in your chart.

When the graph is completed, look to see if there seems to be any relation between the number of letters in the word and the number of vowels it has. Use the space provided under the graph to describe what you find.

Individual Data

| Letters in Word | Vowels in Word | Letters in Word | Vowels in Word |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |
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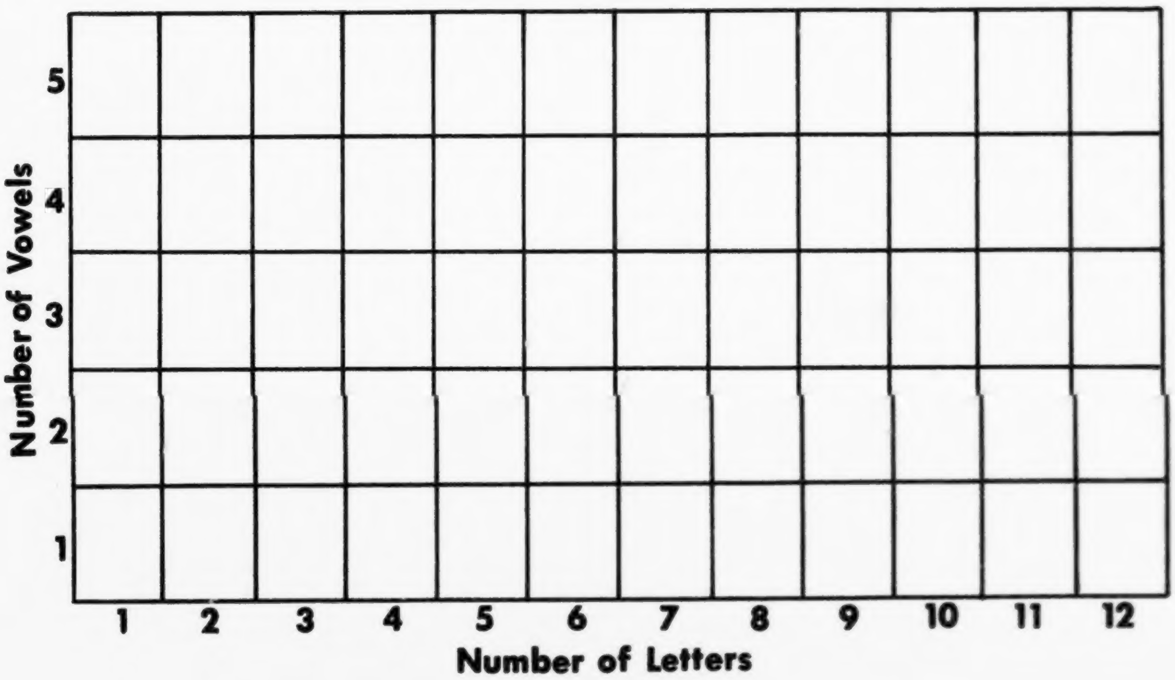
Activity 3–*Continued* Name ______________________________

*WORD LENGTHS AND NUMBER OF VOWELS*

Relation Graph



Describe the relation that seems to be shown by the points on your graph. If there seems to be no relation, say so and explain why. ______________________________

______________________________

______________________________

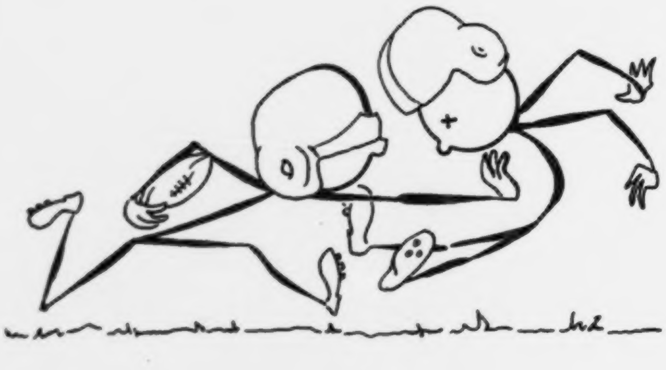
______________________________

*Football Roster*

*STATE UNIVERSITY*

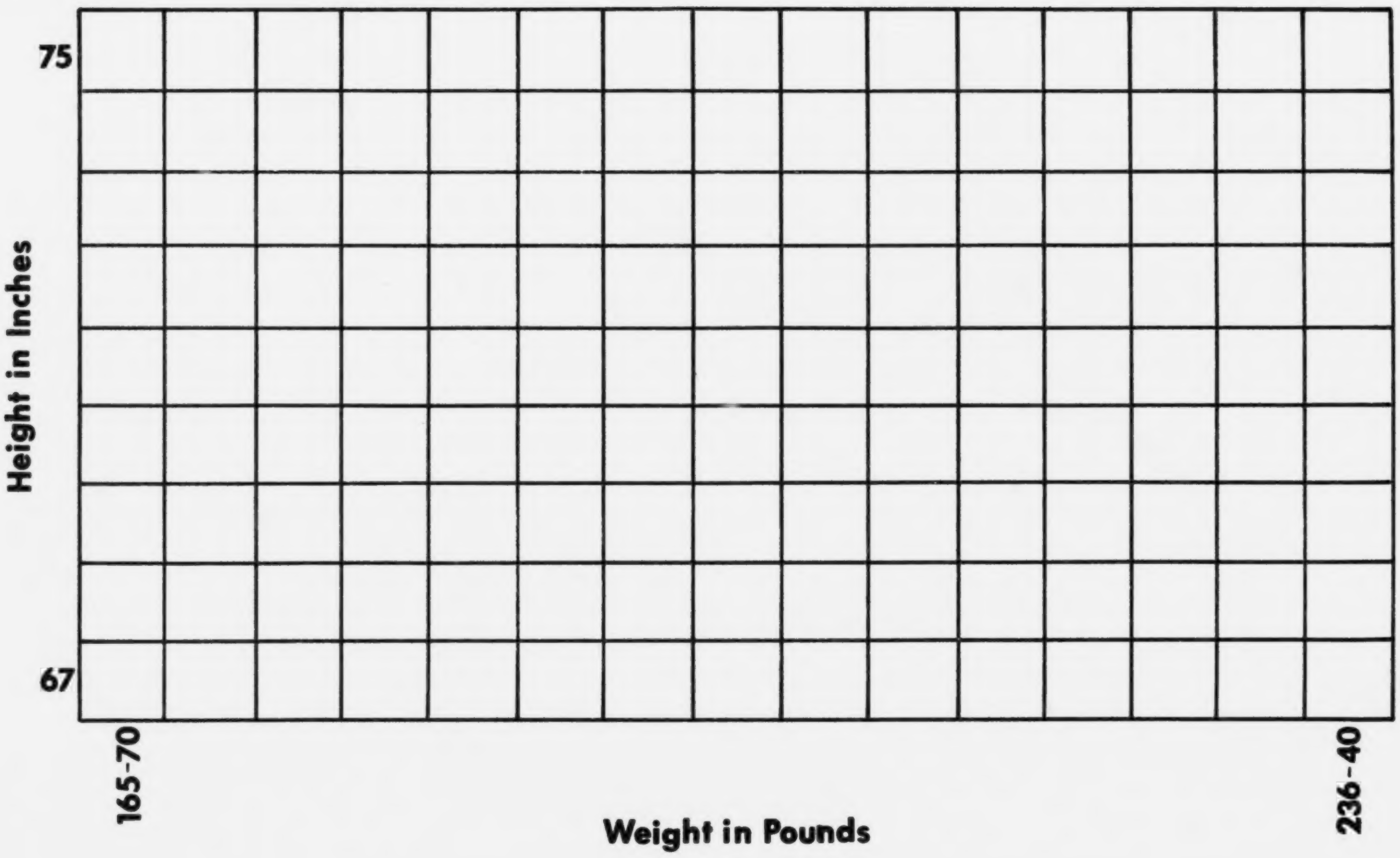
| Number | Height | Weight | Grade Average |
|---|---|---|---|
| 15 | 73 | 170 | A |
| 70 | 72 | 205 | B |
| 72 | 73 | 210 | B |
| 67 | 71 | 205 | C |
| 43 | 71 | 196 | C |
| 61 | 71 | 205 | B |
| 78 | 72 | 220 | A |
| 74 | 71 | 203 | B |
| 76 | 72 | 215 | C |
| 38 | 69 | 175 | B |
| 51 | 74 | 190 | C |
| 28 | 72 | 180 | A |
| 27 | 70 | 173 | D |
| 30 | 69 | 175 | B |
| 54 | 69 | 205 | B |
| 53 | 72 | 200 | C |
| 25 | 70 | 168 | B |
| 75 | 72 | 205 | A |
| 60 | 70 | 195 | D |
| 20 | 72 | 170 | A |

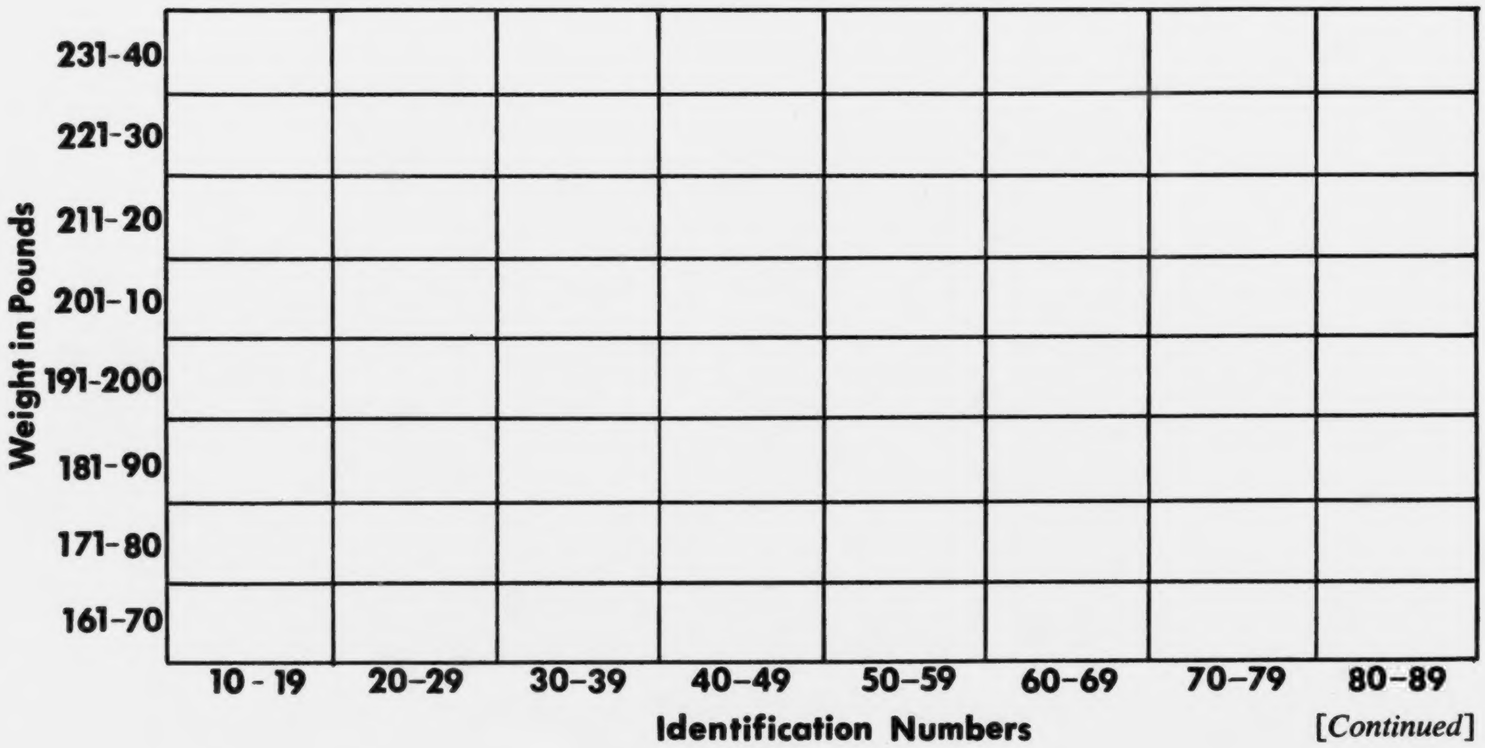
| Number | Height | Weight | Grade Average |
|---|---|---|---|
| 47 | 72 | 200 | B |
| 24 | 71 | 173 | C |
| 22 | 69 | 165 | B |
| 19 | 71 | 173 | B |
| 42 | 70 | 188 | A |
| 34 | 74 | 175 | D |
| 36 | 70 | 195 | D |
| 18 | 71 | 175 | B |
| 65 | 73 | 210 | C |
| 31 | 70 | 167 | B |
| 62 | 68 | 200 | C |
| 33 | 71 | 182 | A |
| 83 | 74 | 232 | A |
| 49 | 71 | 182 | B |
| 88 | 75 | 240 | B |
| 58 | 70 | 190 | C |



Activity 1 Name ______________________________

Using data from the Football Roster sheet, complete the two graphs below and a third one, at the top of the next page.





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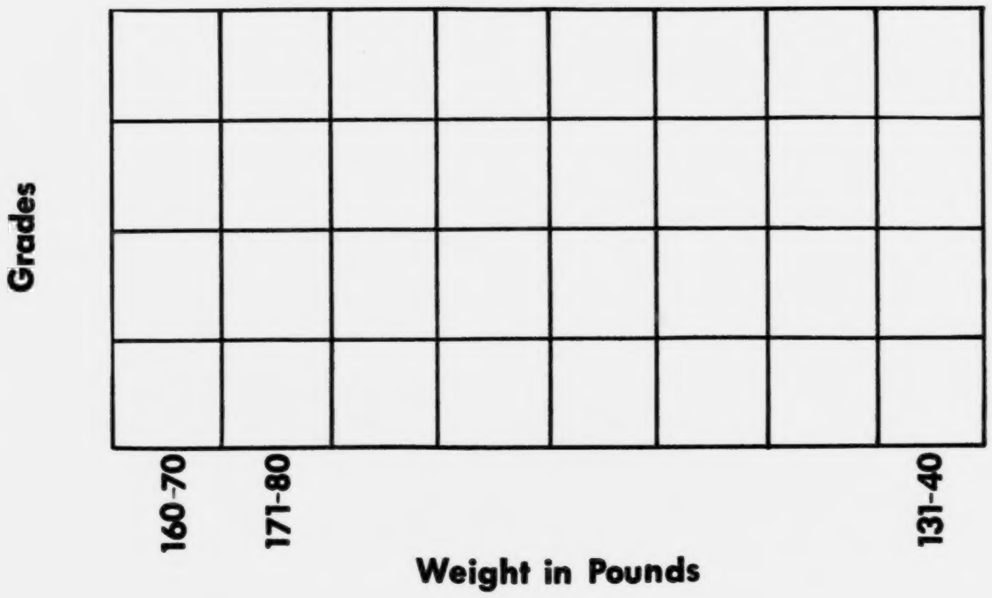
Activity 1—*Continued* Name ____________________

Grades and Weight



Use the grid at the right to make any other comparison that you would like to consider.

[*Continued*]

Activity 1–*Continued* Name ______________________________

Using the data sheet again, complete the frequency tables below.

| | Height | Tally | Frequency |
|---|---|---|---|
| Shortest = | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| Tallest = | | | |

| Weight | Tally | Frequency |
|---|---|---|
| 165–70 | | |
| 171–75 | | |
| 176–80 | | |
| 181–85 | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| 236–40 | | |

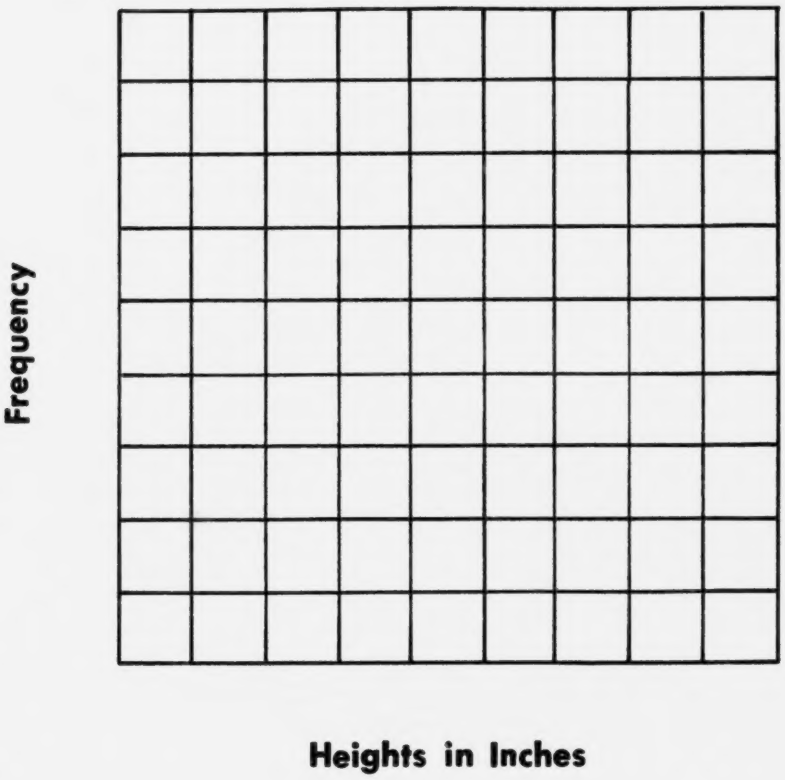
| Grade | Tally | Frequency |
|---|---|---|
| | | |
| | | |
| | | |
| | | |

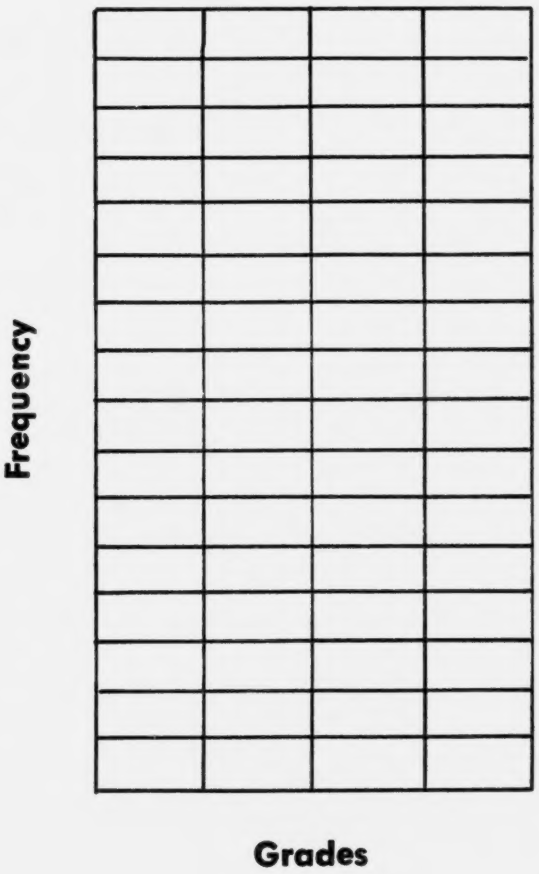
[*Continued*]

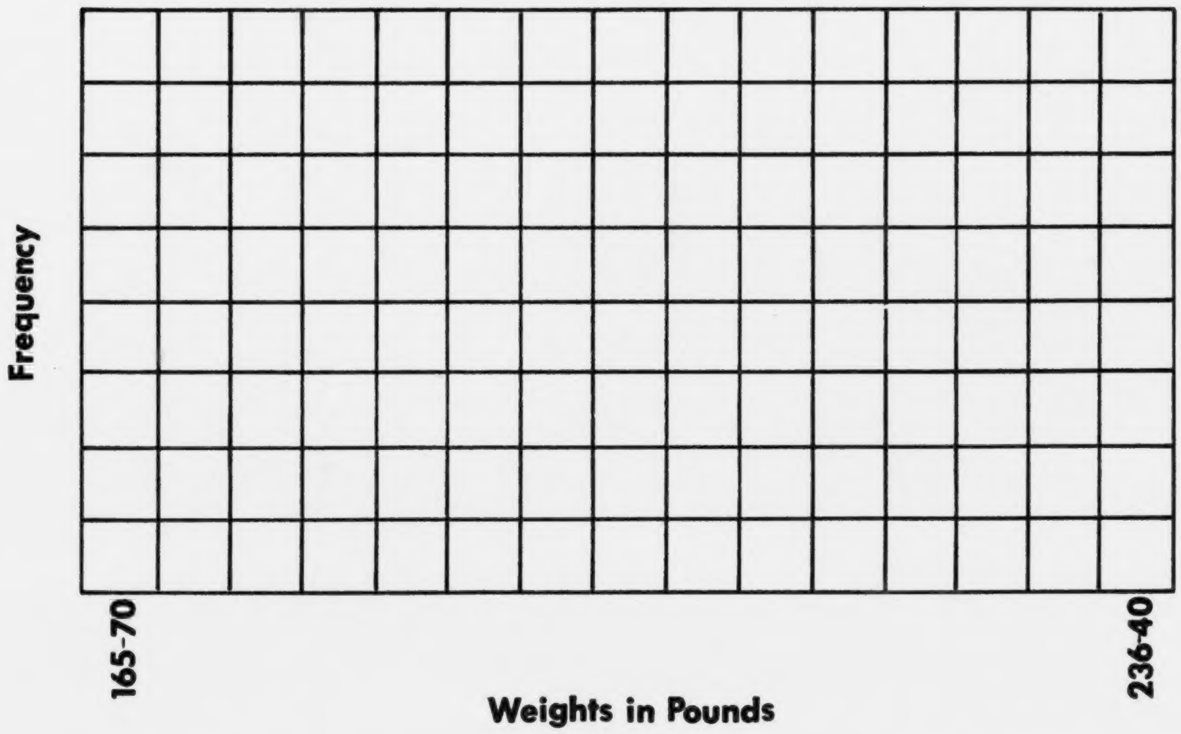
CULMINATING EXPERIENCE UNIT 11, EXPERIENCE 5

Activity 1—*Continued* Name ______________________________

Using the frequency tables you have just made, complete the graphs below.







Activity 2 Name ________________________________

## Summary Sheet

1. What is the height of the tallest player? __________
2. What is the height of the shortest player? __________
3. The most common height is __________.
4. Which player is the lightest? ______
5. Which player is the heaviest? ______
6. From your frequency graph, what would you consider to be an average weight for the team? __________
7. Are these players good students? ______
8. How many have an A average? ______
9. More players have a(n) ______ average than any other.
10. The average for the team is in which range? Circle one.

    A–B B–C C–D D–F
11. What relations did you find?
    - *a*) Does there appear to be any relation between height and weight? ______

      If so, state some specific relation. ________________________________________

      ____________________________________________________________
    - *b*) Does there appear to be any relation between the players' numbers and their weight?

      ______ If so, state some specific relation. ________________________________

      ____________________________________________________________
    - *c*) Is there any relation between grades and weight? ______

      If so, state some specific relation. ________________________________________

      ____________________________________________________________
12. Give a description of the typical player on the team using the information you have obtained about his height, weight, and grades. ________________________________

    ____________________________________________________________

## RECORDING DIGITS AND SUMS OF LAST TWO DIGITS

UNIT 12, EXPERIENCE 1

Activity 1

Name ______________________________

Follow the directions your teacher gives you.

### TRIAL 1

My guess __________

Actual sums ______ ______ ______ ______ ______

______ ______ ______ ______ ______

My guess matched the actual sum __________ times.

### TRIAL 2

My guess __________

Actual sums ______ ______ ______ ______ ______

______ ______ ______ ______ ______

My guess matched the actual sum __________ times.

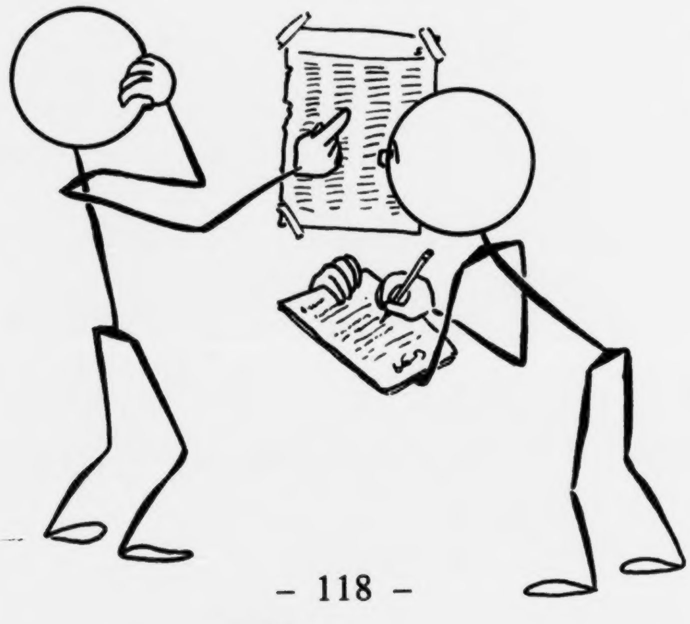
### TRIAL 3

My guess __________

Actual sums ______ ______ ______ ______ ______

______ ______ ______ ______ ______

My guess matched the actual sum __________ times.



## RECORDING DIGITS AND SUMS OF LAST TWO DIGITS

UNIT 12, EXPERIENCE 1

Activity 2

Name ______________________________

Use your page from the telephone directory to answer the questions.

1. Put your finger on the page without looking. Move your finger to the nearest phone number. What are the last 2 digits? _____ and _____.
2. The sum of these digits is _____.
3. Use the 30 phone numbers following this first one. On the chart below, make a tally mark to show each sum you get by adding the last 2 digits of each phone number. When you finish, add up your tally marks and record the totals in the last column.

| Sum of Last Two Digits | Tally | Total |
|---|---|---|
| 0 | | |
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |
| 9 | | |
| 10 | | |
| 11 | | |
| 12 | | |
| 13 | | |
| 14 | | |
| 15 | | |
| 16 | | |
| 17 | | |
| 18 | | |

4. Why does the table include 0? ______________________________
5. Why is 18 the largest sum included in the table? ______________________________

   ______________________________
6. Write six pairs of digits which have the sum 5. (Count 4, 1 and 1, 4 as two different pairs.) ______________________________

## RECORDING DIGITS AND SUMS OF LAST TWO DIGITS

Activity 3

Name ______________________________

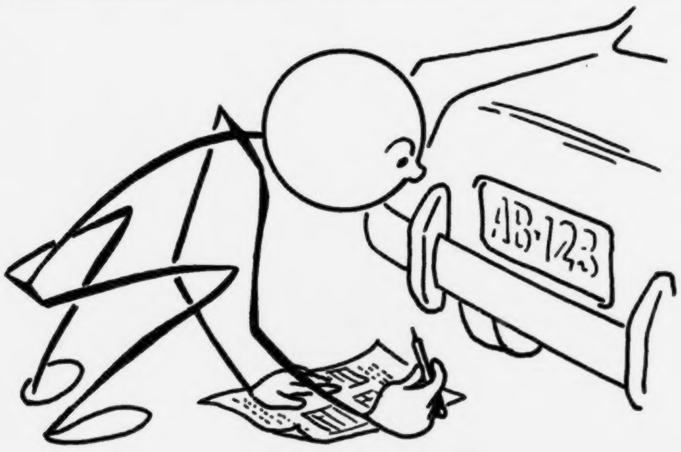
1. Record the license-plate numbers of 10 cars. Disregard letters—write down only the numerals.

___________ ___________ ___________ ___________ ___________

___________ ___________ ___________ ___________ ___________

Now tally the last digits and the sums of the last 2 digits.

| Last Digit | Tally of Times You Got This Digit |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |



| Sum of Last Two Digits | Tally of Times You Got This Sum |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| 13 | |
| 14 | |
| 15 | |
| 16 | |
| 17 | |
| 18 | |

2. Is there a best guess for the last digit of a license-plate number?

______ yes ______ no

3. Is there a best guess for the sum of the last 2 digits of that number?

______ yes ______ no

4. Which sum do you think would occur most often if you looked at thousands of license plates? ______

Activity 1 Name ______________________________

What happens when you flip a coin 20 times? Does the same side always land up? Try it and see. Flip your coin. Notice that one side is "heads" and the other is "tails." If heads turns up, put a tally mark in the H row below. If tails turns up, put a tally mark in the T row. Make 20 tosses, then fill in the totals.

| Side | Tally | Total |
|---|---|---|
| H | | |
| T | | |

Which side came up more often? ______ Which side do you think will turn up more often if you make 30 more flips? ______

Now flip the coin 30 more times and record your results below.

| Side | Tally | Total |
|---|---|---|
| H | | |
| T | | |

Which side came up more often this time? ______ Look at the totals from both sets of flips. If you add all the heads together and all the tails together, which turns up more often? ______

Now exchange coins with a neighbor or your teacher and flip the new coin 30 times. Record your results below.

| Side | Tally | Total |
|---|---|---|
| H | | |
| T | | |

Which side came up more often this time? ______

If you have time, make another chart and try someone else's coin. In looking over all your results, does it appear that one side comes up more often than the other? ______

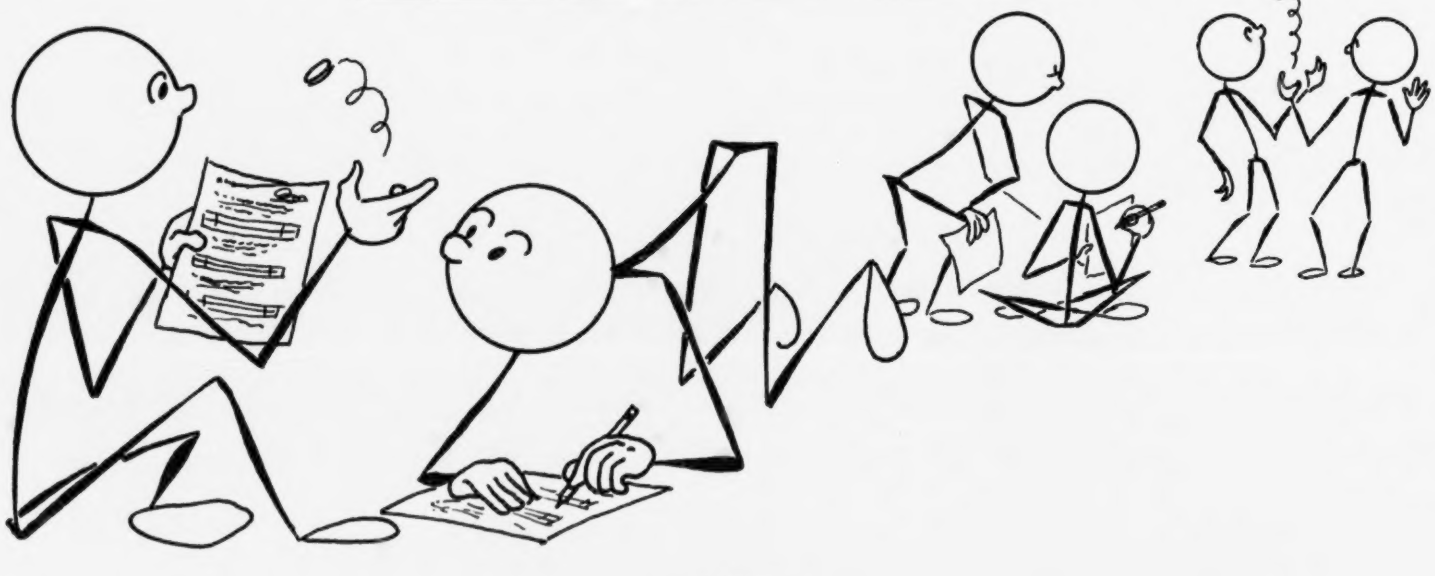
Activity 2 Name ______________________________

What happens when you flip 2 coins at the same time? Try it and see. As you complete each flip of the 2 coins, tally your result below. Do it 40 times.

| The Coins Fall | Tally | Total |
|---|---|---|
| H,H | | |
| H,T or T,H | | |
| T,T | | |

1. Which way did the coins fall most often? __________
2. Why do you think they fell this way most often? ______________________________
______________________________________________
3. Was there much difference in the number of times you got H,H and the number of times you got T,T? _____
4. How do you explain this? ______________________________
______________________________________________
5. Enter below the results you might expect to get if you tossed these 2 coins 4,000 times.

| The Coins Fall | Total |
|---|---|
| H,H | |
| H,T or T,H | |
| T,T | |



ROLLING DICE UNIT 12, EXPERIENCE 3

Activity 1 Name ______________________________

Throw the die. What number faces up? ______ Throw it again. What number came up this time? ______ What other numbers could come up? ______________________

EXPERIMENT 1: Try to find out whether one number turns up on your die more often than any other number. Do you think one will? ______ If so, which one do you expect to turn up most often? ______

The table lists all the ways a die can turn up. Every time you throw the die, tally the number that turns up in the appropriate row. Keep throwing the die until one number has 10 tally marks after it. Then find your total for each number.

| Face of Die | Tally | Total |
|---|---|---|
| ⚀ | | |
| ⚁ | | |
| ⚂ | | |
| ⚃ | | |
| ⚄ | | |
| ⚅ | | |

EXPERIMENT 2: Let's do the same thing again. Toss the die until one number comes up 10 times. Do you think it will be the same number as before? ______

| Face of Die | Tally | Total |
|---|---|---|
| ⚀ | | |
| ⚁ | | |
| ⚂ | | |
| ⚃ | | |
| ⚄ | | |
| ⚅ | | |

[*Continued*]

Activity 1–*Continued* Name ______________________________

EXPERIMENT 3: Exchange dice with your neighbor or your teacher and see what results you get with the new die. Do this experiment as you did the others, tossing the die until one number has come up 10 times.

| Face of Die | Tally | Total |
|---|---|---|
| ⚀ | | |
| ⚁ | | |
| ⚂ | | |
| ⚃ | | |
| ⚄ | | |
| ⚅ | | |

Now take the information from each of your 3 charts and write the totals in the chart below. Then get the grand totals by adding the 3 totals for each face.

| Face of Die | Total #1 | Total #2 | Total #3 | Grand Total (#1+#2+#3) |
|---|---|---|---|---|
| ⚀ | | | | |
| ⚁ | | | | |
| ⚂ | | | | |
| ⚃ | | | | |
| ⚄ | | | | |
| ⚅ | | | | |

Use the table you have just filled in to answer these questions.

1. Did each of the 6 numbers come up at least once in each experiment? ______
2. Look at the grand totals. Did any number turn up twice as often as another one? ______
3. If you were to throw your die once more, would you know what number to expect to turn up? ______

ROLLING DICE UNIT 12, EXPERIENCE 3

Activity 2 Name ________________________________

Shake the dice and roll them. What sum do you get on the two dice? ______ Shake them again and roll them. What sum did you get this time? ______ Roll the dice 100 times. Record your sums in the box below. When the same sum comes up more than once, use tally marks to show the number of times it comes up. Keep track of your throws by marking off, in the block on the right, each number as you throw. Put a line through one way for the first 50 throws and the other way for the second 50 throws. For example, on toss 1 do this: $\cancel{1}$; on toss 51 do this: $\xcancel{1}$.

| Sums on Dice | | |
|---|---|---|
| | | |
| | | |
| | | |
| | | |

| Number of Throws | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |

In the chart below, arrange your sums from smallest to largest. After each sum put the total number of times it came up in your experiment. Did you miss any possible sums in your first 100 tosses? If so, put them in their proper place in the chart and record the number of times as 0.

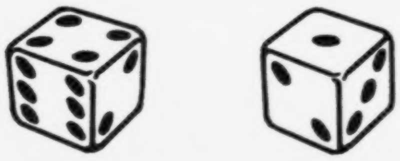
| Sums | Times Found in 100 Tosses | Tally | New Total |
|---|---|---|---|
| Smallest: | | | |
| | | | |
| Largest: | | | |

[*Continued*]

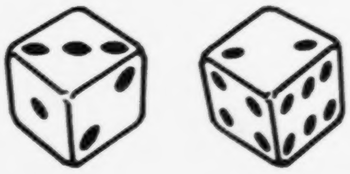
Activity 2–*Continued* Name ______________________________

Look back at your chart for this activity. What sum(s) did you get most often? ______________ How many times did you get it (them)? ______________________ What sum(s) did you get least often or not at all? ______________

Now let's try something a little different. Try to throw sums of five:

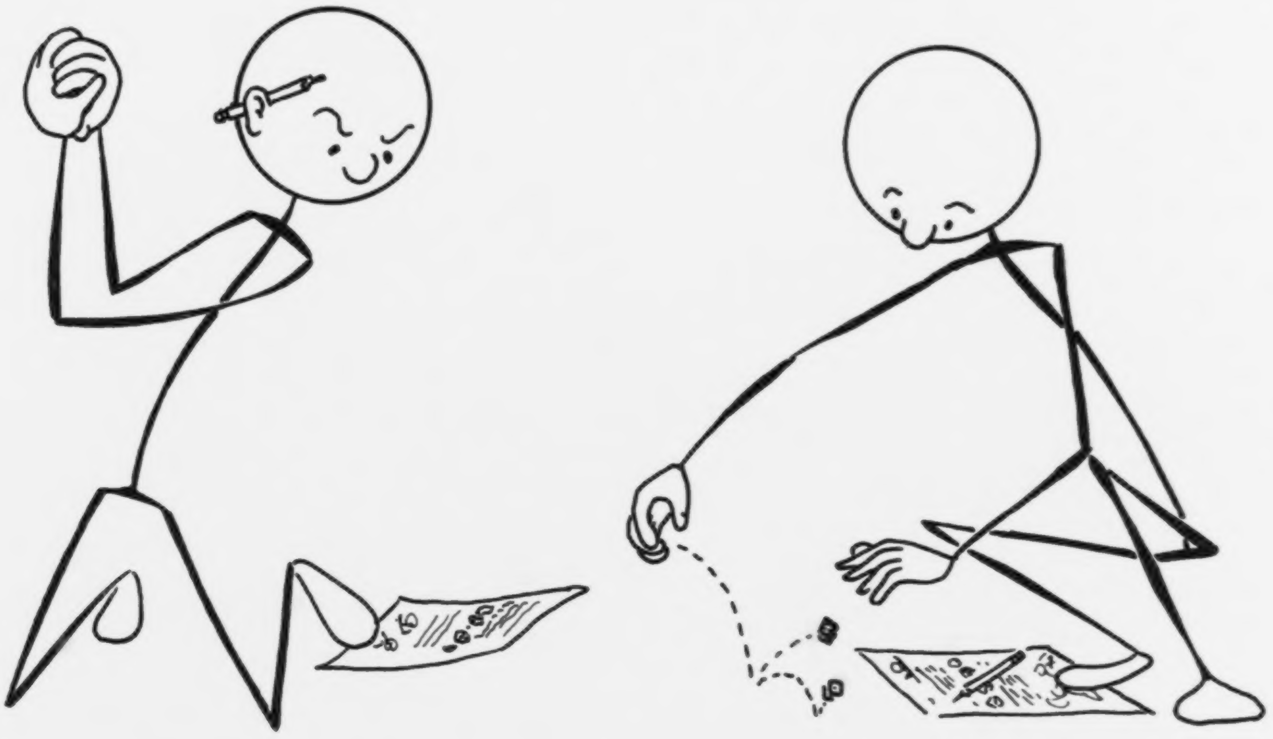


**or**



Keep throwing the dice until you have thrown a sum of five 10 times. In the tally column in the chart keep a record of all the sums that turn up while you are trying to throw 10 sums of five.

Now look at the tally column. Which sum has the most tally marks after it? _____ Which sum has the fewest tally marks after it? _____ What sum do you think is your "lucky sum"? _____ How many times do you think you'd have to throw to get it? _____ Try. How many throws did it take? _____ Do you think it would take that long to get it again? _____ Try it and see! How many throws did it take? _____



ROLLING DICE UNIT 12, EXPERIENCE 3

Activity 3 Name ______________________________

As you have seen, some sums seem to occur more often than others. Study your table from Activity 2. Use the first column of the table below to arrange the sums in order from those you found most often to those you found least often. Then throw the dice another 50 times and tally your results in the middle column. (Do more throws if you have time.) Then find your total for each sum.

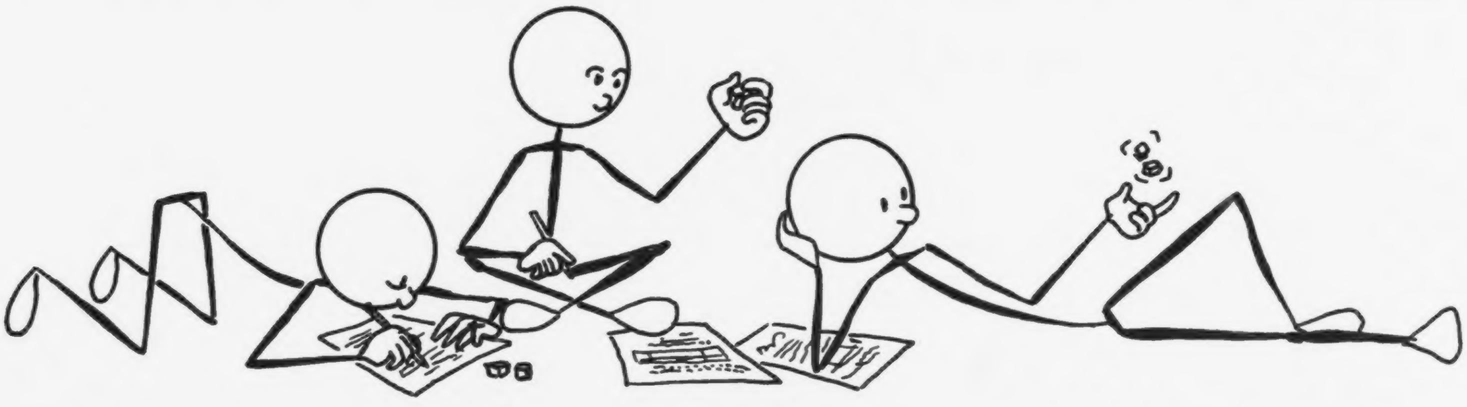
| Sum on Dice | Tally | Total |
|---|---|---|
| Most often: | | |
| Least often: | | |

Do your sums seem to be in the right order? ______ What changes would you make in the order? ______________________________

Do you think that if you threw your dice a few more times, your results might change? ______

Look at the final totals in your tables for Activities 2 and 3 and then answer these questions.

1. Which sum occurred most often? ______
2. Which sum occurred least often? ______
3. What sums seemed to come up about the same number of times? ______________
4. The next time you roll a pair of dice, are you more likely to get a sum of 3 or a sum of 8? ______ Why? ______________________________



## DRAWING BEADS

## UNIT 12, EXPERIENCE 4

Activity 1

Name ______________________________

Draw 3 beads out of your can. What colors are they? ____________________________

Put them back in, stir the beads around with your finger, and draw 3 more. What colors are they? ______________________________ Let's use letters to stand for the colors. If you get 2 reds and 1 green, write RRG. If you get a red, a blue, and a green, write RBG. Continue drawing 3 beads at a time without looking into the can. Record below the colors you get each time. Then put the beads back into the can, stir them up, and draw 3 more. Make 20 draws.

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________
6. ____________
7. ____________
8. ____________
9. ____________
10. ____________
11. ____________
12. ____________
13. ____________
14. ____________
15. ____________
16. ____________
17. ____________
18. ____________
19. ____________
20. ____________

Now make a list of all the *different* color combinations you have gotten so far. Also write down the number of times you got each combination.

| Color Combination | Tally of Times You Got the Combination | Total |
|---|---|---|
| | | |

Now make another 20 draws using the same method and record your findings below.

21. ____________
22. ____________
23. ____________
24. ____________
25. ____________
26. ____________
27. ____________
28. ____________
29. ____________
30. ____________
31. ____________
32. ____________
33. ____________
34. ____________
35. ____________
36. ____________
37. ____________
38. ____________
39. ____________
40. ____________

[*Continued*]

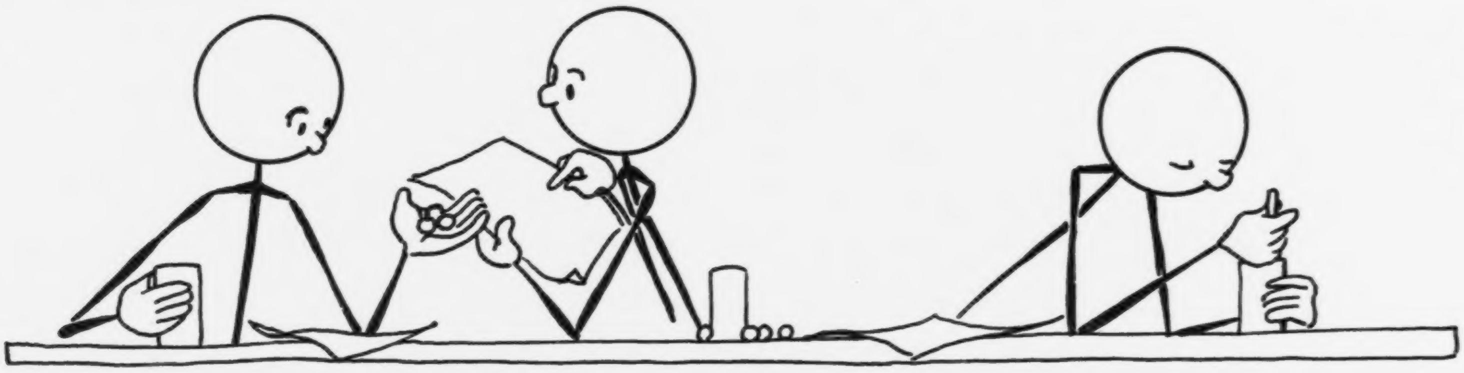
Activity 1–*Continued* Name ______________________________

Did any combinations come up that you didn't find in your first 20 drawings? ______
If so, what are they? ____________________ Add these to the chart on the first page of this activity. Also put in tally marks, so that you can see how many times you got each combination.

Make 20 more draws and record them below.

41. ____________
42. ____________
43. ____________
44. ____________
45. ____________
46. ____________
47. ____________
48. ____________
49. ____________
50. ____________
51. ____________
52. ____________
53. ____________
54. ____________
55. ____________
56. ____________
57. ____________
58. ____________
59. ____________
60. ____________

Did you get any new combinations this time? ______ If so, what are they? ______________ ____________________ Add them to your chart and place tally marks after each one for as many times as you got it in draws 41–60.



Use your chart to answer these questions.

1. What color combination(s) did you draw most often? ____________________
2. What color combination(s) did you draw least often? ____________________

Answer these questions after you compare your findings with those of your classmates.

1. Did you draw all the possible combinations? ____________________
2. If not, which ones are you missing? ____________________
3. If you were to draw 3 beads at random from the can, what color(s) would you expect them to be? ____________________

## DRAWING BEADS

UNIT 12, EXPERIENCE 4

Activity 2

Name ______________________________

Name ______________________________

The can contains 36 beads. There is a different number of beads for each color. For example, there could be 18 red, 10 blue, and 8 green beads. Take turns making trial drawings and try to decide the color of the greatest number of beads and the color of the least number of beads. (No fair counting first!)

Draw 3 beads. What colors are they? ______________________________ Put them back in. Do you think that one drawing is enough to let you decide the color of the greatest number of beads? ______

Make 10 draws of 3 beads each, returning the beads before each new drawing. List the color combinations that you get.

1. __________ 3. __________ 5. __________ 7. __________ 9. __________

2. __________ 4. __________ 6. __________ 8. __________ 10. __________

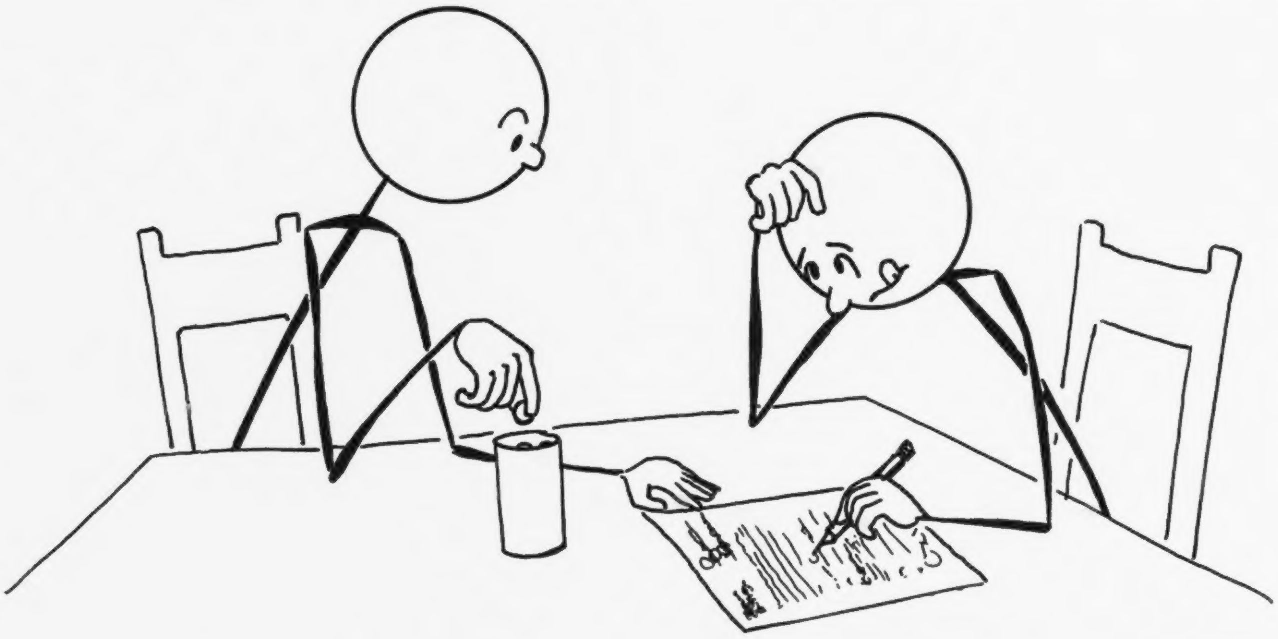
What color do you think most of the beads are? ______________

CONCLUSION 1: There are fewer ______________ (color) beads and more ______________ (color) beads than any other color.

Make 10 more draws of 3 beads at a time and see if these results make you want to change Conclusion 1.

1. __________ 3. __________ 5. __________ 7. __________ 9. __________

2. __________ 4. __________ 6. __________ 8. __________ 10. __________



[*Continued*]

Activity 2–*Continued*

Name ____________________

Name ____________________

Look at your results so far. Then complete Conclusion 2.

CONCLUSION 2: We think that there are fewer ____________ beads and more ____________ beads than any other color.

Now make one more check.

| | | | | |
|---|---|---|---|---|
| 1. ________ | 3. ________ | 5. ________ | 7. ________ | 9. ________ |
| 2. ________ | 4. ________ | 6. ________ | 8. ________ | 10. ________ |

CONCLUSION 3: Looking at all 30 drawings, we think there are fewer ____________ beads and more ____________ beads than any other color.

If Conclusions 1, 2, and 3 agree, or if 2 and 3 agree, record your Final Decision below and ask your teacher for another can to decode. If Conclusions 2 and 3 do not agree, make at least 20 more drawings before making your final decision.

| | | | | |
|---|---|---|---|---|
| 1. ________ | 5. ________ | 9. ________ | 13. ________ | 17. ________ |
| 2. ________ | 6. ________ | 10. ________ | 14. ________ | 18. ________ |
| 3. ________ | 7. ________ | 11. ________ | 15. ________ | 19. ________ |
| 4. ________ | 8. ________ | 12. ________ | 16. ________ | 20. ________ |

FINAL DECISION: The color of the greatest number of beads is ______________.

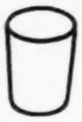
The color of the fewest number of beads is ______________.

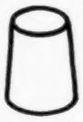
(We used the can labeled _____ to make our decision.)

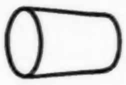
TOSSING OBJECTS UNIT 12, EXPERIENCE 5

Activity 1 Name ______________________________

Which way do you think a paper or plastic cup will land if you just toss it into the air and let it fall? Put an X under the position you think it will land in most often.


_________


_________


_________

Put a 0 under the way you think it will land least often.

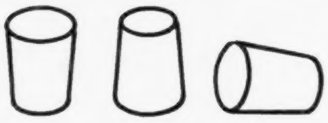
Now toss your cup a few times and see what happens. Make a tally mark in one of the columns to record the way it lands each time. Toss the cup 20 times.

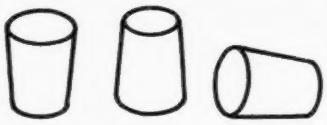
| **Bottom** | **Top** | **Side** |
|---|---|---|
| | | |

Would you like to make a prediction now? If so, circle your prediction.

**Most Likely** **Least Likely**

PREDICTION 1:





Make another 20 trials and record your results here.

| **Bottom** | **Top** | **Side** |
|---|---|---|
| | | |

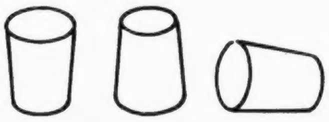
[*Continued*]

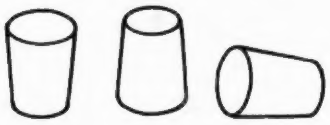
Activity 1—*Continued* Name ____________________

Study the information from both experiments and then make your second prediction.

**Most Likely** **Least Likely**

PREDICTION 2:





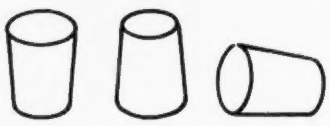
Make some more tosses. Make at least 20, but more if you have time.

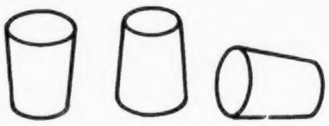
| **Bottom** | **Top** | **Side** |
|---|---|---|
| | | |

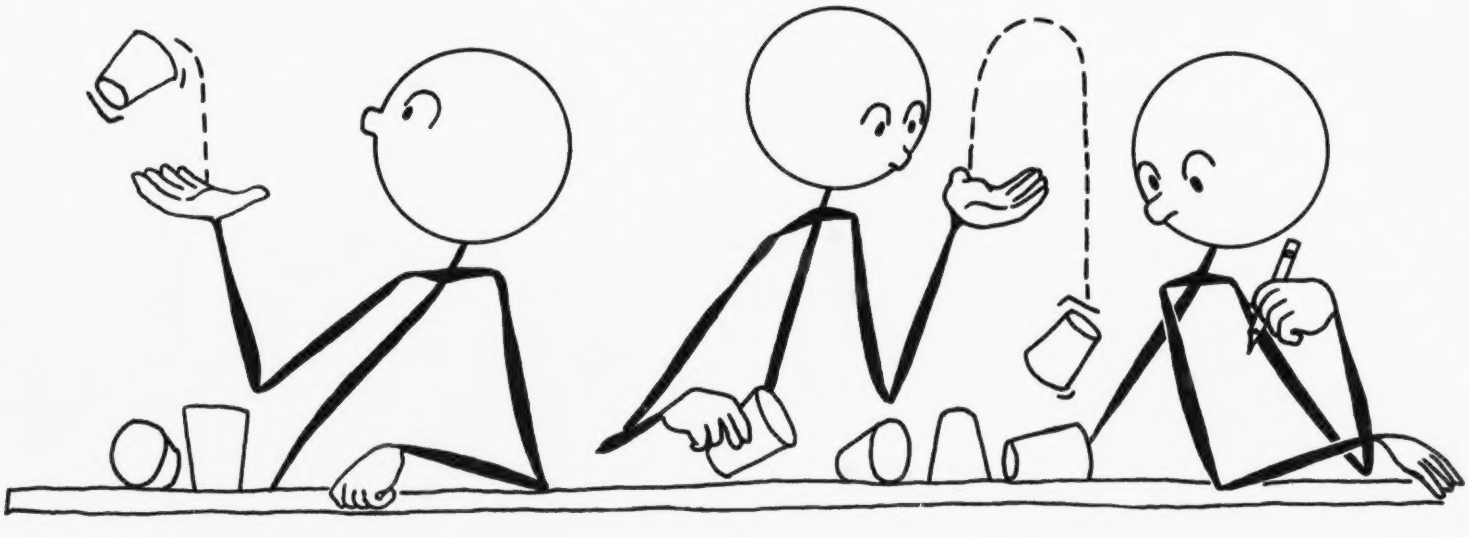
FINAL PREDICTION: I have made __________ tosses of the cup.

On the basis of these tosses, I have decided the following.

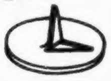
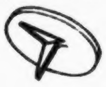
**Most Likely** **Least Likely**







Activity 2 Name ______________________________

Have you ever tossed a thumbtack in the air and had it land point down, sticking into the floor? __________ It doesn't happen very often, does it? How do you think a thumbtack will usually land, like this  or like this  ? Circle one.

EXPERIMENT 1: Toss the tack 25 times. Show the way that it lands each time by placing a tally mark in the correct column of row 1.

| **Experiment** | **Lands Up** | **Lands on Side** | **Lands Down** |
|---|---|---|---|
| **1** | | | |
| **2** | | | |
| **3** | | | |

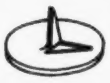
Which way did it land most often? ____________________

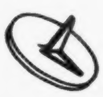
EXPERIMENT 2: Make another 25 tosses and record them in row 2. Then look at the results of your first 50 tosses. Which way has the tack landed most often now? ____________________ Did it come up one way twice as often as any other way? ______ If so, which way? ____________________ Do you think you can predict the way that this tack will usually land? ______ If so, which way? ____________________

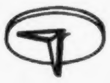
EXPERIMENT 3: If you didn't make a prediction after Experiment 2, do another 50 tosses and record the results in row 3.

If you decided to make a prediction after Experiment 2, check it by doing the 50 tosses.

FINAL CONCLUSION: I have come to the following conclusion based on my tosses. The most likely way for a thumbtack to land is:







My data indicate that this thumbtack is no more likely to land one way than another. ______

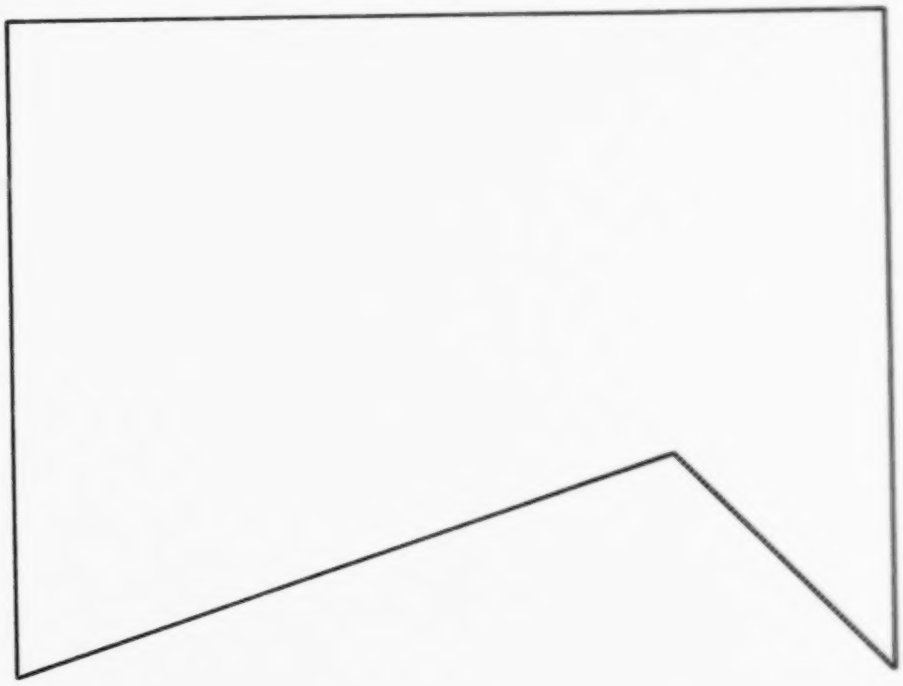
Would a different thumbtack make any difference? ______

Activity 1

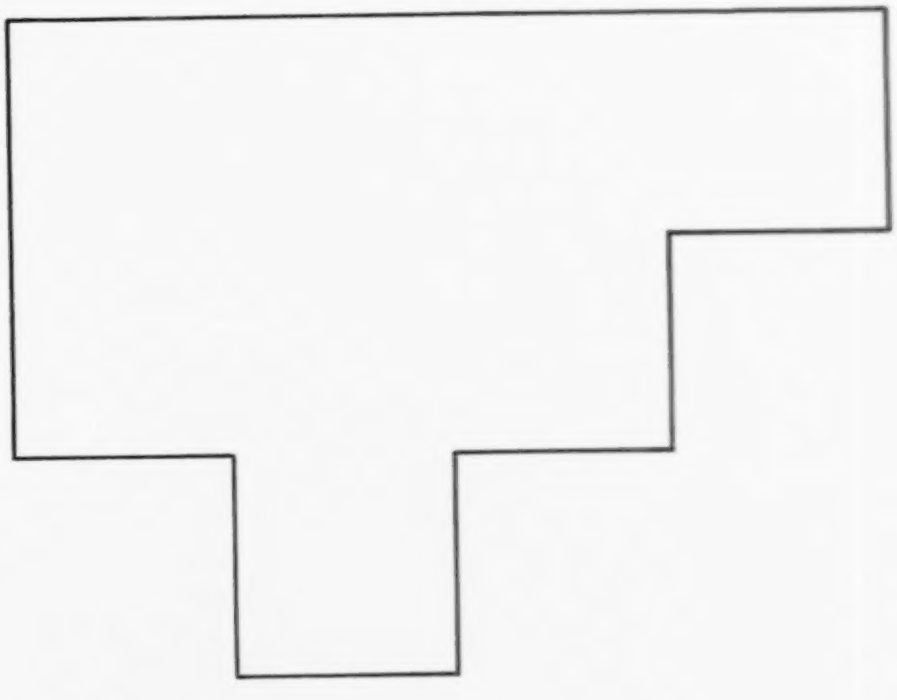
Name ______________________________

Practice making some shapes with a rubber band on your geoboard, then copy on the geoboard each of the pictures shown on these pages:

1



2



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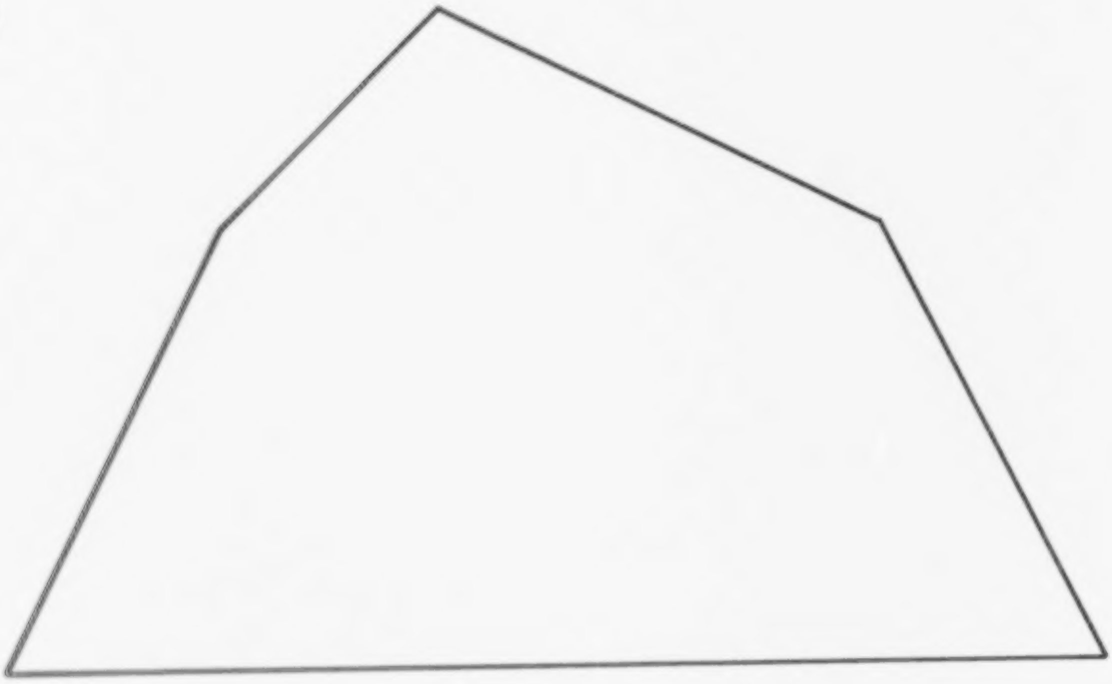
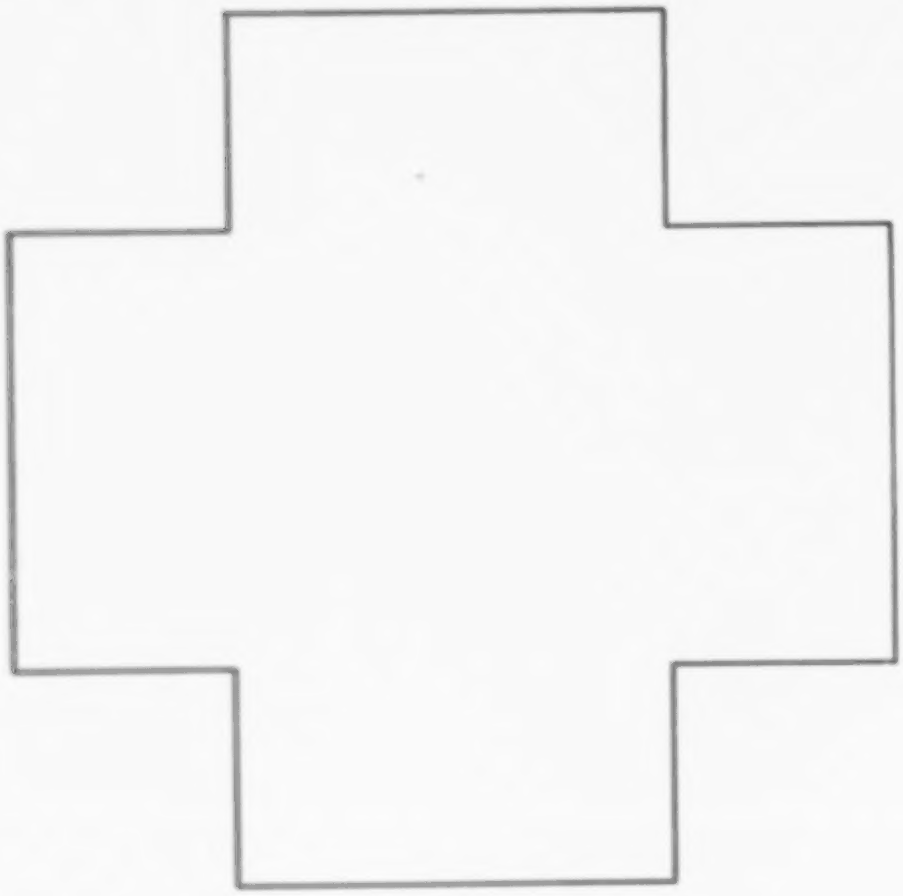
Activity 1–*Continued* Name ______________________________

**3**



**4**



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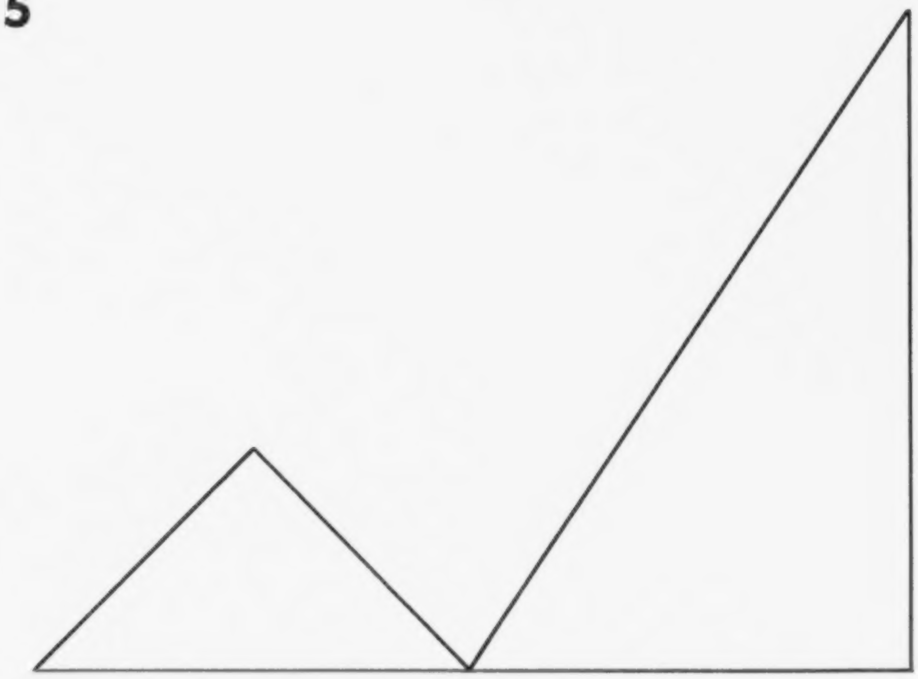
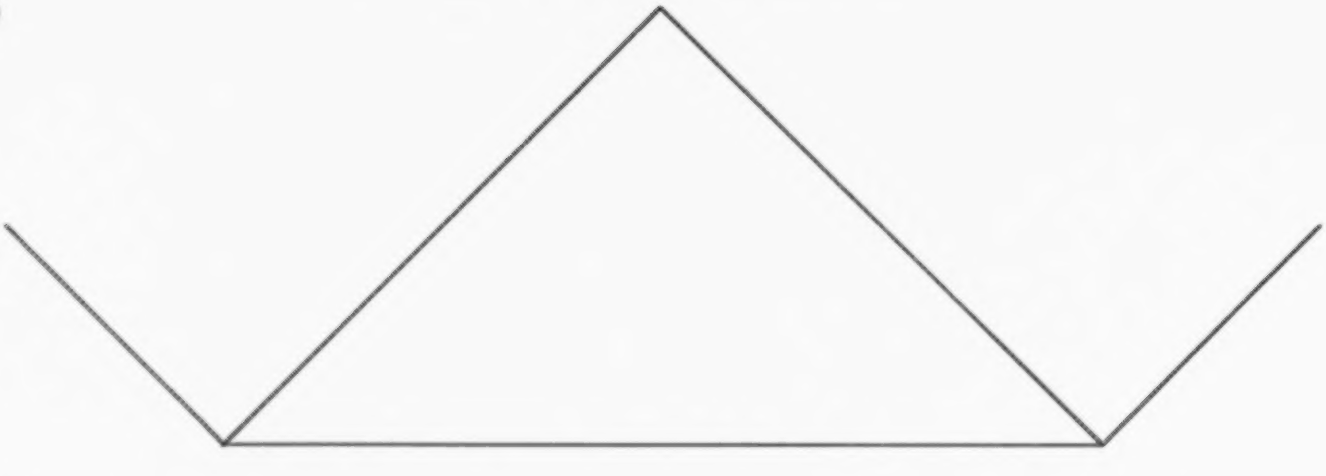
Activity 1–*Continued*

Name ______________________________

**5**



**6**



Now draw pictures of three other shapes you can make on the geoboard. Show them to your teacher.

## SHAPES LIKE A RUBBER BAND

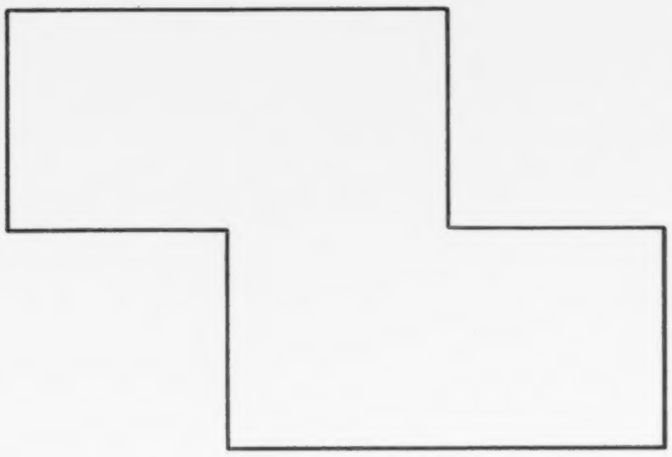
UNIT 13, EXPERIENCE 1

Activity 2

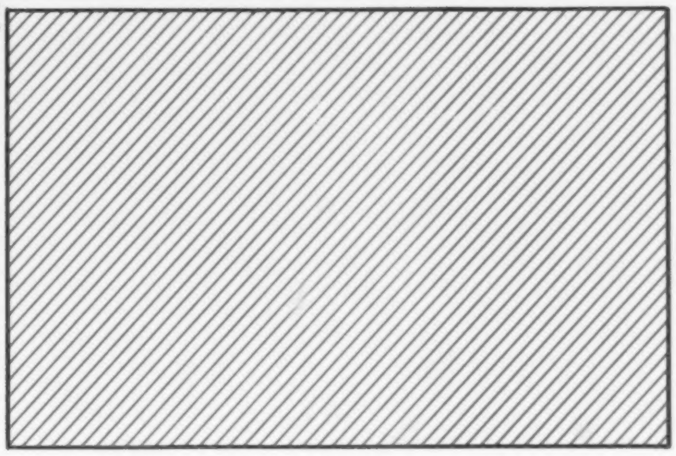
Name ______________________________

Two kinds of pictures are shown here. Not all of them can be copied on the geoboard. Write "Can" beside each picture you can copy on the geoboard.

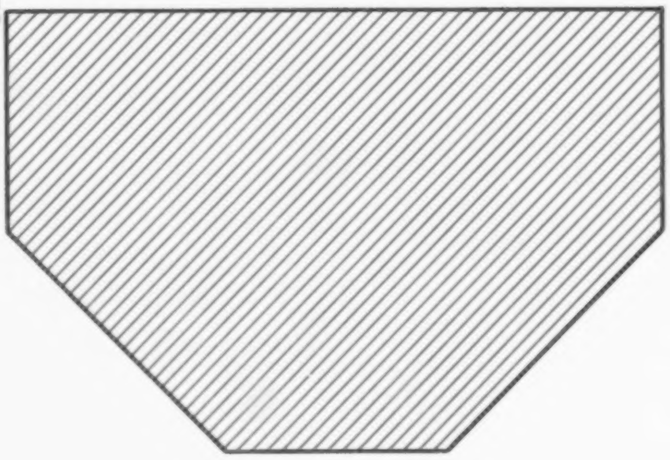
**1**



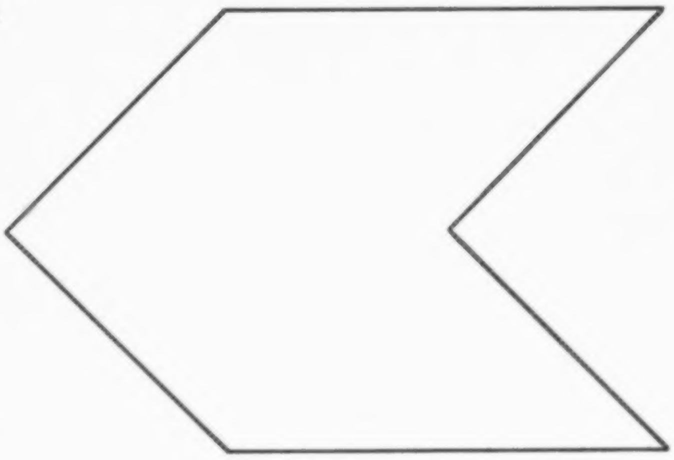
**2**



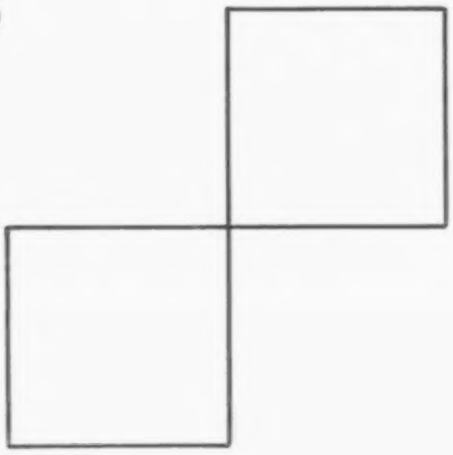
**3**



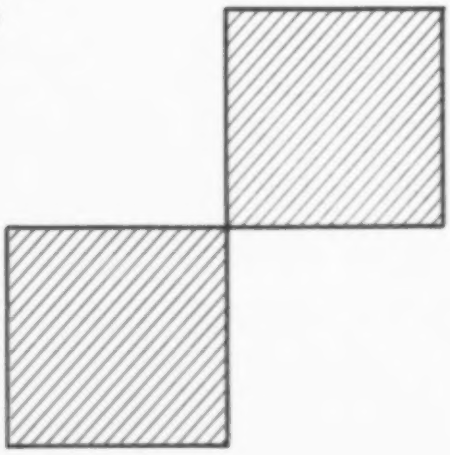
**4**



**5**



**6**

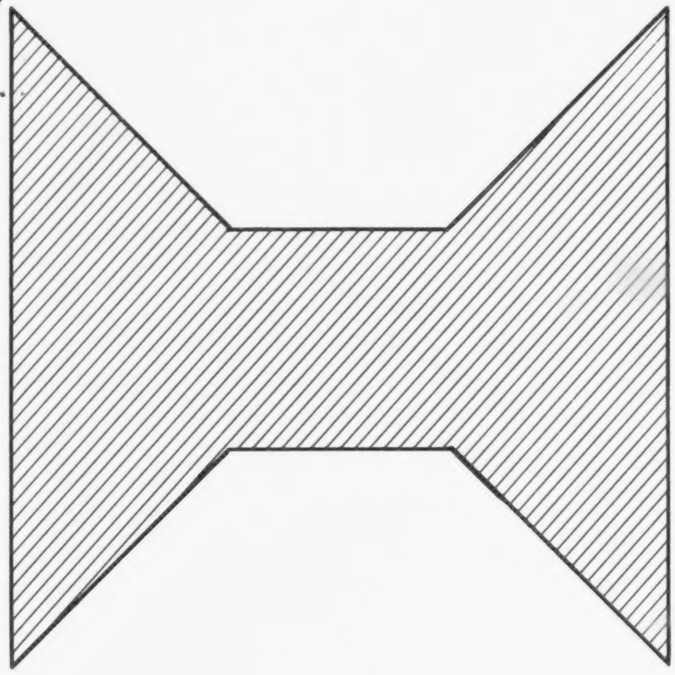


*[Continued]*

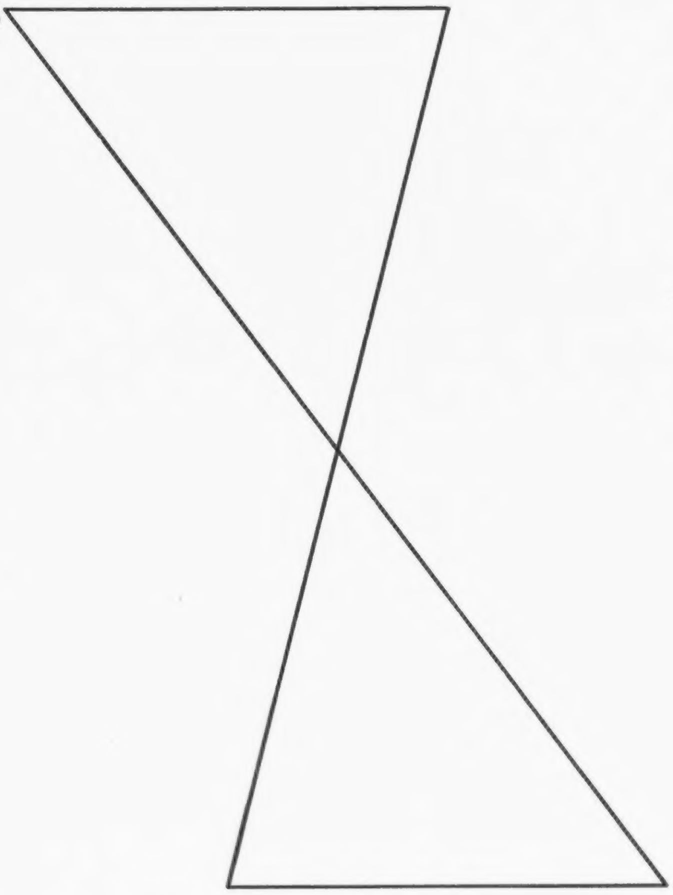
Activity 2–*Continued*

Name ________________________________

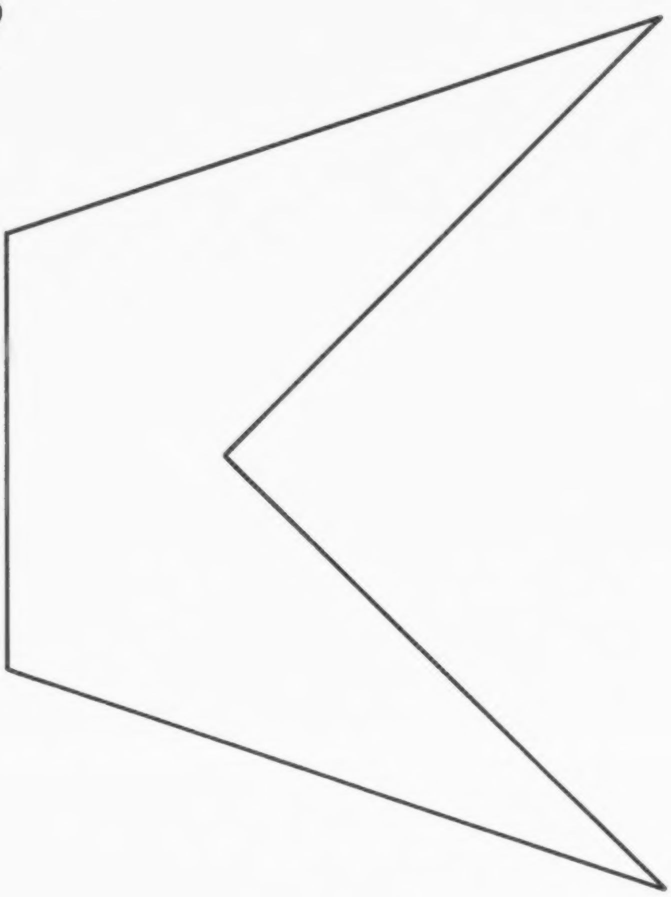
**7**



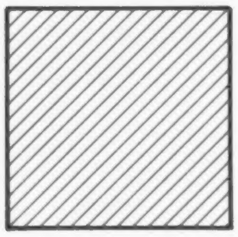
**8**



**9**



**10**



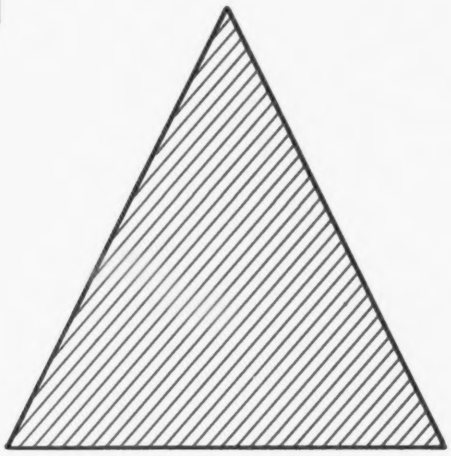
Now have the teacher check your work.

SHAPES LIKE A RUBBER BAND UNIT 13, EXPERIENCE 1

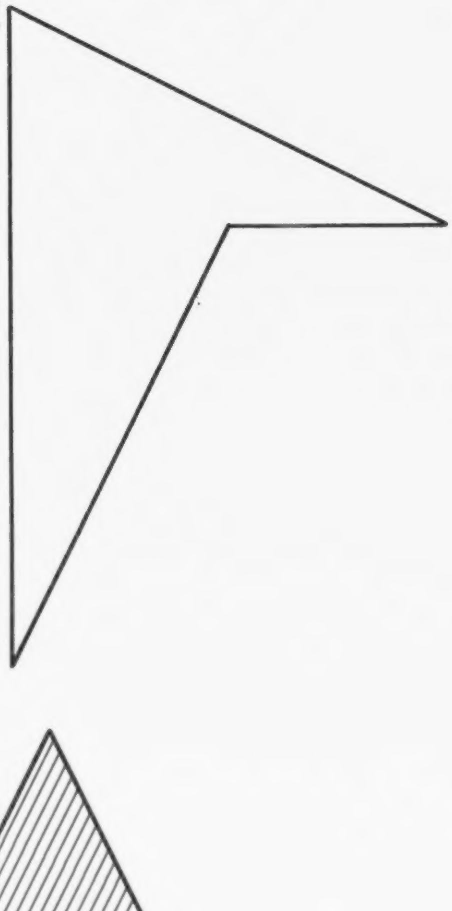
Activity 3 Name ______________________________

Write "G" beside each picture that you can make on the geoboard. When you have finished, ask your teacher to check your work.

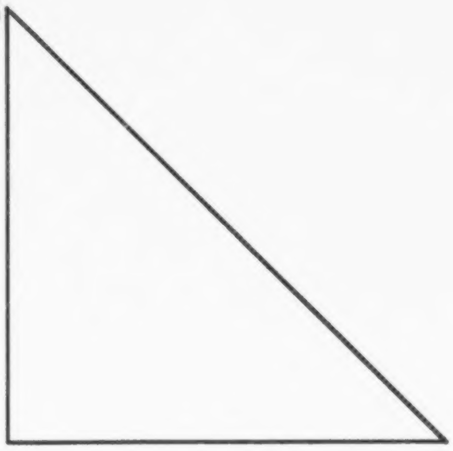
**1**



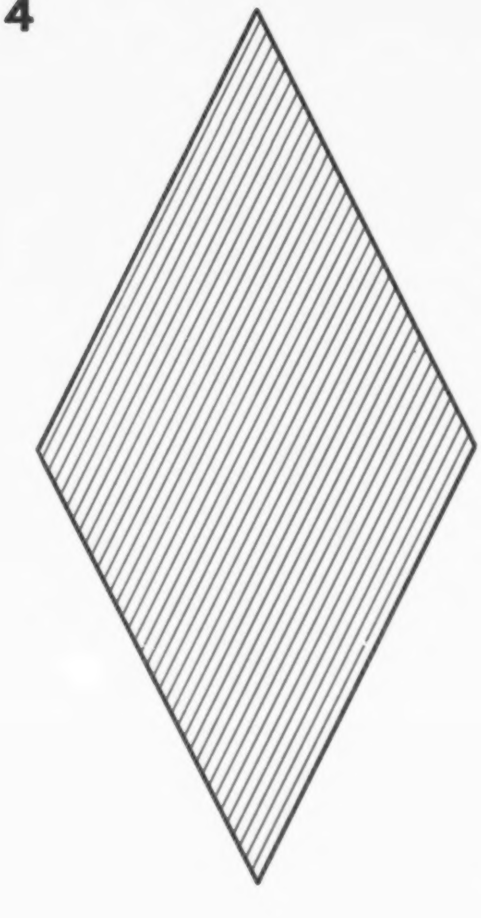
**2**



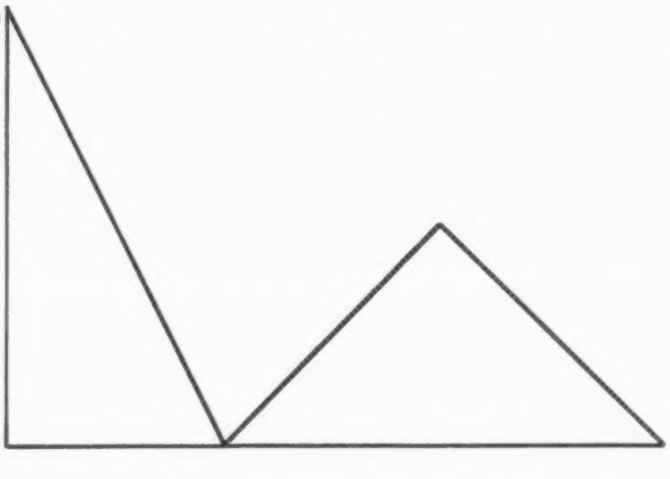
**3**



**4**



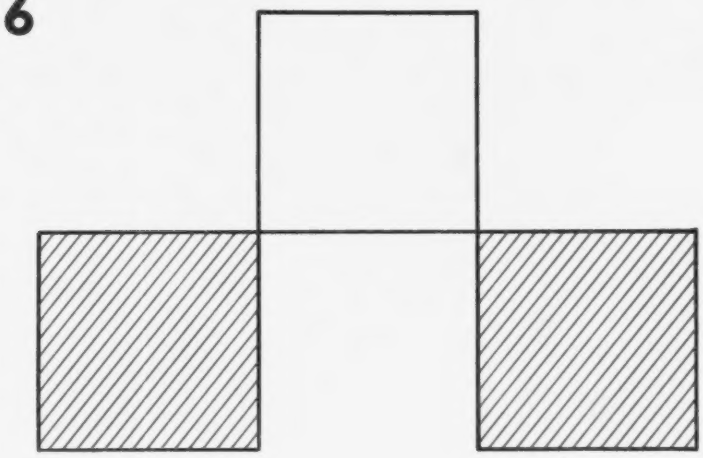
**5**



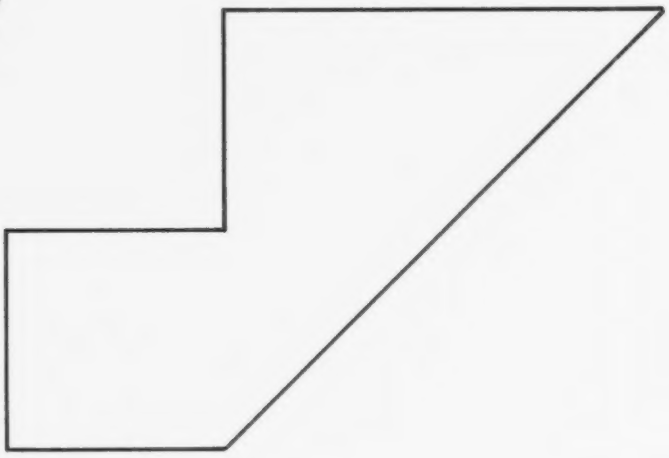
[*Continued*]

Activity 3–*Continued* Name ________________________

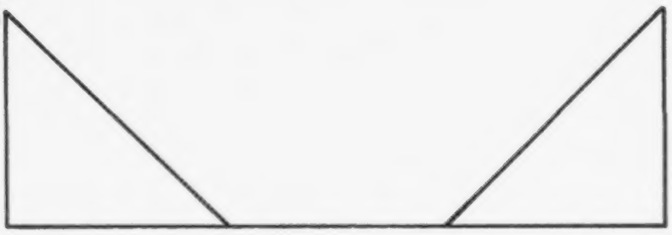
**6**



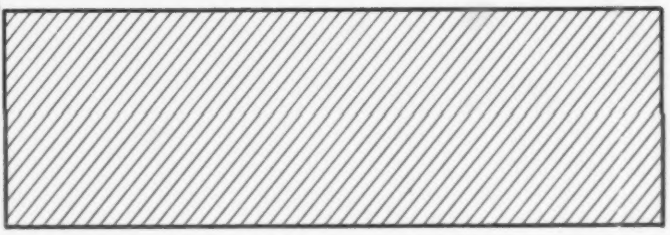
**7**



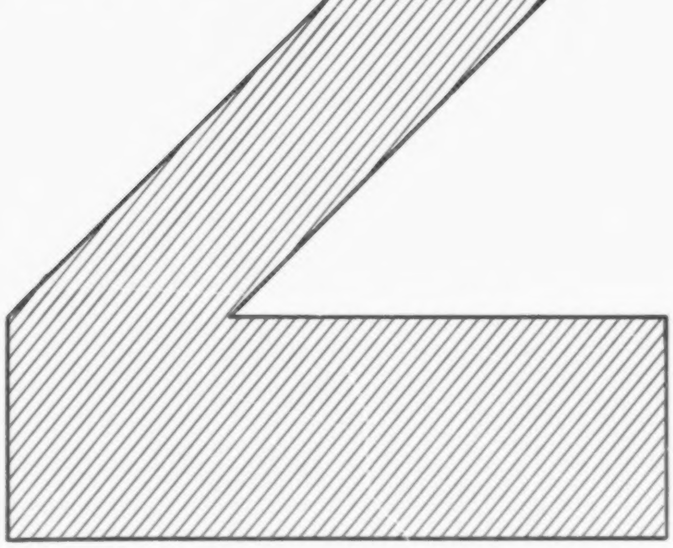
**8**



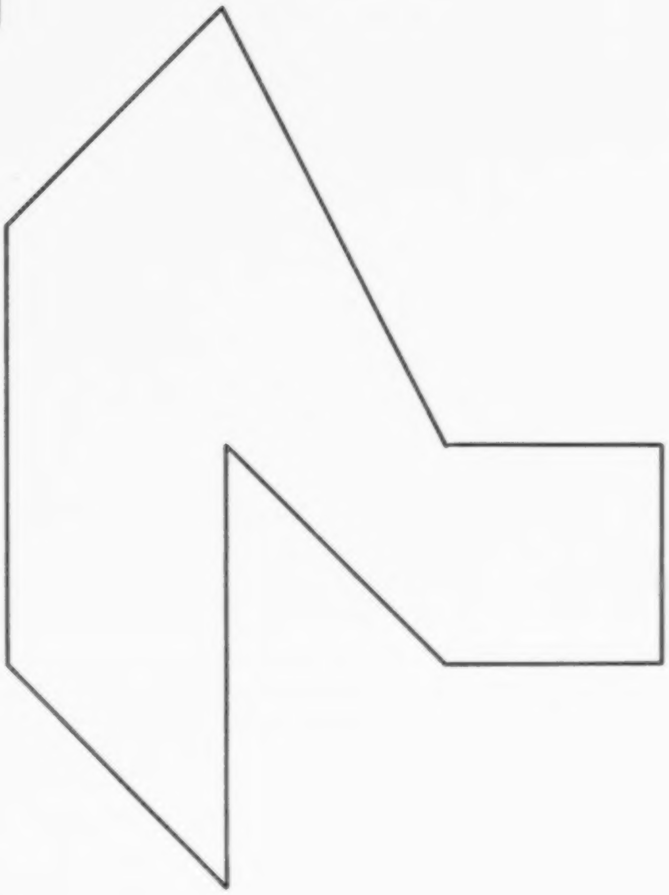
**9**



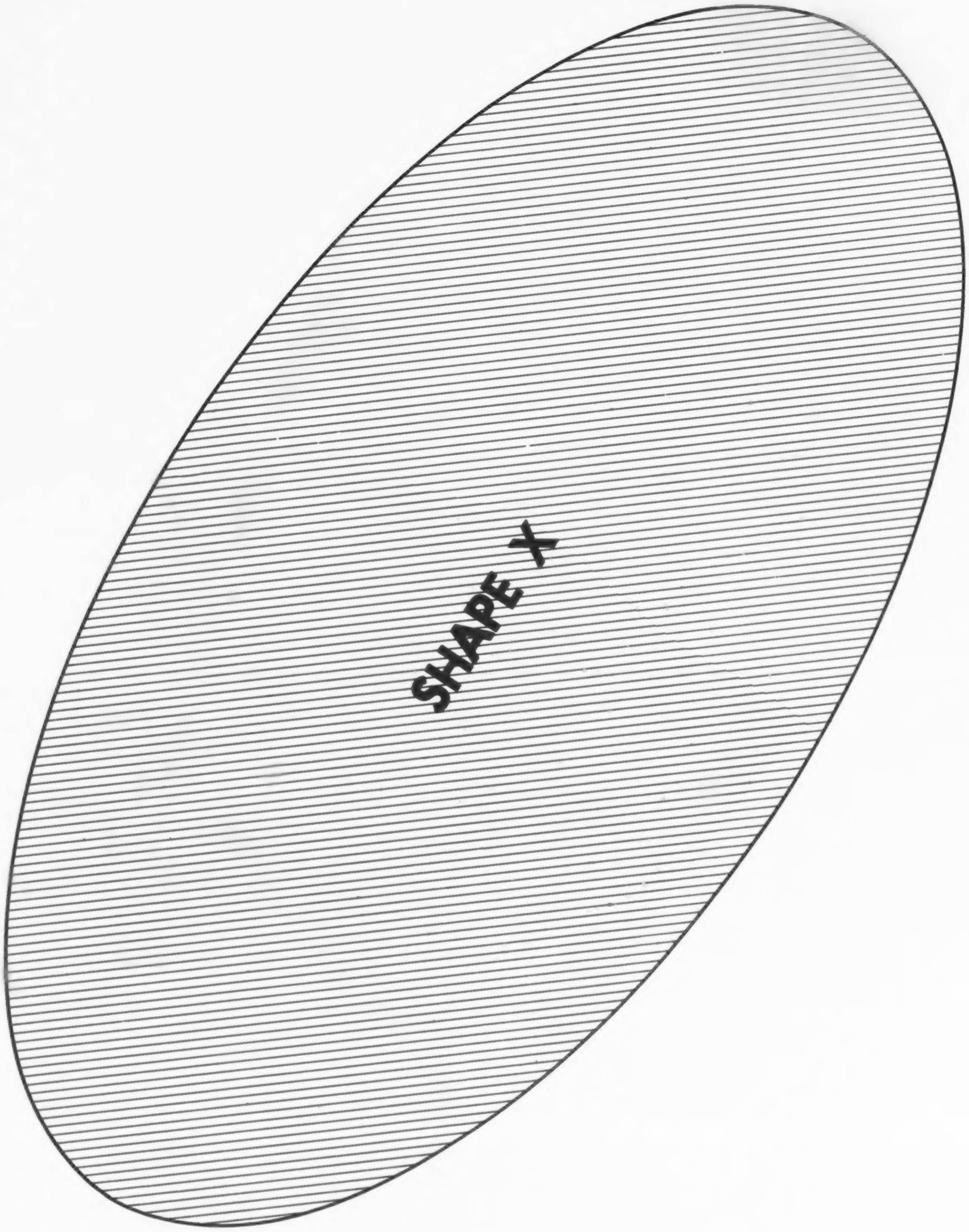
**10**



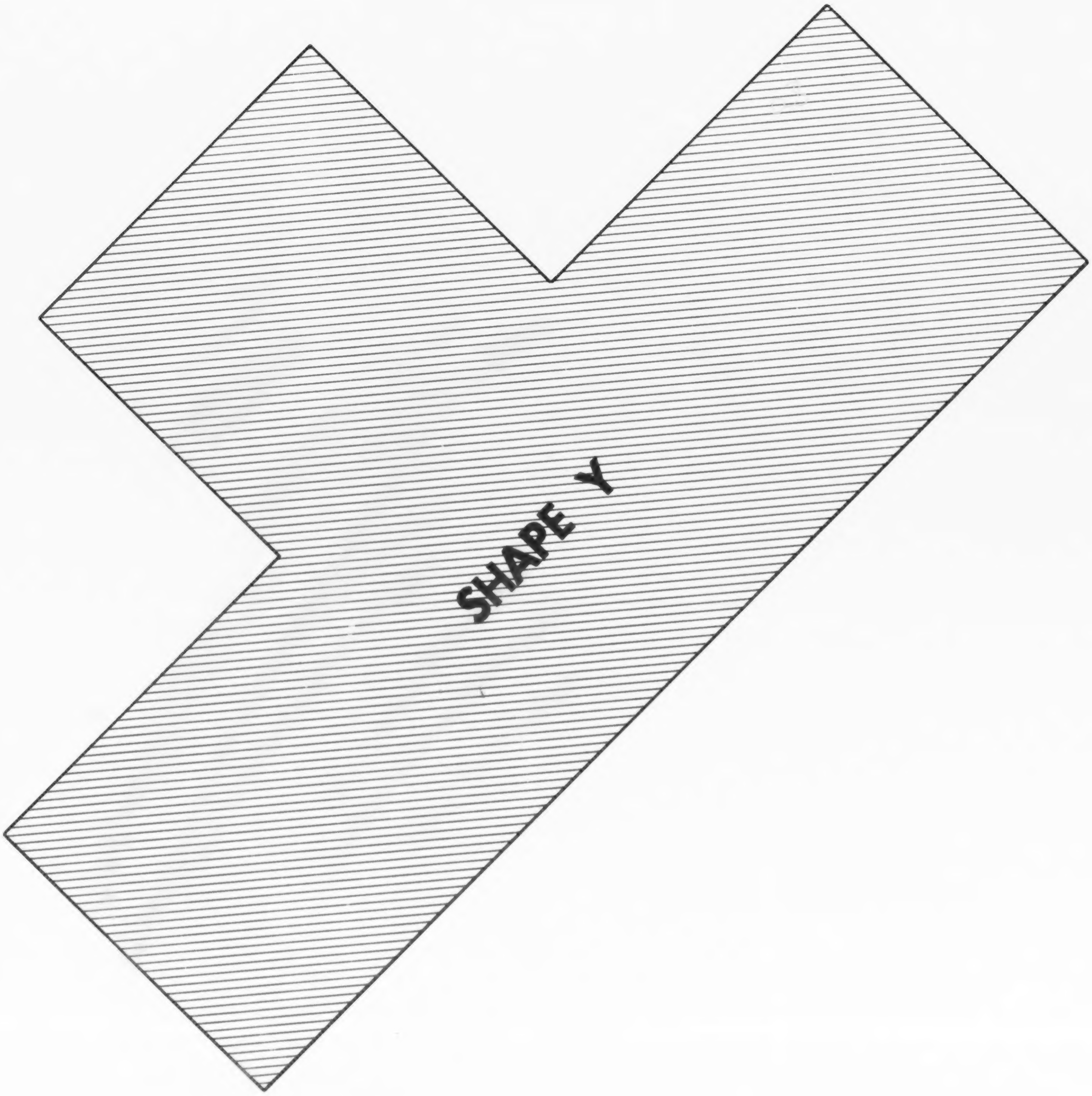
**11**



**Teacher Materials**



***Teacher Materials***



SHAPES LIKE SHADOWS OF PIECES OF CARDBOARD

UNIT 13, EXPERIENCE 2

Activity 1

Name ________________________________

Part 1

1. Take out the cardboard called "Shape X." Use the lamp to make a shadow of Shape X on a piece of paper. Color the shadow with the crayon. Be sure to color all the shadow.
2. Turn the cardboard a little to make a different shadow, on a new piece of paper. Color it.
3. Make some more shadows of Shape X on different pieces of paper and color them. Try to make them all different.

Part 2

1. Now take Shape Y and make some shadows of it. Make the shadows different and use different pieces of paper.
2. Go on to Part 3.

Part 3

1. Now take another piece of cardboard and cut a shape from it.
2. Make some different shadows from this cardboard shape on different pieces of paper. Color all these shadows.

Part 4

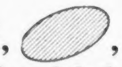
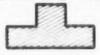
1. Look at all the pictures you have colored. Can you tell which piece of cardboard made each shadow?
2. Have your teacher check your work.

## SHAPES LIKE SHADOWS OF PIECES OF CARDBOARD

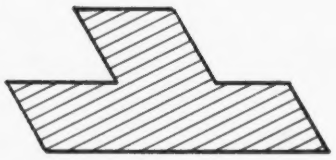
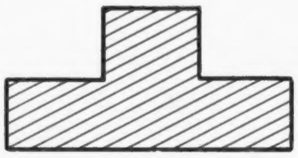
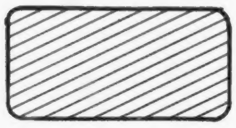
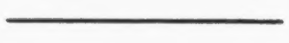
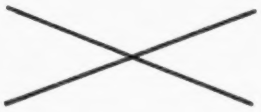
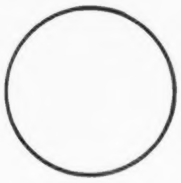
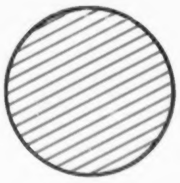
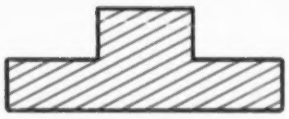
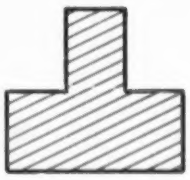
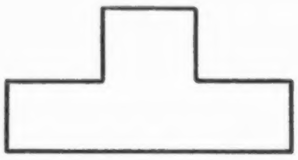
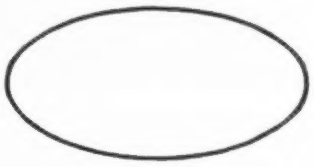
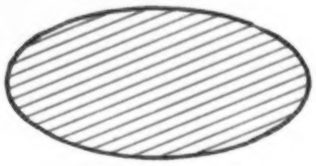
UNIT 13, EXPERIENCE 2

Activity 2

Name ______________________________

Mark each of the pictures with an X, if it is a shadow of Shape X, , or with a Y, if it is a shadow of Shape Y, . If it can be made with either, mark it with both an X and a Y. Do not mark any picture that is not a shadow of X or Y.

If you are not sure of the answers, use the lamp and cardboard patterns to find out. When you have finished, have your teacher check your work.

| | | |
|---|---|---|
| **1**  | **2**  | **3**  |
| **4**  | **5**  | **6**  |
| **7**  | **8**  | **9**  |
| **10**  | **11**  | **12**  |

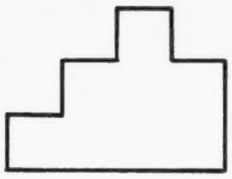
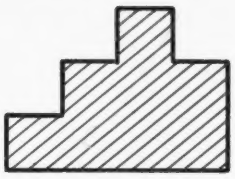
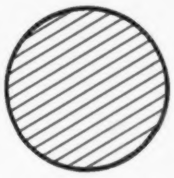
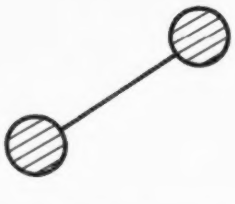
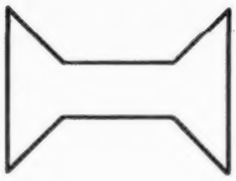
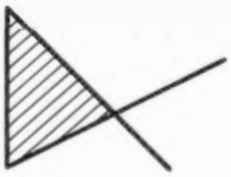
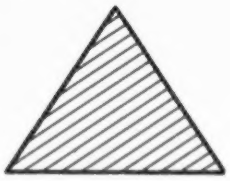
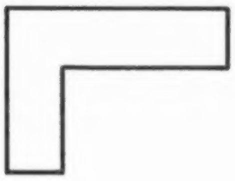
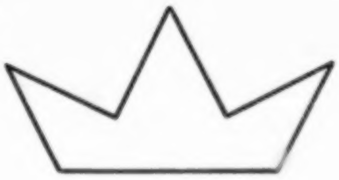
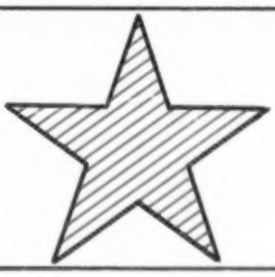
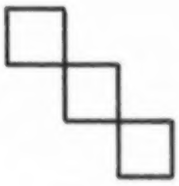
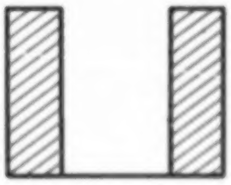
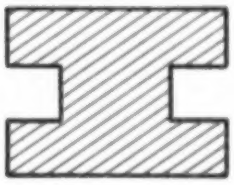
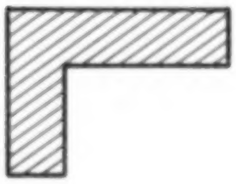
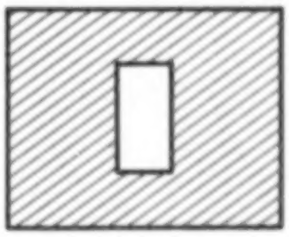
## SHAPES NOT LIKE A RUBBER BAND AND NOT LIKE A PIECE OF CARDBOARD

UNIT 13, EXPERIENCE 3

Name ________________________________

Three kinds of pictures are shown below. Write "G" in each box that contains a picture you can make, in a larger size, on the geoboard. Write "C" in each box that contains a picture that could be a shadow of a piece of cardboard. Do not write anything in the other boxes.

When you have finished, have your teacher check your work.

| | | |
|---|---|---|
| **1**  | **2**  | **3**  |
| **4**  | **5**  | **6**  |
| **7**  | **8**  | **9**  |
| **10**  | **11**  | **12**  |
| **13**  | **14**  | **15**  |

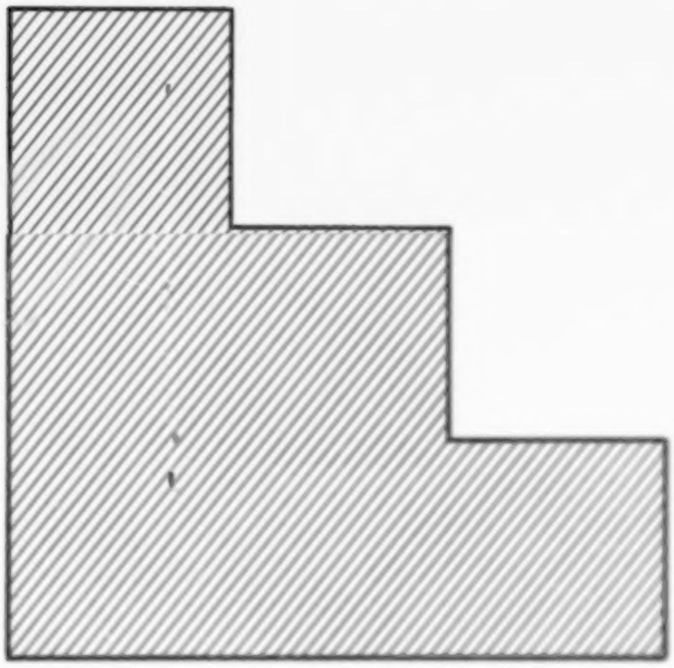
Name ______________________________

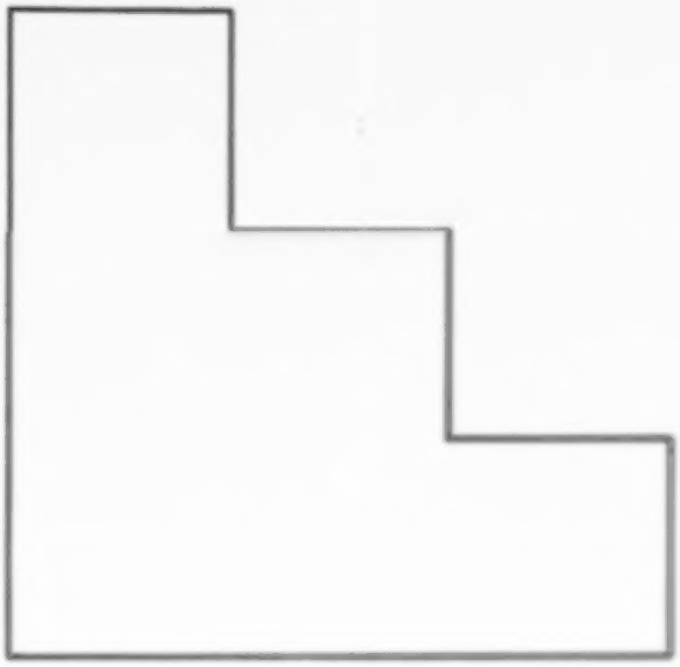
Use the pieces of cardboard and the little sticks to make the shapes shown on these pages. Use cardboard to make the ones like shadows and sticks to make the ones like a rubber band.

Make your figures on top of the pictures so they will be just alike. Then write the answers to the questions.

When you have finished, have your teacher check your answers.

1.





How many pieces of cardboard did you use? _____

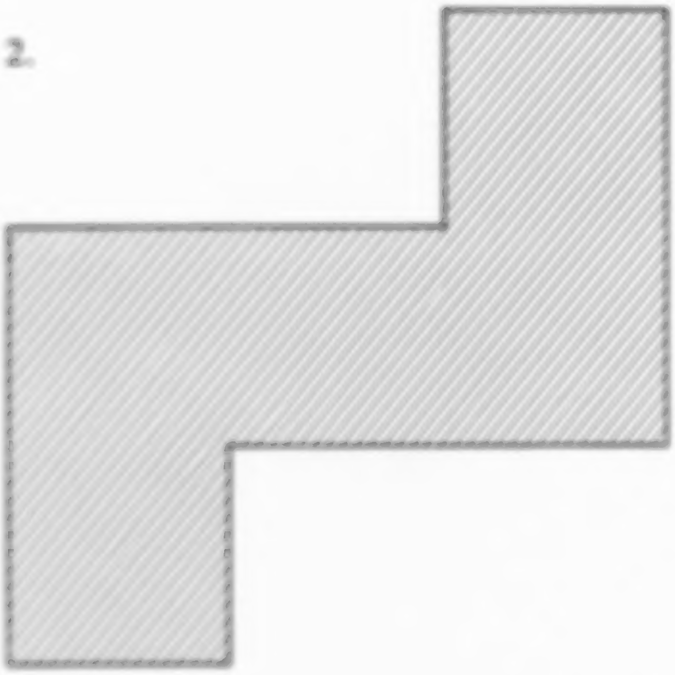
How many sticks did you use? _____

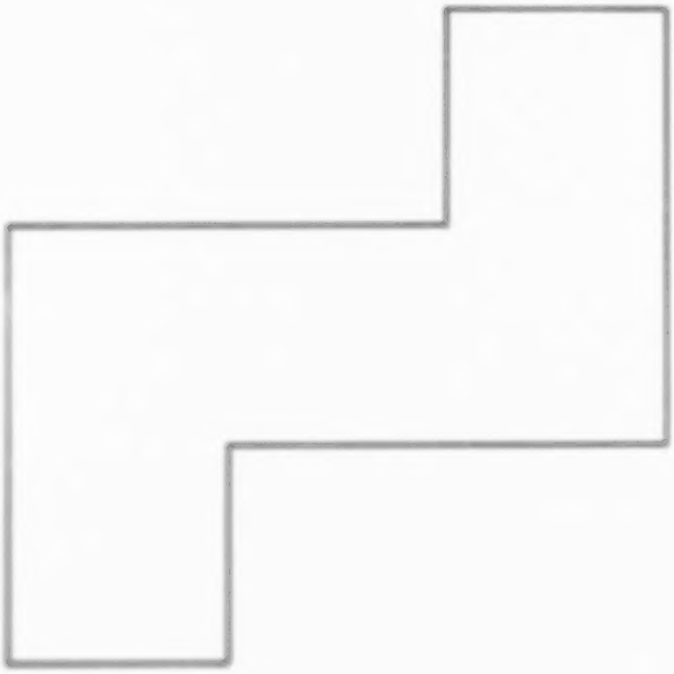
Did you use more cardboard or more sticks? ____________

[Continued]

*Continued* Name ______________________________

2.

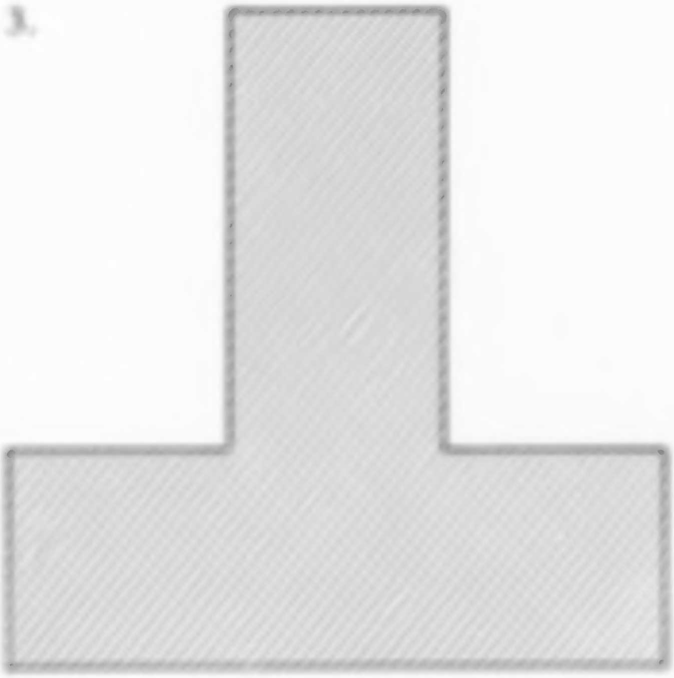


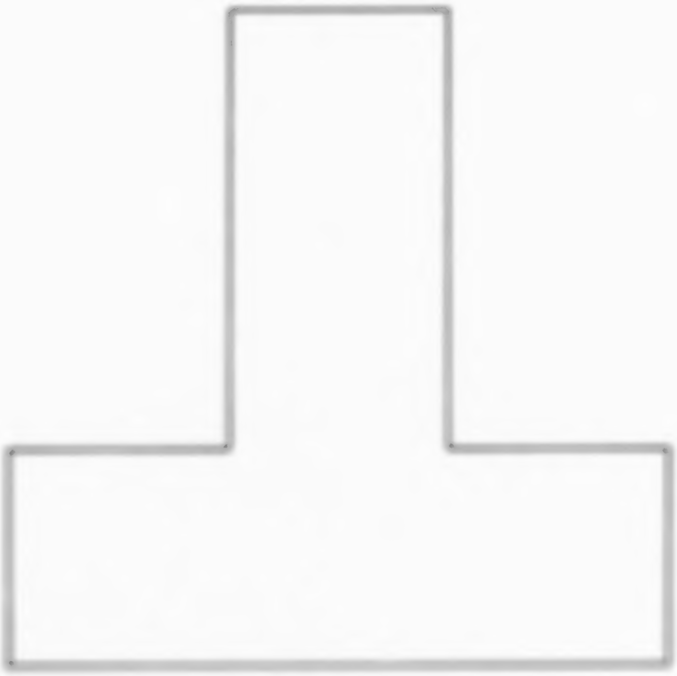


How many pieces of cardboard did you use? _____ How many sticks? _____

Did you use more cardboard or more sticks? _____________

3.





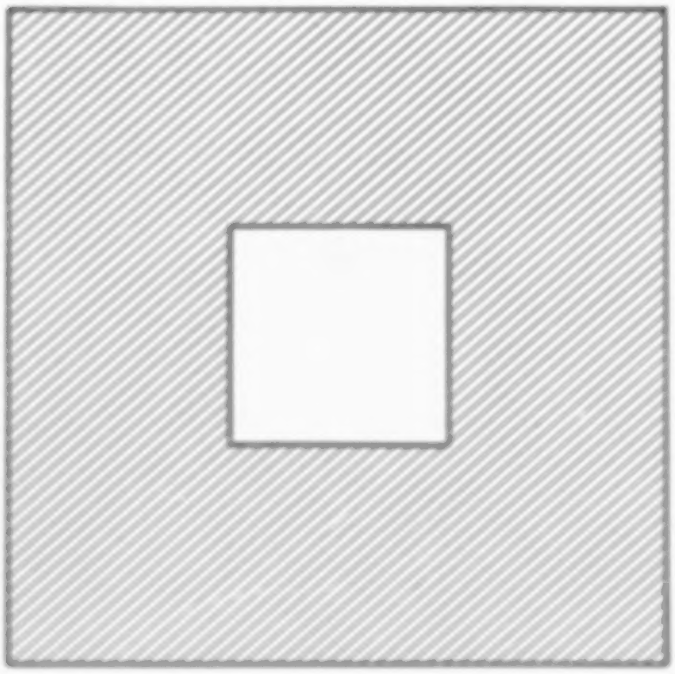
How many pieces of cardboard did you use? _____ How many sticks? _____

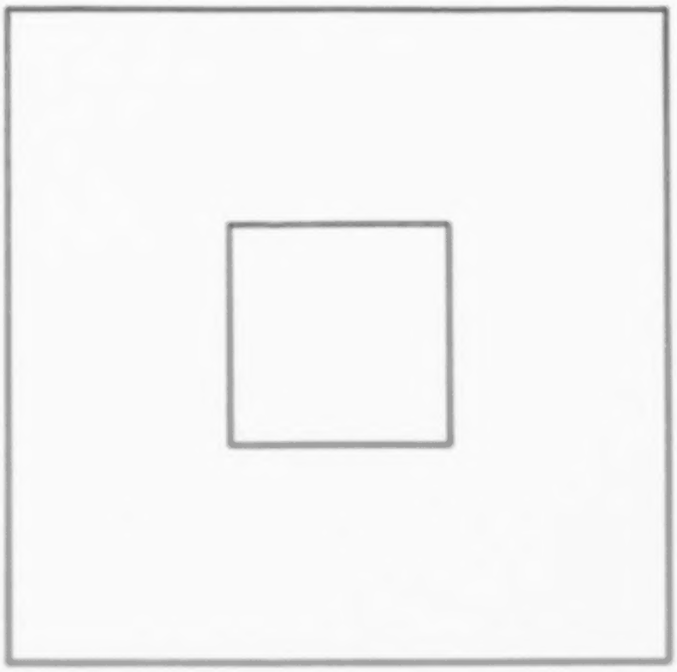
Did you use more cardboard or more sticks? _____________

[Continued]

*Continued* Name ______________________________

4.

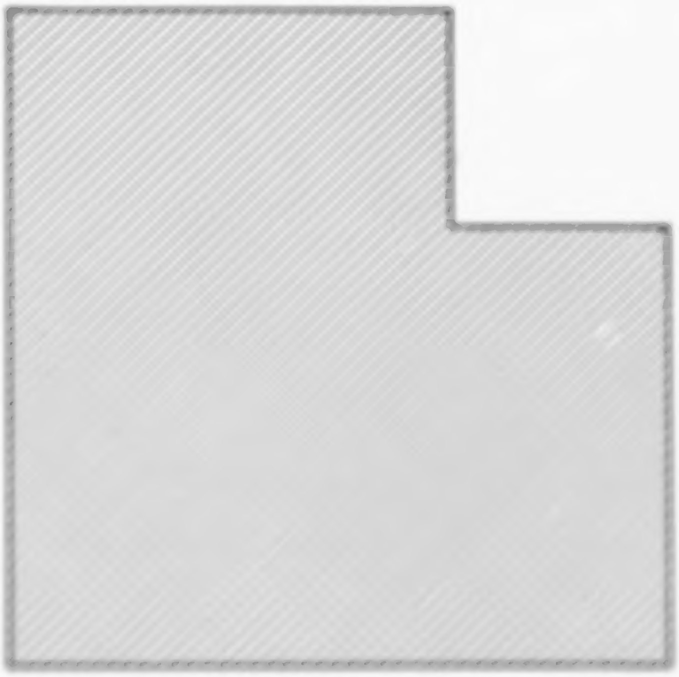


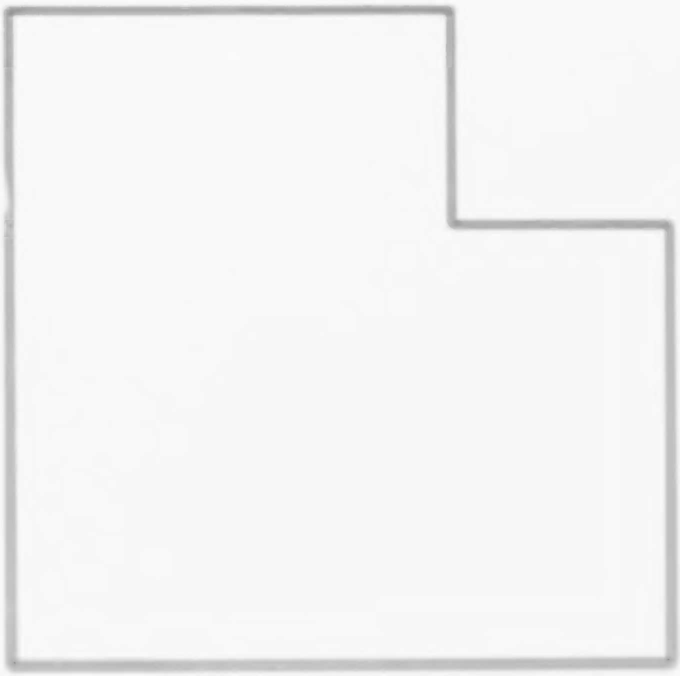


How many pieces of cardboard did you use? _____ How many sticks? _____

Did you use more cardboard or more sticks? _____________

5.





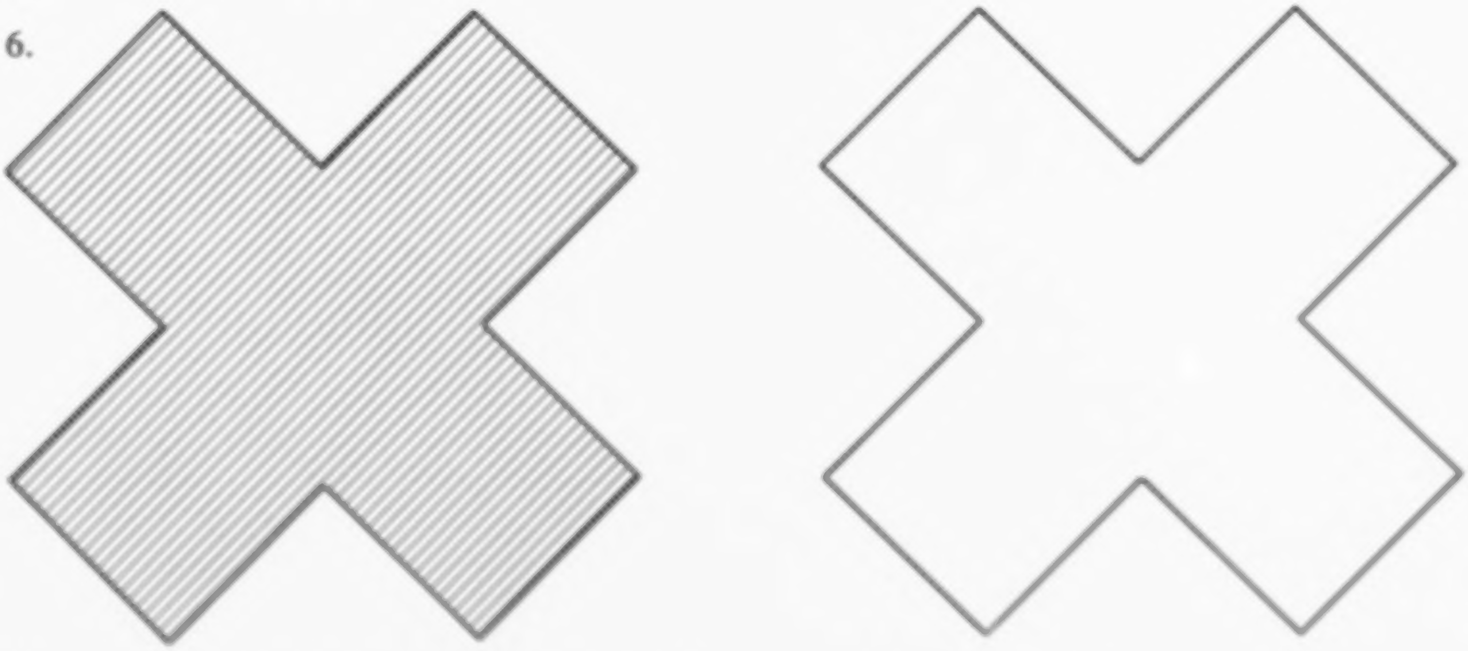
How many pieces of cardboard did you use? _____ How many sticks? _____

Did you use more cardboard or more sticks? _____________

[Continued]

*Continued* Name ______________________

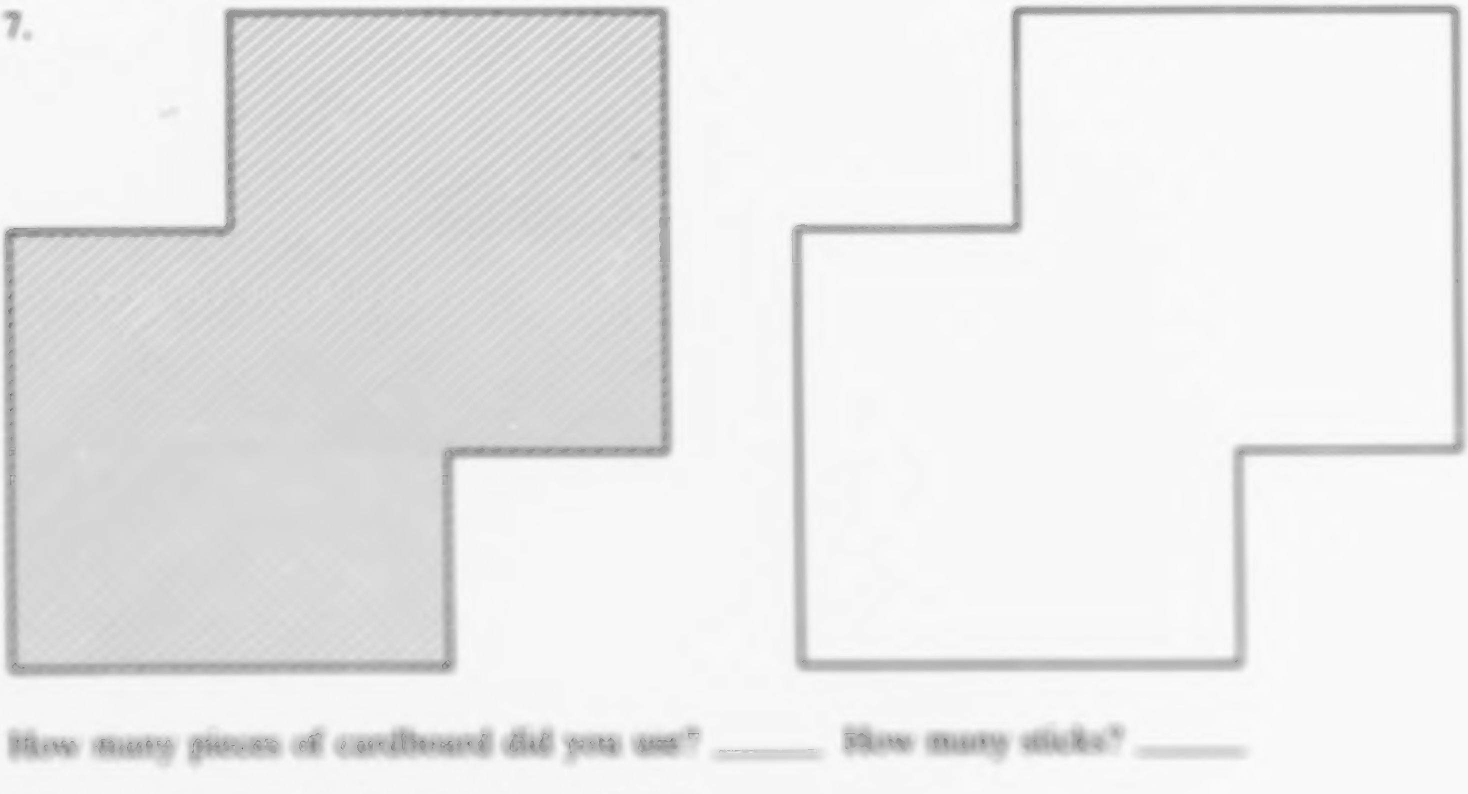
6.



How many pieces of cardboard did you use? _____ How many sticks? _____

Did you use more cardboard or more sticks? ____________

7.



How many pieces of cardboard did you use? _____ How many sticks? _____

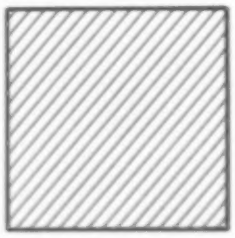
Did you use more cardboard or more sticks? ____________

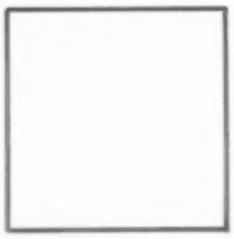
MORE REPLICAS OF GEOMETRIC FIGURES UNIT 13, EXPERIENCE 5

Name ______________________________

Use the pieces of cardboard and the sticks to make the pictures, then write the answers to the questions. When you are done, have your teacher check your answers.

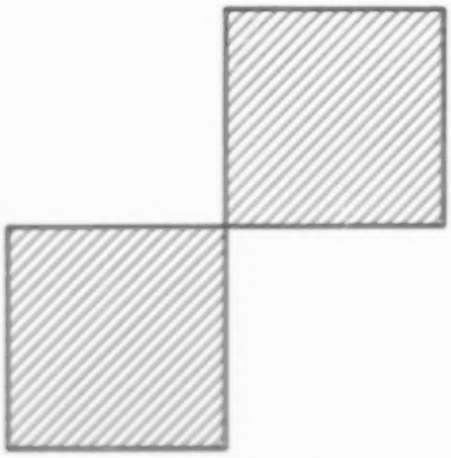
1.

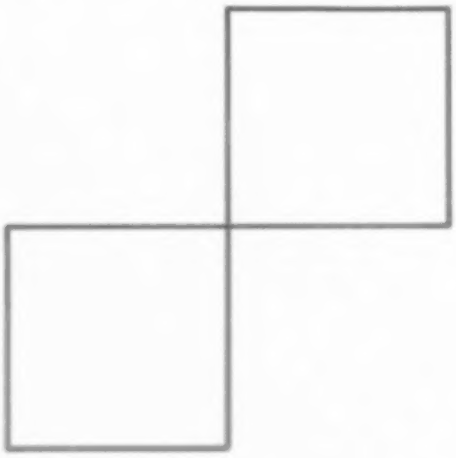




How many pieces of cardboard did you use? _____ How many sticks? _____

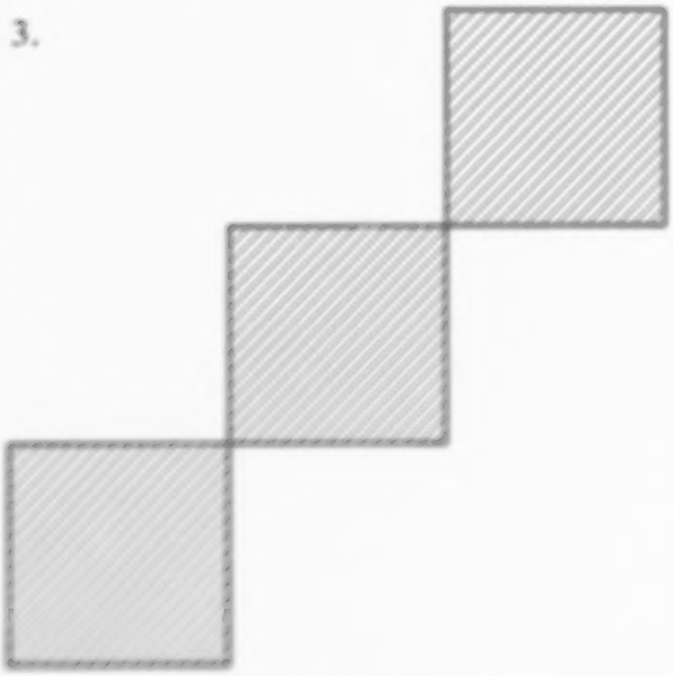
2.

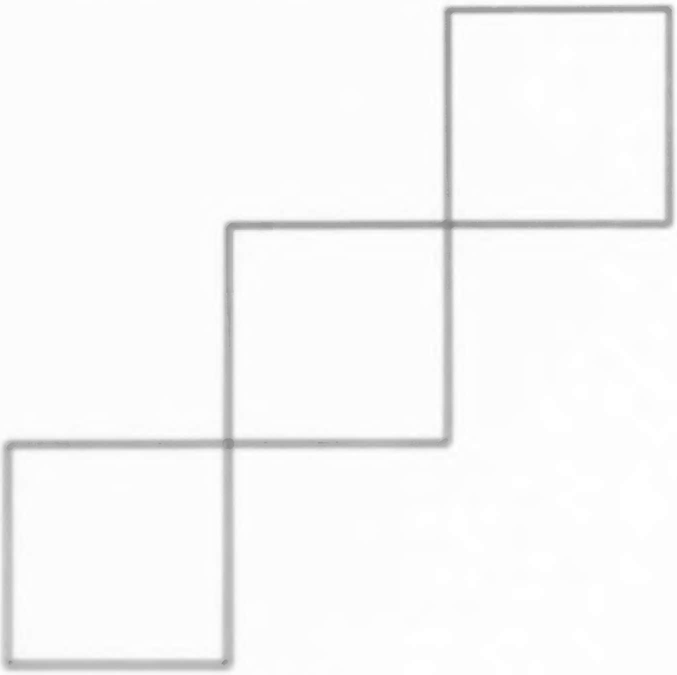




How many pieces of cardboard did you use? _____ How many sticks? _____

3.



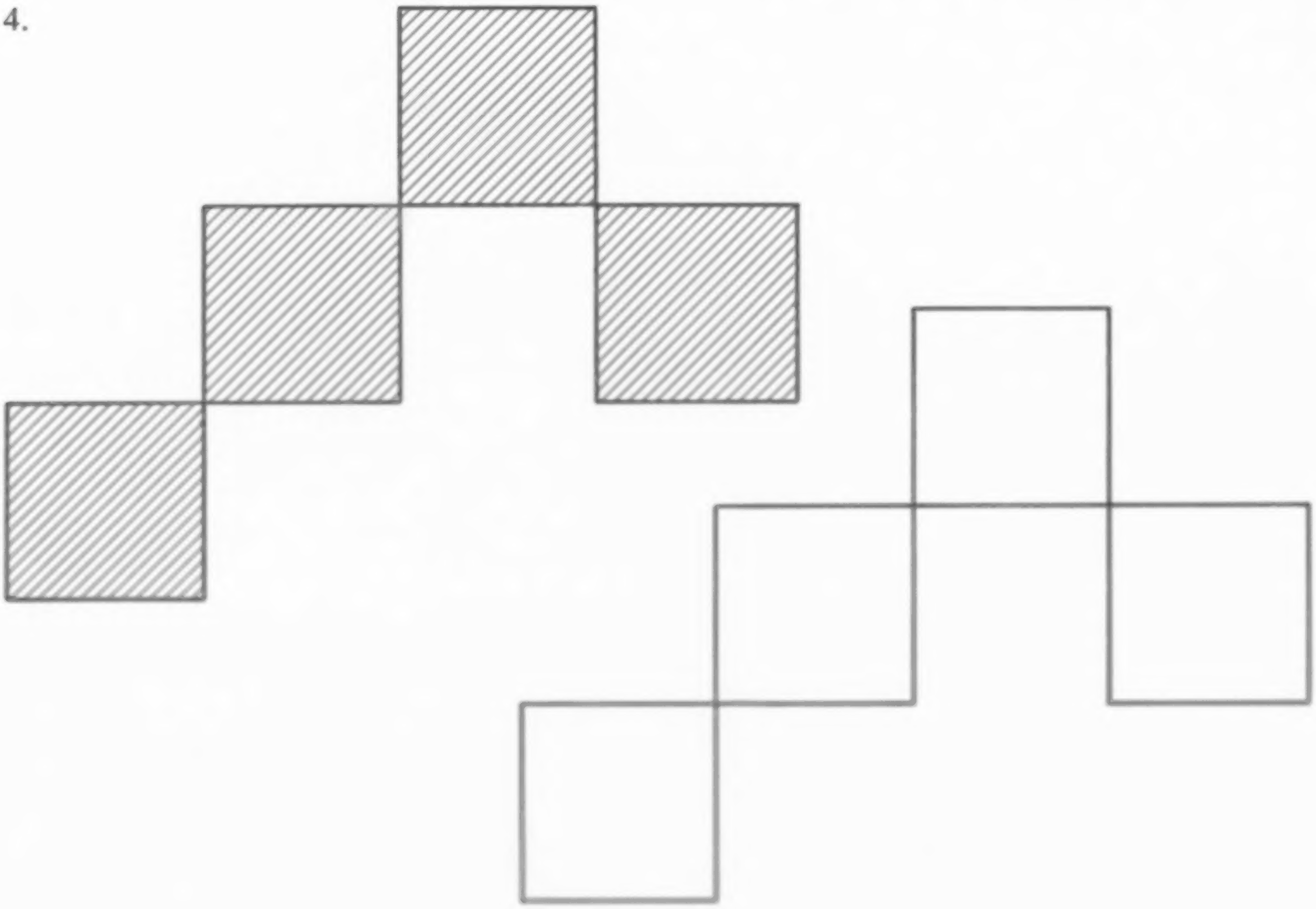


How many pieces of cardboard did you use? _____ How many sticks? _____

[Continued]

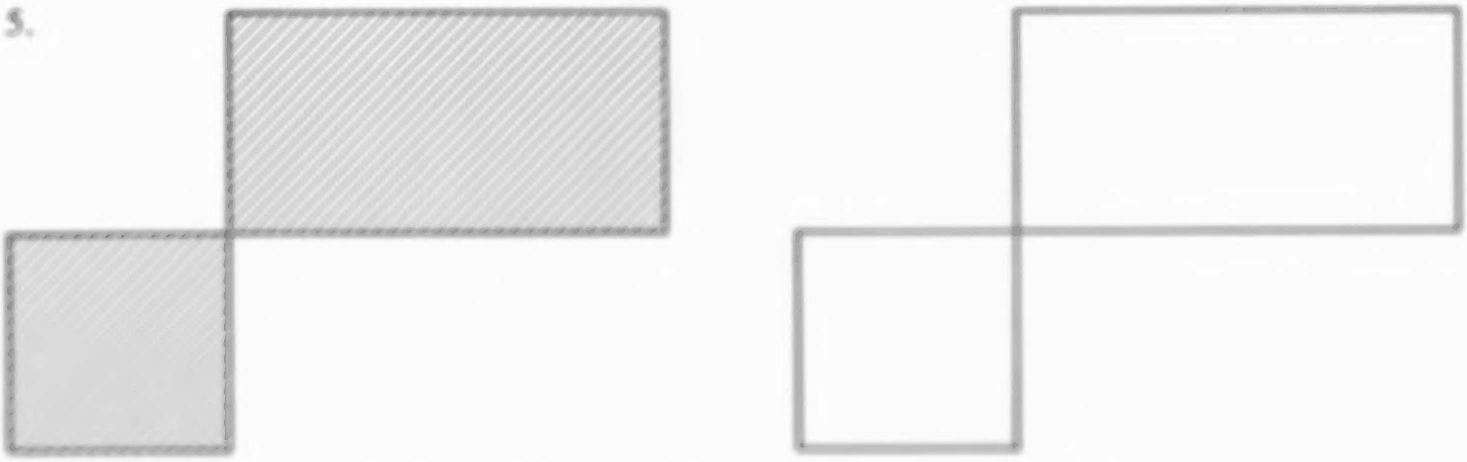
*Continued* Name ____________________

4.



How many pieces of cardboard did you use? _____ How many sticks? _____

5.



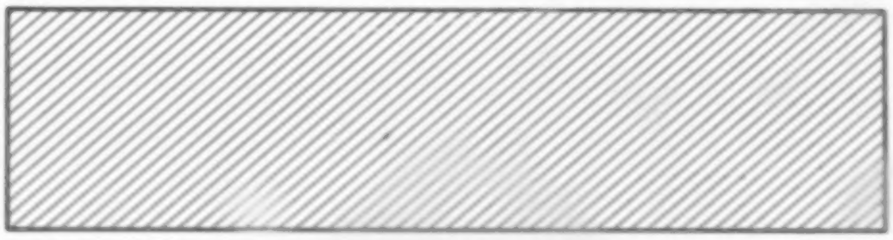
How many pieces of cardboard did you use? _____ How many sticks? _____

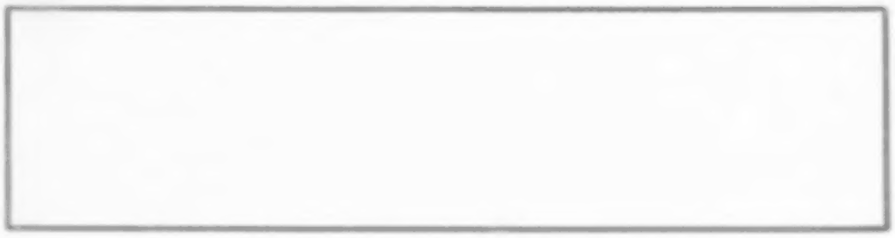
[*Continued*]

MORE REPLICAS OF GEOMETRIC FIGURES UNIT 13, EXPERIENCE 5

*Continued* Name ______________________________

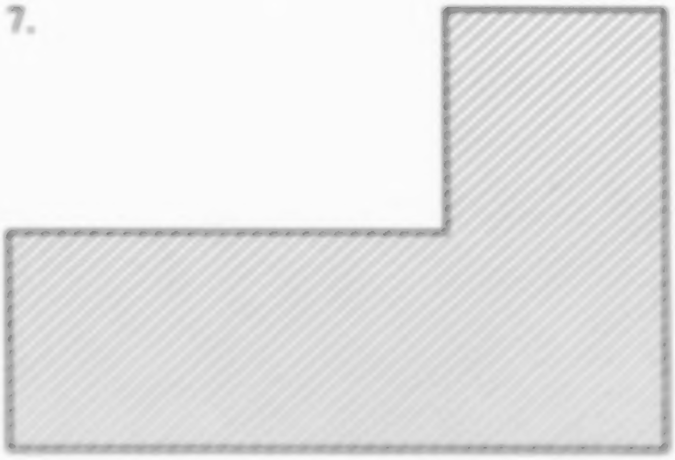
6.

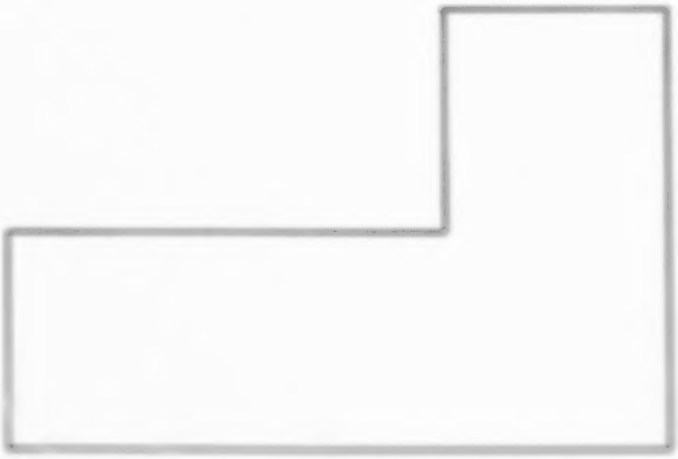




How many pieces of cardboard did you use? _____ How many sticks? _____

7.





How many pieces of cardboard did you use? _____ How many sticks? _____

(Continued)

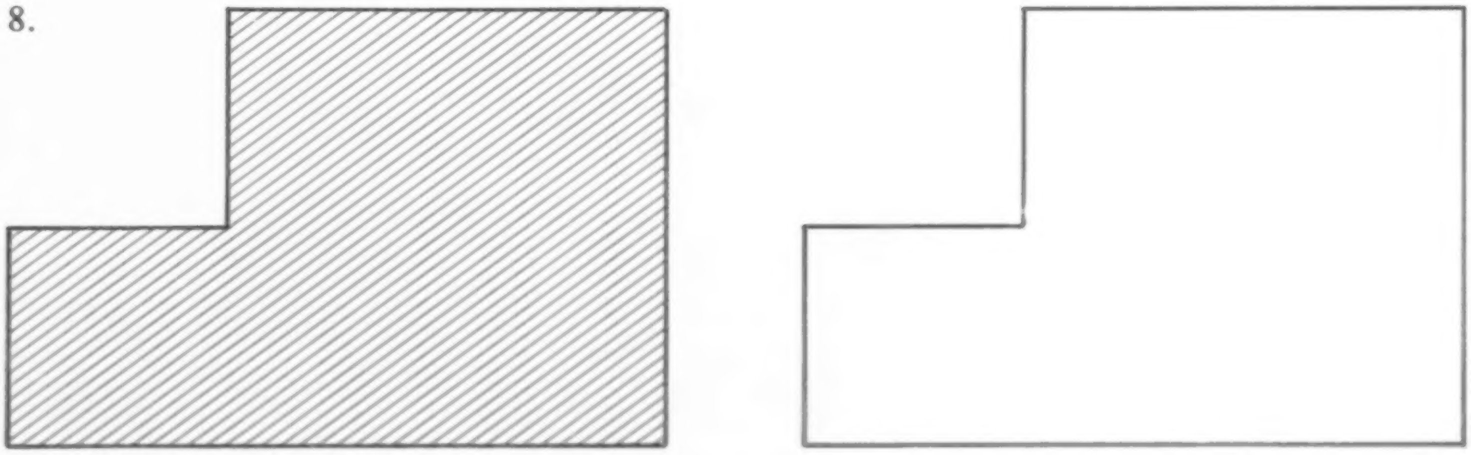
## MORE REPLICAS OF GEOMETRIC FIGURES

UNIT 13, EXPERIENCE 5

*Continued*

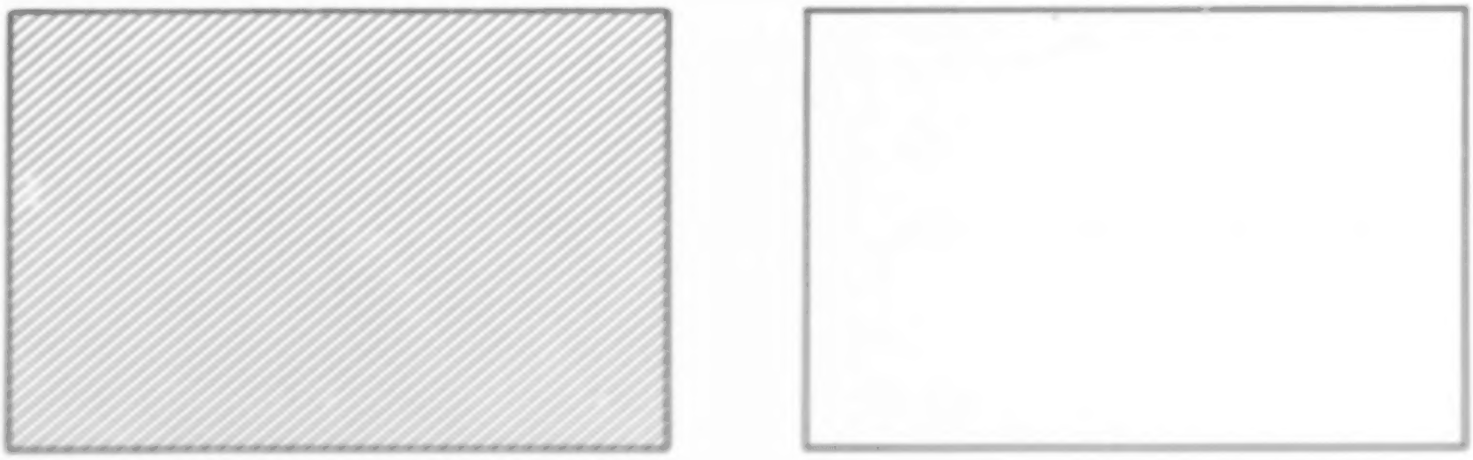
Name ______________________________

8.



How many pieces of cardboard did you use? _____ How many sticks? _____

9.



How many pieces of cardboard did you use? _____ How many sticks? _____